

# UNBALANCED TRANSPORTATION PROBLEM

Algorithm  $\Rightarrow$

✓  $\rightarrow$  Take input of cost, source, destination  
as vectors

✓  $\rightarrow$  Check if balanced/unbalanced (Demand  
== supply)

$\rightarrow$  If unbalanced  $\Rightarrow$   $\hookrightarrow$  balanced

Check  $\Rightarrow$

if demand < supply  
 $\rightarrow$  create dummy destination

else  
 $\rightarrow$  Can't solve as demand > supply

Ex (balanced transportation problem)

	8	
23		4 3 2 4
	16	2 3 2 3
12		1 1 2 2
	13	
9		
	7	

Ex (Unbalanced transportation problem)

	30	
50		4 3 2 4
	25	2 3 2 3
40		1 1 2 2
	35	
70		
	40	
<hr/>	<hr/>	
160	130	

① Create dummy destination  
with cost 0

	30				
50		4	3	2	4
	25	2	3	2	3
40		1	1	2	2
	35				0
70					...
	40				
	30				
	<hr/>				
	160				
	<hr/>				
	160				

→ North-West Corner Rule: <sup>0</sup>

	0	<del>5</del>	0	0	0
	<del>30</del>	<del>25</del>	<del>35</del>	<del>40</del>	<del>30</del>
0	<del>20</del>	<del>50</del>	30	20	0
0	<del>35</del>	<del>40</del>	0	5	35
0	<del>30</del>	<del>70</del>	0	0	40
			0	0	30

||

→ Minimum Cost Rule

[source] [destination]

	30
50	25
40	35
70	40
	30
<hr/>	<hr/>
160	160

4	3	<del>2</del>	4	<del>0</del>
2	3	2	<del>3</del>	<del>0</del>
<del>1</del>	<del>1</del>	2	<del>2</del>	0

	0	0	0	<del>0</del>	0
0	<del>15</del>	<del>50</del>	<del>30</del>	<del>25</del>	<del>15</del>
			35	10	30

0	<del>15</del>	<del>40</del>		25	15
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0	<del>15</del>	<del>40</del>	<del>70</del>	30	25	15
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if dest == 0  
break

(4) 2 1

1) min = 4 loc = 0

2) min = 2 loc = 1

3) min = 1 loc = 2

				4	3	2	<del>4</del>	0
				2	3	<del>2</del>	<del>3</del>	0
				<del>1</del>	1	<del>2</del>	<del>2</del>	0
		0	0	20		200		
	<del>30</del>	<del>25</del>	<del>35</del>	<del>40</del>		30		
30	<del>50</del>	0	0	0	20	30		
0	<del>20</del>	<del>40</del>	0	0	20	20		
<del>15</del>	<del>40</del>	<del>70</del>	30	25	15			
0								

① 4 2 1      ② 3 3 1

loc = 2

4	3	2	4	0
2	3	2	3	0
1	1	2	2	0

0				
<del>30</del>	25	35	40	30

50

40

40 ~~70~~      30

	8	4 3 2 4
23		2 3 2 3
	16	1 1 2 2

12

13

9

$u[\text{supply}]$

7

$v[\text{demand}]$

	8	16	13	7
23	8 11	8 12	0 13	7 14
12	0 21	0 22	12 23	0 24
9	0	8 32	1	0

$$C_{11} = u_1 + v_1$$

$$4 = u_1 + v_1$$

$$C_{12} = u_1 + v_2$$

$$3 = u_1 + v_2$$

$$C_{14} = u_1 + v_4$$

$$4 = u_1 + v_4$$

$$C_{12} = u_1 + v_2$$

$$2 = u_2 + v_3$$

$$C_{14} = u_1 + v_4$$

$$u_1 = 0$$

$$v_1 = 4$$

$$C_{23} = u_2 + v_3$$

$$C_{32} = u_3 + v_2$$

$$C_{33} = u_3 + v_3$$

	8	16	13	7
23	8	15	0	0
12	0	22 1	23 11	0
9	0 <sup>*</sup> 31	0 32	2 33	7

$$u_1 = 0$$

$$u_2 = 0$$

$$u_3 = 0$$

$$v_1 = 4$$

$$v_2 = 3$$

$$= 2$$

$$u = 2$$

$$4 \ 3 \ 2 \ 4$$

$$2 \ 3 \ 2 \ 3$$

$$1 \ 1 \ 2 \ 2$$

we cant get  
pts like 3,1  
as it doesnt  
form a  
cycle  
u,v helps in  
setting  
cycle

$$C_{13} = u_1 - v_3$$

$$2 - 0 - 2 = 0$$

$$C_{14} = u_1 - v_4$$

$$4 - 0 - 2 = 2$$

$$C_{21} = u_2 - v_1$$

$$2 - 0 - 4 = -2$$

$$C_{32} = u_3 - v_2$$

$$1 - 0 - 3 = -2$$

$$C_{24} = u_2 - v_4$$

$$3 - 0 - 2 = 1$$

$$C_{31} = u_3 - v_1$$

$$1 - 0 - 4 = -3$$

	8	16	13	7
23	8	15	0	0
12	0	1	11	0
9	0	0	2	7

loc = 2, 1

lc = 0  
mi = 2 (loc i)  
mj = 1 (loc j)

Sign loop

a	a	a	a	1	1	0	0
a	a	a	a	0	1	1	0
a	a	a	a	0	0	1	1

k = 0  
i = 0

j = 0 lc = 1  
j = 3 lc = 2

0	1	0	0
0	1	1	0
0	0	1	0

if only 1 basic cell in neighbourhood column then its zero  
next rows same algorithm

lc ≠ 1  
lc = 0

0	0	0	0
0	1	1	0
0	0	1	0

i = 1

j = 0 lc = 0  
j = 3 lc = 2  
lc ≠ 1  
lc = 0

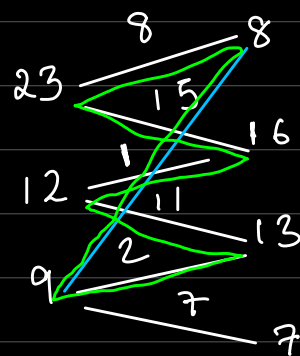
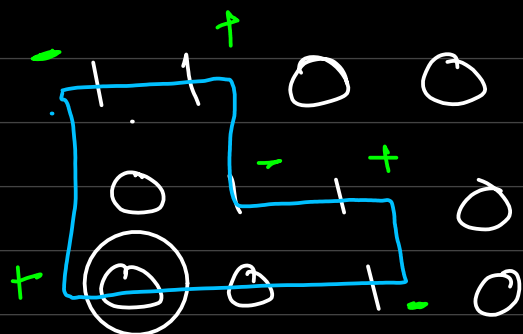
0	1	0	0
0	1	1	0
0	0	1	1

0	1	0	0
0	1	1	0
0	0	1	0



$k=1$   
 $k \leq 2$

$1, 2$   
 $1 \quad 1 \quad 0 \quad 0$   
 $0 \quad 1 \quad 1 \quad 0$   
 $0 \quad 0 \quad 1 \quad 1$



	8	16	13	7
23	-8	15 <sup>+</sup>	0	0
12	0	1 <sup>-</sup>	11 <sup>+</sup>	0
9	+0 <sup>8</sup>	0	2 <sup>-</sup>	7

8, 1, 2

	8	16	13	7
23	7	16	0	0
12	0	0	12	0
9	1	0	1	7

$4 \quad 3 \quad 2 \quad 4$   
 $2 \quad 3 \quad 2 \quad 3$   
 $1 \quad 1 \quad 2 \quad 2$

$$u_1 = 0$$

$$2 - (u_1 + v_3) = 2 - (0 + 5) = -3$$

$$u_2 = -3$$

$$4 - (u_1 + v_4) = 4 - (0 + 5) = -1$$

$$u_3 = -3$$

$$2 - (u_2 + v_1) = 2 - (-3 + 4) = 0$$

$$v_1 = 4$$

$$3 - (u_2 + v_2) = 3 - (-3 + 3) = 3$$

$$v_2 = 3$$

$$v_3 = 5$$

$$v_4 = 5$$

$$\begin{array}{cccc|cccc} N & N & -3 & -1 & 1 & 1 & 0 & 0 \\ 1 & 3 & 2 & 1 & 0 & 0 & 1 & 0 \\ N & 1 & 2 & N & 1 & 0 & 1 & 1 \end{array}$$

		0	1	2	3
		<del>250</del>	350	400	200
0	300	0	300	0	0
1	400	250	0	0	150
2	500	0	50	400	50

3 1 7 4  
2 6 5 9  
8 3 3 2

3, 2, 8

3, -1, 8

$$3 - (-7) = 10$$

u2  
v

$$7 - (0 + 1) = 6$$

$$4 - (0 + 0) = 4$$

$$6 - (9 + 1) = -4$$

$$5 - (9 + 1) = -5$$

0 1 0 0  
1 0 0<sup>\*</sup> 1  
0 1 1 1

0 0 0 0  
0 0 0<sup>\*</sup> 1  
1 1  
0 0

0	1	0	1
0	1	0	1 (1,3)
0	0	0	0

a	a	a	a	0	1	1	1	0	1	0	1
a	a	a	a	1	1	0	0	0	1	0	1
a	a	a	a	1	0	0	0	0	0	0	0

sign[1][3] = +

i = 1

j = 0

a a a a

a a a +

a a a a

a a a a

a - a +

a a a a

a + a a

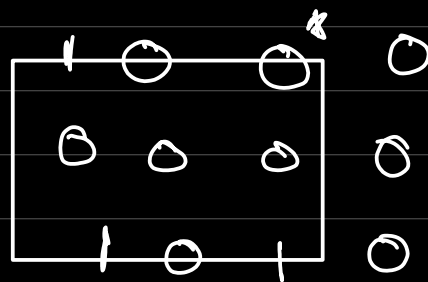
a - a +

a a a a

i = 0

a

1	1	0*	0
0	0	1	0
1	0	1	1



$$c[0][0] = 1$$

$$j = 0$$

-1	0	2	0
0	0	0	0
1	0	1	0

$$c[2][0] = 1 \quad i = 2$$

-1	0	2	0
0	0	0	0
1	0	1	0

cycle[2]

3-1  
1  
2

1 2 3

$$\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{array}$$

$$\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{array}$$

$$k=1$$

$$i=2$$

$$j=2$$

$$i=1$$

$$j=2$$

$$\begin{array}{cccc} 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccc} 0 & 0 & 1 & 0 \end{array}$$

$$\begin{array}{cccc} 1 & 0 & -1 & 0 \end{array}$$

$$i=1$$

$$j=1$$

$$\begin{array}{cccc} 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccc} 0 & -1 & 1 & 0 \end{array}$$

$$\begin{array}{cccc} 1 & 0 & -1 & 0 \end{array}$$

$$i=0$$

$$j=1$$

$$\begin{array}{cccc} 0 & 1 & 0 & 0 \end{array}$$

$$\begin{array}{cccc} 0 & -1 & 1 & 0 \end{array}$$

$$\begin{array}{cccc} 1 & 0 & -1 & 0 \end{array}$$

0 6 0 0

0 1 1 0

0 1 1 0

$i=2$

$i=2$

$j=2$

0 0 0 0

0 1 1 0

0 1 -1 0

$j=2$

$i=1$

0 0 0 0

0 1 1 0

0 1 -1 0

$j=1$

$j=1$

0 0 0 0

0 -1 1 0

0 1 -1 0

0 7 13 3

8 0 0 4

0 9 0 0