Useful functions

These functions defines the system that will be studied.

Definition of the Atom states

There are 3 states : 1 - ground; 2 - excited; 3 - Rydberg.

In[1]:=
```
AtomState = {"1", "2", "3"};
SystemState[n_] := Flatten[Table[AtomState, {p, 1, n}]]
SystemState[2]
```

Out[3]=
```
{1, 2, 3, 1, 2, 3}
```

In[4]:=
```
AvailableStates[n_] := Sort[Permutations[SystemState[n], {n, n}]]
AvailableStates[2]
```

Out[5]=
```
{{1, 1}, {1, 2}, {1, 3}, {2, 1}, {2, 2}, {2, 3}, {3, 1}, {3, 2}, {3, 3}}
```

In[6]:=
```
Anagram[n_] := Module[{liste = AvailableStates[n]},
  (* take the list of available states for n atoms *)
  Module[{counts = Table[Table[ (* make a table containing ... *)
       Count[i, State], {State, AtomState}]
     (* ... the number of times a "State" appears in ... *)
     , {i, liste}]}, (* ... each available configuration *)
   Table[ (* make the output table containing ... *)
    First[Position[counts, j]][[1]] (* ... the position
     of the first occurance of an equivalent configuration *)
    , {j, counts}]
  ]
 ]
```

In[7]:=
```
Anagram[2]
```

Out[7]=
```
{1, 2, 3, 2, 5, 6, 3, 6, 9}
```

Density matrix

Then, one defines the density matrix of N atoms system.

In[8]:=
```
(* The density matrix is hermitian, then it has to obey to the following rules,
the diagonal elements are real because of initial conditions *)
SymRules[Num_] := Drop[DeleteDuplicates[Flatten[
    Table[If[m ≠ n, ρ[n, m][t] -> Conjugate[ρ[m, n][t]], 0], {m, 1, 3^Num}, {n, m}]]], 1]
SymRules[
 1]
```

Out[9]=
```
{ρ[1, 2][t] → Conjugate[ρ[2, 1][t]],
 ρ[1, 3][t] → Conjugate[ρ[3, 1][t]], ρ[2, 3][t] → Conjugate[ρ[3, 2][t]]}
```

In[10]:=
```
SymRules2[Num_] := Module[{anagram = Anagram[Num]},
  Flatten[
   Table[ρ[n, m][t] -> ρ[anagram[[n]], anagram[[m]]][t], {m, 1, 3^Num}, {n, 1, 3^Num}]]
  ]
SymRules2[1]
```

Out[11]=
```
{ρ[1, 1][t] → ρ[1, 1][t], ρ[2, 1][t] → ρ[2, 1][t], ρ[3, 1][t] → ρ[3, 1][t],
 ρ[1, 2][t] → ρ[1, 2][t], ρ[2, 2][t] → ρ[2, 2][t], ρ[3, 2][t] → ρ[3, 2][t],
 ρ[1, 3][t] → ρ[1, 3][t], ρ[2, 3][t] → ρ[2, 3][t], ρ[3, 3][t] → ρ[3, 3][t]}
```

In[12]:=
```
Dens3Level[Num_] := Array[ρ, {3^Num, 3^Num}];
Dens[Num_] := Block[{DensityMat},
  DensityMat = Array[ρ, {3^Num, 3^Num}];
  Table[DensityMat[[x, y]][t], {x, 1, 3^Num}, {y, 1, 3^Num}]
  ]
Dens[1] // MatrixForm
```

Out[14]//MatrixForm=

$$\begin{pmatrix} \rho[1,1][t] & \rho[1,2][t] & \rho[1,3][t] \\ \rho[2,1][t] & \rho[2,2][t] & \rho[2,3][t] \\ \rho[3,1][t] & \rho[3,2][t] & \rho[3,3][t] \end{pmatrix}$$

Rydberg blockade

Here one evaluates the number of atomes in Rydberg states

In[15]:=
```
FindStates[Num_, State_, Val_] := Block[{p, q},
  p = AvailableStates[Num];
  q = Length[p];
  Flatten[Position[Table[Count[p[[m]], State] == Val, {m, 1, q}], True]]]
```

In[16]:=
```
Pr[Num_, Nryd_] := Sum[ρ[i, i][t], {i, FindStates[Num, "3", Nryd]}];
Pr[2, 0]
```

Out[17]=
```
ρ[1, 1][t] + ρ[2, 2][t] + ρ[4, 4][t] + ρ[5, 5][t]
```

In[18]:=
```
Px[Num_, Nryd_, state_] := Sum[ρ[i, i][t], {i, FindStates[Num, state, Nryd]}];

FindStates[2, "2", 2][[1]]
```

```
5
```

Liouville - von Neumann equation (Lindbald form)

In this section, we will write the master equation of Molmer's paper (equation (1) of 1302.0682v1 on ariv.org).

Matrix definition

The main part of this section is here, we compute the σ matrices defined in Molmer's paper. Then, we easly define the operators using these matices.

In[19]:=
```
(* The famous σμν^k matrix of Molmer's paper: transition operator for atom j *)
σμν[Num_, k_, mu_, nu_] := Table[
   If[
    i[[k]] == mu (* initial state *)
     &&
      j[[k]] == nu (* final state *)
     &&
      Drop[i, {k}] == Drop[j, {k}] (* the other atoms does not change *)
    , 1, 0], {i, AvailableStates[Num]}, {j, AvailableStates[Num]}];
```

Hamiltonian

In[20]:=
```
H[Num_] := Sum[Ω1 / 2 * (σμν[Num, i, "1", "2"] + σμν[Num, i, "2", "1"]) +
     Ω2 / 2 * (σμν[Num, i, "2", "3"] + σμν[Num, i, "3", "2"]), {i, Num}];
```

Detunings

In[21]:=
```
ω0[Num_] := Table[Sum[If[AvailableStates[Num][[m, r]] == "2", δ1, 0] +
     If[AvailableStates[Num][[m, r]] == "3", δ1 + δ2, 0], {r, 1, Num}], {m, 1, 3^Num}]
DiagonalMatrix[ω0[2]] // MatrixForm
```

Out[22]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta1+\delta2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\,\delta1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\,\delta1+\delta2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta1+\delta2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\,\delta1+\delta2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\,\delta1+2\,\delta2 \end{pmatrix}$$

Dipole - dipole interaction

In[27]:=
```
Vaa[Num_] := Δ * Sum[
     Sum[σμν[Num, k1, "3", "3"].σμν[Num, k2, "3", "3"], {k1, 1, k2 - 1}], {k2, 1, Num}];
Vaa[2] // MatrixForm
```

Out[28]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Delta \end{pmatrix}$$

Liouvillian

In[37]:=
```
(* We can notice that it requires a density matrix *)
Leg[Dens_, Num_] :=
 1 / 2 * Γge * (Sum[2 * σμν[Num, i, "1", "2"].Dens.σμν[Num, i, "2", "1"] -
     σμν[Num, i, "2", "2"].Dens - Dens.σμν[Num, i, "2", "2"], {i, 1, Num}] )
Lre[Dens_, Num_] := 1 / 2 * Γre *
  (Sum[2 * σμν[Num, i, "2", "3"].Dens.σμν[Num, i, "3", "2"] -
     σμν[Num, i, "3", "3"].Dens - Dens.σμν[Num, i, "3", "3"], {i, 1, Num}] )

Leg[Dens[1], 1] // MatrixForm
Tr[Lre[Dens[2], 2]] // Simplify (* Must be zero, because by definition,
 the trace of the density should not change and be unitary *)
```

Out[39]//MatrixForm=

$$\begin{pmatrix} \Gamma ge\,\rho[2,2][t] & -\frac{1}{2}\Gamma ge\,\rho[1,2][t] & 0 \\ -\frac{1}{2}\Gamma ge\,\rho[2,1][t] & -\Gamma ge\,\rho[2,2][t] & -\frac{1}{2}\Gamma ge\,\rho[2,3][t] \\ 0 & -\frac{1}{2}\Gamma ge\,\rho[3,2][t] & 0 \end{pmatrix}$$

Out[40]= 0

Liouville - von Neumann equation

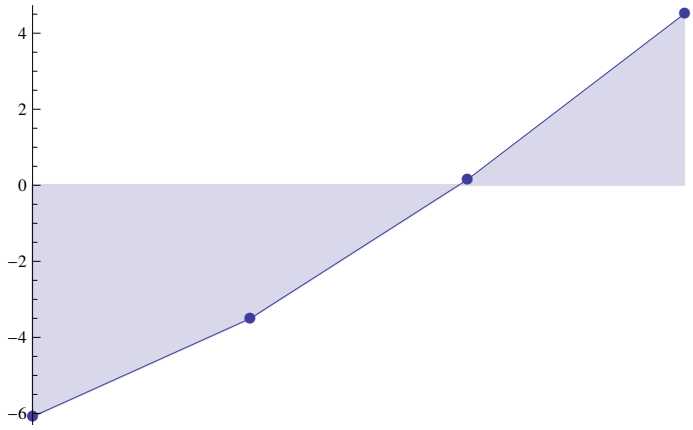
We are now ready to write the master equation.

In[41]:=
```
(* This function outputs the Optical Bloch equations,
one can specify Decay=(True or False) *)
Equations[Num_, Decay_] := Block[{Trans, DensityMat, Sym, Liouvillian, ρdot},
  Sym = SymRules[Num];
  Trans = H[Num] + DiagonalMatrix[ω0[Num]] + Vaa[Num];
  DensityMat = Dens[Num];
  Liouvillian = Leg[DensityMat, Num] + Lre[DensityMat, Num];

  ρdot = (-I * (Trans.DensityMat - DensityMat.Trans) +
      If[Decay, Liouvillian, DiagonalMatrix[Table[0 , {m, 1, 3^Num}]]]) /. Sym;

  Table[D[ρ[m, n][t], t] == ρdot[[m, n]], {m, 1, 3^Num}, {n, m}] (*ρ[m,n]'[t]*)

 ]

(* This code is optimized:
  For 3 atomes it takes 1sec, Ilya's program takes 7sec
  For 4 atomes it takes about 1min40, Ilya's program takes 5min *)
Timing[Equations[1, True];][[1]]
```

```
0.002886
```

The complexity seems to be exponential

```
DiscretePlot[Log[Timing[Equations[k,True];][[1]]],{k,1,4},Joined→True,PlotMarkers→
Automatic,Axes→{False,True}]
```




```
OpticalBloch1Decay = Equations[1, True];
OpticalBloch2Decay = Equations[2, True];
OpticalBloch3Decay = Equations[3, True];

OpticalBloch1NoDecay = Equations[1, False];
OpticalBloch2NoDecay = Equations[2, False];
OpticalBloch3NoDecay = Equations[3, False];
(*OpticalBloch4NoDecay=Equations[4,True];
OpticalBloch4Decay=Equations[4,False];*)
```

Intitial conditions

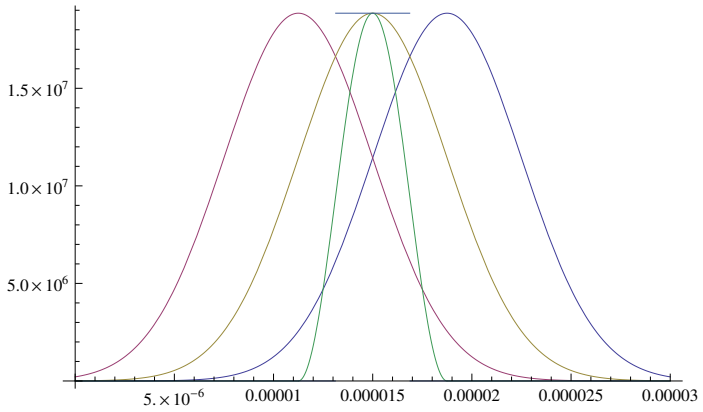
```
Sigs[Num_] := Table[Table[KroneckerDelta[m, p] KroneckerDelta[n, q],
    {p, 1, 3^Num}, {q, 1, 3^Num}], {m, 1, 3^Num}, {n, m}];
InitialConditions[Num_] := Table[Dens3Level[Num][[m, n]][0] ==
    If[(m == 1 && n == 1), KroneckerDelta[m, n], 0], {m, 1, 3^Num}, {n, m}];
InitialExcitedConditions[Num_] := Module[
   {AllExcited = FindStates[Num, "2", Num][[1]] }, Table[Dens3Level[Num][[m, n]][0] ==
     If[(m == AllExcited && n == m), KroneckerDelta[m, n], 0], {m, 1, 3^Num}, {n, m}]];
Vars[Num_] := Flatten[Table[Dens3Level[Num][[m, n]][t], {m, 1, 3^Num}, {n, m}]];
InitialConditionsRydStates = Table[Dens3Level[2][[m, n]][0] ==
    If[(m == 9 && n == 9), 2 KroneckerDelta[m, n], 0], {m, 1, 3^2}, {n, m}];
InitialConditionsPStates = Table[Dens3Level[2][[m, n]][0] ==
    If[(m == 5 && n == 5), 2 KroneckerDelta[m, n], 0], {m, 1, 3^2}, {n, m}];
```

Experiments

Now, we can solve the master equation in several cases (with and without decay, changing the number of atoms)

Pulses

```
Ωge[t0_, σt_] := Ω0 * Exp[ -(t - t0 / 2 - σt)^2 / (2 σt^2) ]

Ωer[t0_, σt_] := Ω0 * Exp[ -(t - t0 / 2 + σt)^2 / (2 σt^2) ]

Chirp[t0_, σt_, ω0_, αt_] :=
  Ω0 * Exp[ -(t - t0 / 2)^2 / (2 σt^2) ] * Cos[ω0 * (t - t0 / 2) + αt * (t - t0 / 2) ^ 2 / 2]

Gauss[t0_, σt_] := Ω0 * Exp[ -(t - t0 / 2)^2 / (2 σt^2) ]

SinSq[t0_, σt_] := Ω0 * Piecewise[{{Cos[ π (t - t0) / (2 σt) ]^2, (t > t0 - σt) && (t < t0 + σt)}}]

Step[t0_, σt_] := Ω0 * Piecewise[{{1, (t > t0 - σt / 2) && (t < t0 + σt / 2)}}];

With[{tf = 30. * 10^(-6)}, Plot[Evaluate[
    Flatten[{Block[{Ω0 = 2 * Pi * 3.0 * 10^6, σt = tf / 8, ω0 = 1, αt = 10^12}, {Ωge[tf, σt],
        Ωer[tf, σt], Gauss[tf, σt], SinSq[tf / 2, σt], Step[tf / 2, σt]}]}]], {t, 0, tf}]]
```



STIRAP Experiments

n atoms function

```
STIRAPn[Numval_, equDecay_, equNoDecay_,
  δ1val_, δ2val_, Δval_, Ω0val_, tfval_, Γgeval_, Γreval_] := (

  {s1, s2} = With[{ (* s1 is the solution with decayn, and s2 is without decay *)
     Num = Numval
    }, {
     With[{
       tf = tfval
      },
     NDSolve[Flatten[{
        Block[{
```

```
          Ω1 = Ωge[tf, σt],
          Ω2 = Ωer[tf, σt],
          σt = tf / 8,
          Ω0 = Ω0val,
          Γge = Γgeval,
          Γre = Γreval,
          δ1 = δ1val,
          δ2 = δ2val,
          Δ = Δval
         }, equDecay], InitialConditions[Num]}],
     Vars[Num], {t, 0, tf}, MaxSteps → 40 000]],
   With[{
     tf = tfval
    }, NDSolve[Flatten[{
       Block[{
         Ω1 = Ωge[tf, σt],
         Ω2 = Ωer[tf, σt],
         σt = tf / 8,
         Ω0 = Ω0val,
         Γge = Γgeval,
         Γre = Γreval,
         δ1 = δ1val,
         δ2 = δ2val,
         Δ = Δval
        }, equNoDecay], InitialConditions[Num]}],
     Vars[Num], {t, 0, tf}, MaxSteps → 40 000]]}];
With[{
  tf = tfval
 },
 Grid[{
   {
    Plot[{
      Pr[Numval, 0] /. s1 , (*with decay*) (* this plots the probability of 0
       atom in Rydberg state for a Numval number of atoms system *)
      Pr[Numval, 1] /. s1 (* this plots the probability of 1 atom
       in Rydberg state for a Numval number of atoms system *)
     }, {t, 0, tf},
     ImageSize → Medium, PlotStyle → Thick,
     Frame → True,
     FrameLabel → {{"Populations", None}, {"Time (s)", None}}, LabelStyle → Bold,
     PlotRange → All],
    Plot[{
      Pr[Numval, 0] /. s2, (*without decay*)
      Pr[Numval, 1] /. s2
     }, {t, 0, tf},
     ImageSize → Medium, PlotStyle → Thick,
     Frame → True,
     FrameLabel → {{"Populations", None}, {"Time (s)", None}}, LabelStyle → Bold,
     PlotRange → All]}
```

```
    , {"Decay", "No Decay"}
   }]
  ]
 )
```

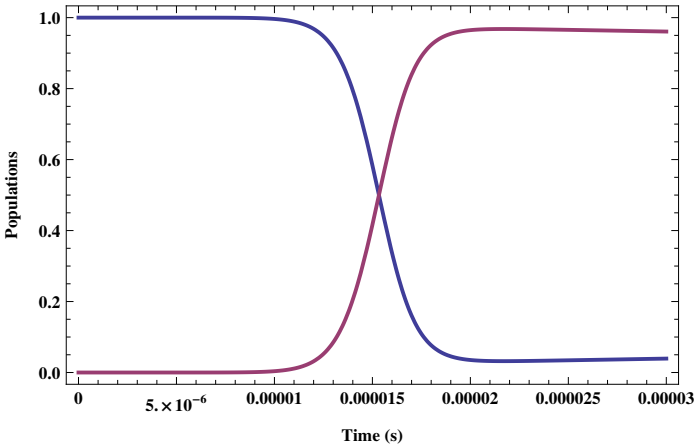
Experiments

```
Pr[2, 0]
Px[2, 2, "3"]
```

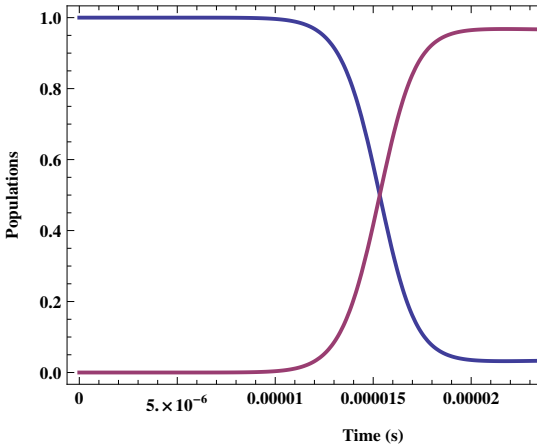
```
ρ[1, 1][t] + ρ[2, 2][t] + ρ[4, 4][t] + ρ[5, 5][t]
```

```
ρ[9, 9][t]
```

```
(* n atoms experiment *)
(* n=1 *)
With[{
  Num = 1, (* n=1 *)
  equDecay = OpticalBloch1Decay, (* n=1 *)
  equNoDecay = OpticalBloch1Decay, (* n=1 *)
  δ1 = 0,
  δ2 = 0,
  Δ = 1 * 10^10,
  Ω0 = 2 * Pi * 3.0 * 10^6,
  tf = 30. * 10^(-6),
  Γge = 38.0 * 10^6,
  Γre = 1 * 10^3
 }, STIRAPn[Num, equDecay, equNoDecay, δ1, δ2, Δ, Ω0, tf, Γge, Γre]]
```

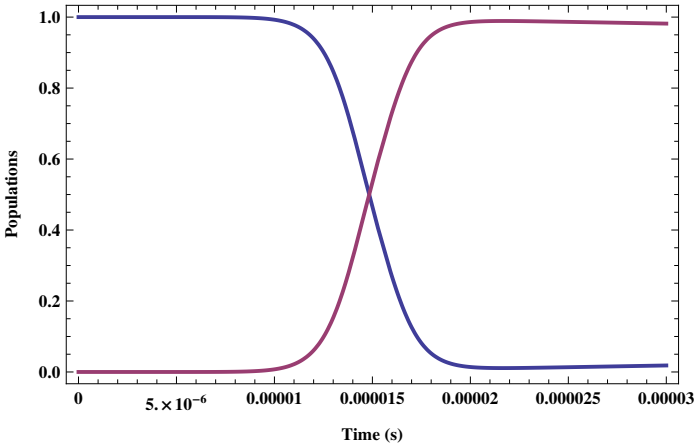


Decay

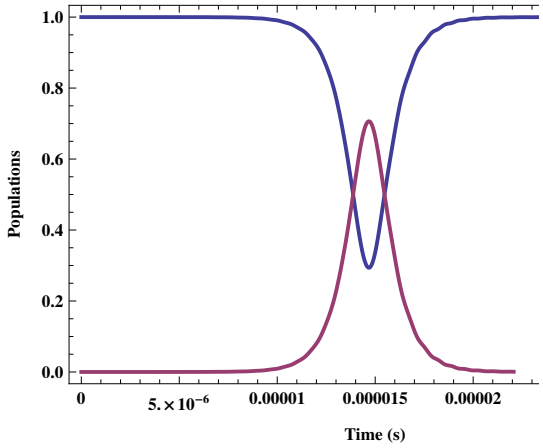


No Decay

```
(* n=2 *)
With[{
  Num = 2, (* n=2 *)
  equDecay = OpticalBloch2Decay, (* n=2 *)
  equNoDecay = OpticalBloch2NoDecay, (* n=2 *)
  δ1 = 0,
  δ2 = 0,
  Δ = 1 * 10^10,
  Ω0 = 2 * Pi * 3.0 * 10^6,
  tf = 30. * 10^(-6),
  Γge = 38.0 * 10^6,
  Γre = 1 * 10^3
 }, STIRAPn[Num, equDecay, equNoDecay, δ1, δ2, Δ, Ω0, tf, Γge, Γre]]
```

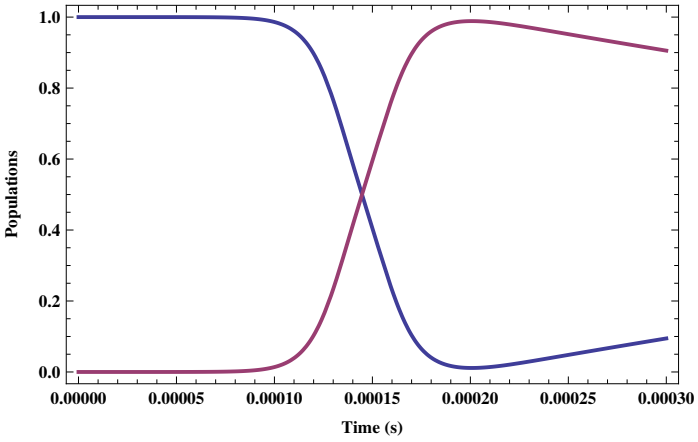


Decay

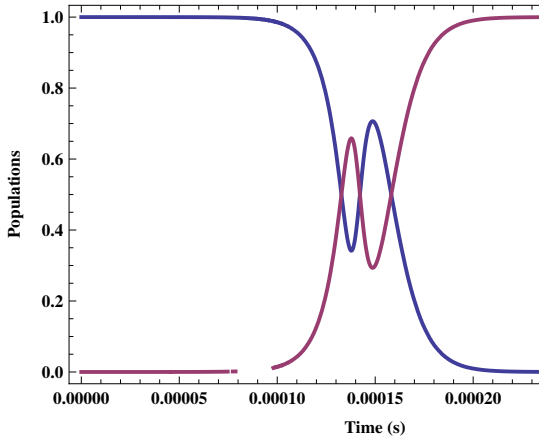


No Decay

```
(* n=3 *)
With[{
  Num = 3, (* n=3 *)
  equDecay = OpticalBloch3Decay, (* n=3 *)
  equNoDecay = OpticalBloch3NoDecay, (* n=3 *)
  δ1 = 0,
  δ2 = 0,
  Δ = 1 * 10^10,
  Ω0 = 2 * Pi * 3.0 * 10^6,
  tf = 30. * 10^(-5), (*adiabaticity violation*)
  Γge = 38.0 * 10^6,
  Γre = 1 * 10^3
 }, STIRAPn[Num, equDecay, equNoDecay, δ1, δ2, Δ, Ω0, tf, Γge, Γre]]
```



Decay



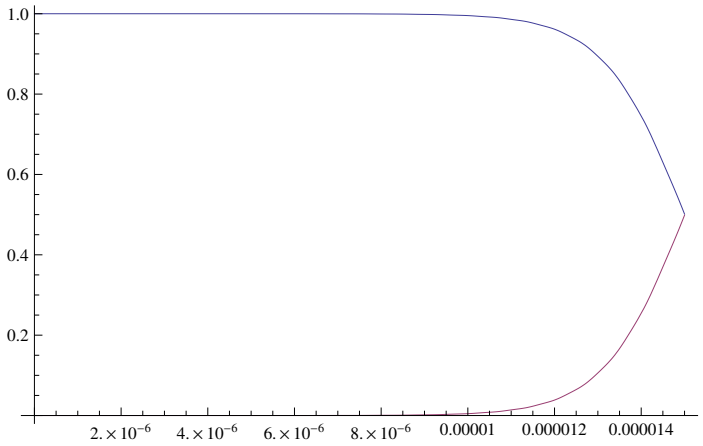
No Decay

```
(* Press "Alt+," to abort,
Mathematica is too long to try to solve the equations for 4 atoms *)
With[{
  Num = 4,
  equDecay = OpticalBloch4Decay,
  equNoDecay = OpticalBloch4NoDecay,
  Δ = 1 * 10^10,
  Ω0 = 2 * Pi * 3.0 * 10^6,
  tf = 30. * 10^(-6),
  Γge = 38.0 * 10^6,
  Γre = 1 * 10^3
 }, STIRAPn[Num, equDecay, equNoDecay, Δ, Ω0, tf, Γge, Γre]]
```

NDSolve::ndsdtc :
The time constraint of 1.` seconds was exceeded trying to solve for derivatives, so the system will be treated as a system of differential–algebraic equations. You can use Method–>{"EquationSimplification"–>"Solve"} to have the system solved as ordinary differential equations. ≫

NDSolve::mconly : For the method IDA, only machine real code is available.
Unable to continue with complex values or beyond floating–point exceptions. ≫

```
(* Tring to split the solving into time steps,
but "MaxSteps->1" seems to be already impossible for 4 atoms *)
s = With[{
     split = 2,
     tf = 30. * 10^(-6),
     Num = 1,
     Decay = True
    }, NDSolve[Flatten[{Block[{Ω1 = Ωge[tf, σt], Ω2 = Ωer[tf, σt],
          Ω0 = 2 * Pi * 3.0 * 10^6,
          σt = tf / 8,
          δ1 = 0,
          δ2 = 0,
          Δ = 1 * 10^10, (* Δ=1*10^6 is a critical value,
          before, there is no blockade, and after there is *)
          Γge = 38.0 * 10^6,
          Γre = 1 * 10^3
           }, OpticalBloch1NoDecay], InitialConditions[Num]}],
     Vars[Num], {t, 0, tf / split}, MaxSteps → Ceiling[40 000 / split]]];

With[{
   split = 2,
   tf = 30. * 10^(-6),
   Num = 1
 }, Plot[{
    Pr[Num, 0] /. s,
    Pr[Num, 1] /. s
  }, {t, 0, tf / split}
  (*,PlotRange→ {0,1}*)
 ]]
```



Chirp Experiments

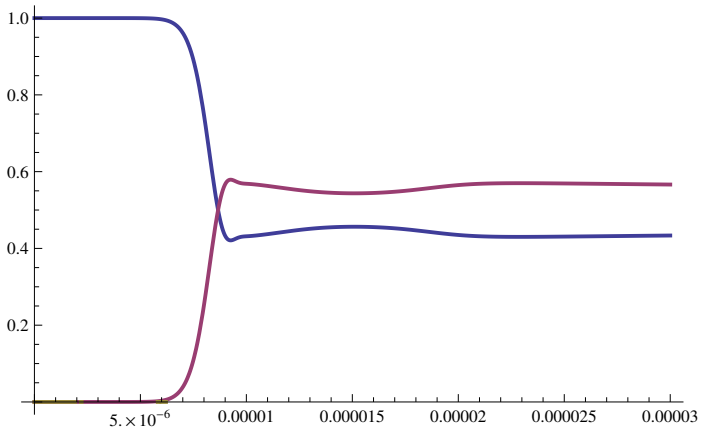
```
CHIRPn[Numval_, equDecay_, Δval_, δval_,
  Ω0val_, αtval_, tfval_, σtval_, Γgeval_, Γreval_] := (

  s = With[{ (* s1 is the solution with decay, and s2 is without decay *)
     Num = Numval
    },
    With[{
      tf = tfval
     },
     NDSolve[Flatten[{
        Block[{
          Ω1 = Gauss[tf, σt] // Simplify,
          Ω2 = Gauss[tf, σt] // Simplify,
          Ω0 = Ω0val,
          σt = tf / 8,
          αt = αtval,
          Γge = Γgeval,
          Γre = Γreval,
          Δ = Δval,
          δ1 = δval + 2 Pi * αtval * (t - tf / 2) ^ 2,
          δ2 = -δ1
         }, equDecay], InitialConditions[Num]}],
      Vars[Num], {t, 0, tf}, MaxSteps → 100 000]]];
  With[{
    tf = tfval,
    Num = Numval
   },
   Plot[{
     Px[Num, 0, "3"] /. s , (* this plots the probability of 0
      atom in Rydberg state for a Numval number of atoms system *)
     Px[Num, 1, "3"] /. s, (* this plots the probability of 1 atom
      in Rydberg state for a Numval number of atoms system *)
     Px[Num, 2, "3"] /. s
    }, {t, 0, tf},
    ImageSize → Medium,
    PlotStyle → Thick,
    PlotRange → All]
  ])
```

```
Px[2, 2, "3"]
```

```
ρ[9, 9][t]
```

```
(* n=1 *)
With[{
    Num = 1, (* n=1 *)
    equDecay = OpticalBloch1Decay, (* n=1 *)
    Δ = 0 * 10^10,
    δ = 0 * 10^6,
    Ω0 = 2 * Pi * 6.0 * 10^6,
    αt = 1 * 10^12,
    tf = 30. * 10^(-6),
    Γge = 38.0 * 10^6,
    Γre = 1 * 10^3
  }, CHIRPn[Num, equDecay, Δ, δ, Ω0, αt, tf, Γge, Γre]];
```

```
(* n=2 *)
With[{
  Num = 2,
  equDecay = OpticalBloch2Decay,
  Δ = 1 * 10^10,
  δ = 1 * 10^4,
  Ω0 = 2 Pi * 6 * 10^6,
  αt = 10^12,
  tf = 30. * 10^(-6),
  Γge = 38.0 * 10^6,
  Γre = 1 * 10^3
 }, CHIRPn[Num, equDecay, Δ, δ, Ω0, αt, tf, 1, Γge, Γre]
]
```



Rabi oscillations

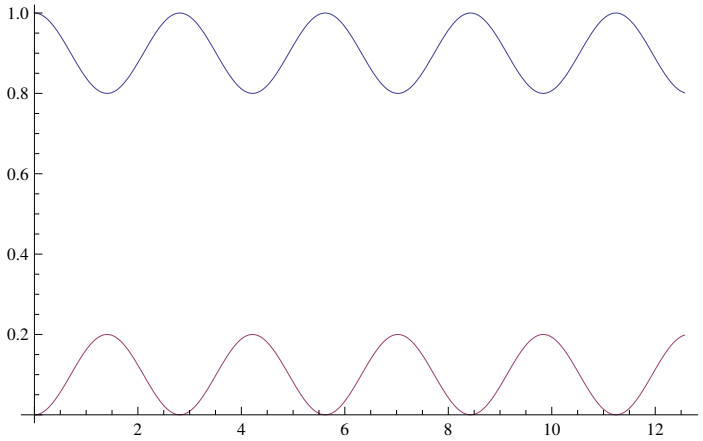
2 level system

This subsection can reproduce the results of this document: http://www.ph.unimelb.edu.au/~-part3/notes/Rabi.pdf using the previously made density matrix formalism. It checks the implementation of the detuning.

```
DSolve[{
  ρ[1, 1]'[t] == 1/2 i Ω1 (ρ[1, 2][t] - ρ[2, 1][t]),
  ρ[1, 2]'[t] == 1/2 i (Ω1 ρ[1, 1][t] - Ω1 ρ[2, 2][t]),
  ρ[2, 1]'[t] == -1/2 i (Ω1 ρ[1, 1][t] - Ω1 ρ[2, 2][t]),
  ρ[2, 2]'[t] == -1/2 i (Ω1 ρ[1, 2][t] - Ω1 ρ[2, 1][t]),
  ρ[1, 1][0] == 1, ρ[1, 2][0] == 0, ρ[2, 1][0] == 0, ρ[2, 2][0] == 0
 }, {ρ[1, 1][t], ρ[1, 2][t], ρ[2, 1][t], ρ[2, 2][t]}, t]
```

$$\left\{\left\{\rho[1,1][t] \to \frac{1}{2}(1+\text{Cos}[t\,\Omega 1]),\ \rho[1,2][t] \to \frac{1}{2}\,\text{i}\,\text{Sin}[t\,\Omega 1],\right.\right.$$
$$\left.\left.\rho[2,1][t] \to -\frac{1}{2}\,\text{i}\,\text{Sin}[t\,\Omega 1],\ \rho[2,2][t] \to \frac{1}{2}(1-\text{Cos}[t\,\Omega 1])\right\}\right\}$$

```
s = With[{
     tf = 4 * Pi,
     Num = 1,
     Decay = True
    }, NDSolve[Flatten[{Block[{Ω1 = 1, Ω2 = 0,
          Δ = 1 * 10^10,
          δ1 = 2, δ2 = 0,
          Γge = 0,
          Γre = 0
           }, OpticalBloch1Decay], InitialConditions[Num]}], Vars[Num], {t, 0, tf}]];
With[{
   tf = 4 * Pi,
   Num = 1
 }, Plot[{
    ρ[1, 1][t] /. s,
    ρ[2, 2][t] /. s
   }, {t, 0, tf}
   (*,PlotRange→ {0,1}*)
 ]]
```

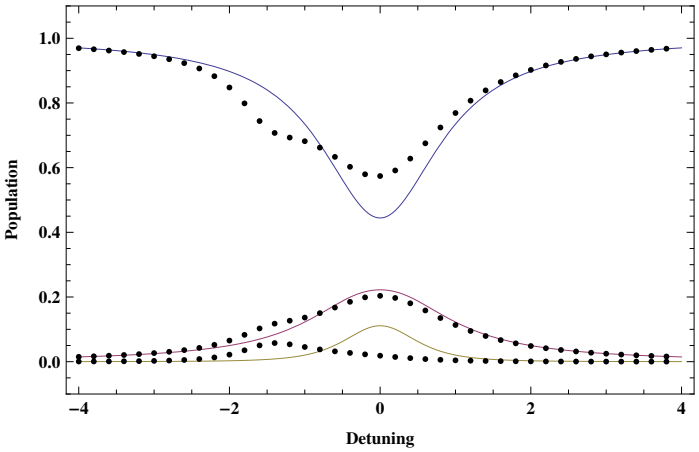


Rydberg blocade criterion

Here we are trying to bring out Rydberg blocade using a parameter so called θ=1-p2/(p1+p2)^2, where p1 is the probability to have one excited atom and p2 is the one to have two atoms excited, for a 2 atoms system.
Indeed if the probability to excitate one single atom is given by p, then p2=p^2 and p1=p(1-p) (and p0=(1-p)^2). θ is then equal to zero. But if there is Rydberg blocade, p2 does not remain equal to p^2, that must change the value of θ and would be the desired criterion.

Spectra

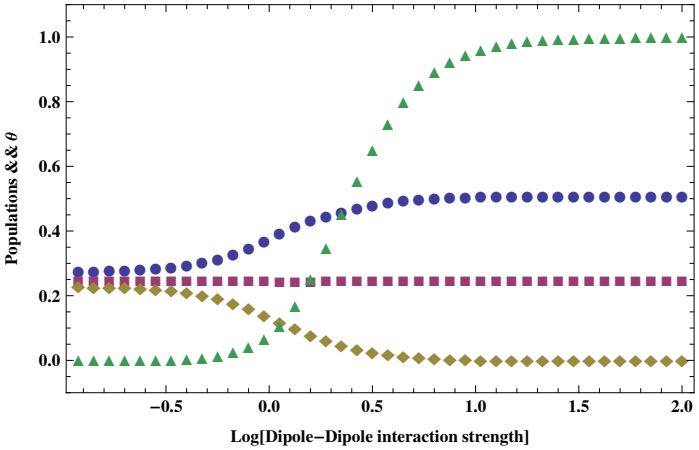
```
Block[{
  n = 40, (* number of points in the spectrum *)
  scale = 5, (* zoom to the center of the spectrum *)
  tf = 4 Pi, (* final time of the experiment,
  this should be long enough because we are considering stationary solutions *)
  Ωval = 1, (* Rabi frequency *)
  Γreval = 1 (* decay value, if it's low, one should increase the final time *)
 },
 For[i = 0, i < n,
  (* the variable i will be used to sweep the detuning of the laser radiation *)
  {s = With[{
      Num = 2(* number of atoms *)
     },
     NDSolve[Flatten[{Block[{Ω1 = 0, (* we are only considering the excited and
           the Rydberg state, there is no interaction with the ground state *)
          Ω2 = Ωval,
          Δ = 3, (* !!!
            dipole-dipole interaction,
          CHANGE THIS VALUE to {1, 3, 5} to see Rdyberg blocade
             !!! *)
          δ1 = 0, δ2 = (i - n / 2) / scale,
          Γge = 0,
          Γre = Γreval
          }, OpticalBloch2Decay (* for 2 atoms *)
        ], InitialExcitedConditions[Num]
        (* we should define new initial conditions, where the only populated
         state is the one where all the atoms are in the excited state *)
       }], Vars[Num], {t, 0, tf}]];
   {ρ55[i]} = Re[ρ[5, 5][t] /. s /. t → tf]; (* 2 atoms in excited state *)
   {ρ66[i]} = Re[ρ[6, 6][t] /. s /. t → tf];
   (* 1 atom in excited state and 1 in Rydberg state *)
   {ρ99[i]} = Re[ρ[9, 9][t] /. s /. t → tf]; (* 2 atoms in Rydberg state *)
  }; i++];

 ρ55Tab = Table[{(i - n / 2) / scale, ρ55[i]}, {i, 0, n - 1}];
 ρ66Tab = Table[{(i - n / 2) / scale, ρ66[i]}, {i, 0, n - 1}];
 ρ99Tab = Table[{(i - n / 2) / scale, ρ99[i]}, {i, 0, n - 1}];

 Plot[{(1 - Ωval^2 / 2 / (Ωval^2 + 2 δ^2 + Γreval^2 / 2))^2, (* (1-p)^2 *)
   (Ωval^2 / 2 / (Ωval^2 + 2 δ^2 + Γreval^2 / 2)) *
    (1 - Ωval^2 / 2 / (Ωval^2 + 2 δ^2 + Γreval^2 / 2)), (* p(1-p) *)
   (Ωval^2 / 2 / (Ωval^2 + 2 δ^2 + Γreval^2 / 2))^2 (* p^2 *)
  }, {δ, -n / (2 * scale), n / (2 * scale)},
  Epilog → Map[Point, {ρ55Tab, ρ66Tab, ρ99Tab}],
  PlotRange → {-0.1, 1.1}, Frame → True, Axes → False, ImageSize → Medium,
  FrameLabel → {{"Population", None}, {"Detuning", None}}, LabelStyle → Bold]
]
```



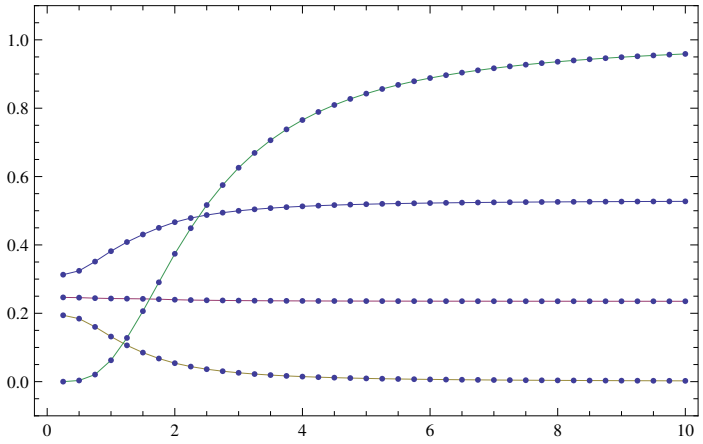
Plotting θ with respect to Δ dipole-dipole interaction parameter

```
Logspace[a_, b_, n_] := 10.0^Range[a, b, (b - a) / (n - 1)]
Linspace[a_, b_, n_] := Range[a, b, (b - a) / (n - 1)]
```

```
(* Semi-log plot *)
Block[{n = 40, init = -1, final = 2, scale = 5, tf = 8 Pi, Ωval = 1, Γreval = 0.25},
 ΔTab = Logspace[init, final, n];
 For[i = 1, i <= n,
  {s = With[{
       Num = 2,
       Decay = True
      }, NDSolve[Flatten[{Block[{Ω1 = 0, Ω2 = Ωval,
            Δ = ΔTab[[i]],
            δ1 = 0, δ2 = 0,
            Γge = 0,
            Γre = Γreval
             }, OpticalBloch2Decay], InitialExcitedConditions[Num]}],
       Vars[Num], {t, 0, tf}]];
   {ρ55[i]} = Re[ρ[5, 5][t] /. s /. t → tf];
   {ρ66[i]} = Re[ρ[6, 6][t] /. s /. t → tf];
   {ρ99[i]} = Re[ρ[9, 9][t] /. s /. t → tf];
   {θ[i]} = Re[1 - (ρ[9, 9][t] / (ρ[9, 9][t] + ρ[6, 6][t]) ^ 2) /. s /. t → tf];
  }; i++];

 ρ55Tab = Table[{init + i * (final - init) / n, ρ55[i]}, {i, 1, n}];
 ρ66Tab = Table[{init + i * (final - init) / n, ρ66[i]}, {i, 1, n}];
 ρ99Tab = Table[{init + i * (final - init) / n, ρ99[i]}, {i, 1, n}];
 θTab = Table[{init + i * (final - init) / n, θ[i]}, {i, 1, n}];

 ListPlot[{ρ55Tab, ρ66Tab, ρ99Tab, θTab}, PlotRange → {-0.1, 1.1},
  Joined → False, PlotMarkers → Automatic , Mesh → All, Frame → True,
  Axes → False, ImageSize → Medium, FrameLabel → {{"Populations && θ", None},
    {"Log[Dipole-Dipole interaction strength]", None}}, LabelStyle → Bold]
]
```



```
(* Linear plot *)
Block[{n = 40, init = 0, final = 10, scale = 5, tf = 4 Pi, Ωval = 1, Γreval = 0.5},
 ΔTab = Linspace[init, final, n];
 For[i = 1, i <= n,
  {s = With[{
       Num = 2,
       Decay = True
      }, NDSolve[Flatten[{Block[{Ω1 = 0, Ω2 = Ωval,
            Δ = ΔTab[[i]],
            δ1 = 0, δ2 = 0,
            Γge = 0,
            Γre = Γreval
             }, OpticalBloch2Decay], InitialExcitedConditions[Num]}],
       Vars[Num], {t, 0, tf}]];
   {ρ55[i]} = Re[ρ[5, 5][t] /. s /. t → tf];
   {ρ66[i]} = Re[ρ[6, 6][t] /. s /. t → tf];
   {ρ99[i]} = Re[ρ[9, 9][t] /. s /. t → tf];
   {θ[i]} = Re[1 - (ρ[9, 9][t] / (ρ[9, 9][t] + ρ[6, 6][t]) ^ 2) /. s /. t → tf];
  }; i++];

 ρ55Tab = Table[{init + i * (final - init) / n, ρ55[i]}, {i, 1, n}];
 ρ66Tab = Table[{init + i * (final - init) / n, ρ66[i]}, {i, 1, n}];
 ρ99Tab = Table[{init + i * (final - init) / n, ρ99[i]}, {i, 1, n}];
 θTab = Table[{init + i * (final - init) / n, θ[i]}, {i, 1, n}];

 ListPlot[{ρ55Tab, ρ66Tab, ρ99Tab, θTab}, PlotRange → {-0.1, 1.1},
  Joined → True, Mesh → All, Frame → True, Axes → False, ImageSize → Medium]
]
```



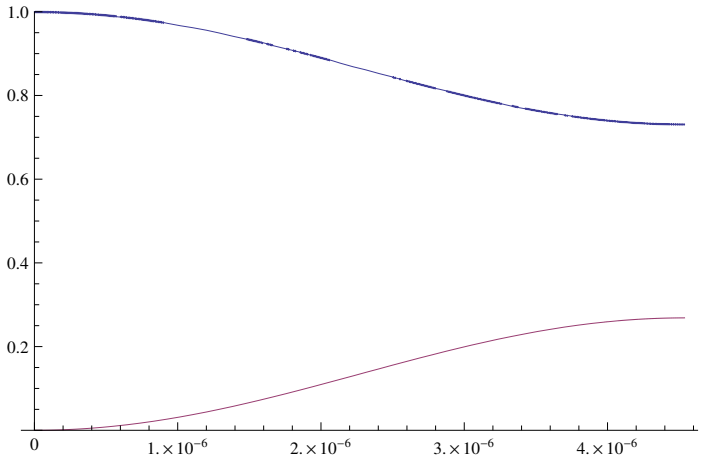
M.Saffman experiment

“For the single atom loading typical parameters currently are : Rb87
5 s–5 p3/2–97 d5/2
sigma + , sigma + coupling to m = 5/2

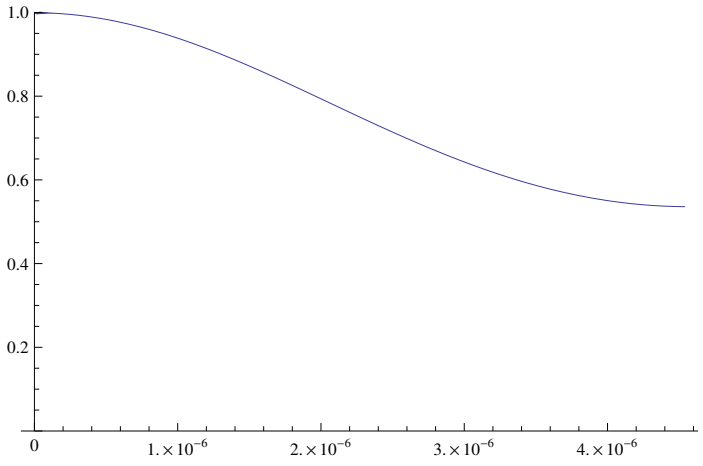
detuning from 5 p3/2, 1.8 GHz
780 nm Rabi frequency 68 MHz
480 nm Rabi frequency 19 MHz
2 - photon Rabi frequency 0.36 MHz

we load from 3–20 atoms into f = 2 and then pi pulse to | r > followed by pi pulse to f = 1.
Our best results are 65 % single atom in f = 1. Our latest data seem to show we actually have better single atom probability in
f = 1 but we have some loss during blow away of f = 2. We are checking this further.We have also done some adiabatic pulse sequences giving similar results with 65 % success rate.”

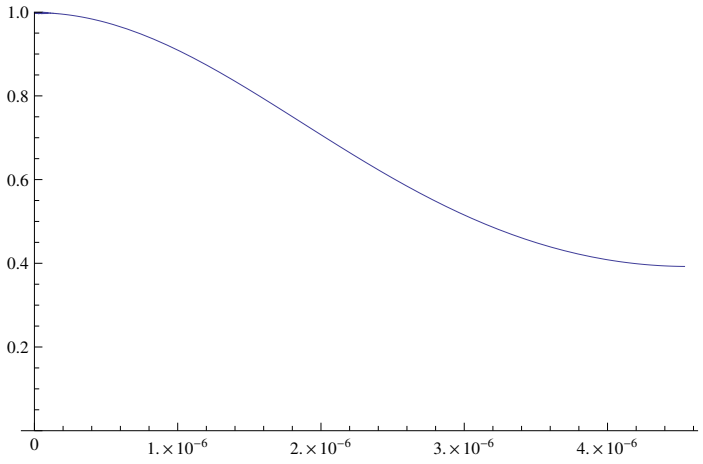
```
Ω1test = 68 * 10^6; Ω2test = 19 * 10^6; δ1test = 1.8 * 10^9;
ΩR = N[Ω1test * Ω2test / (2 * δ1test)];
ΔR = N[(Ω2test^2 - Ω1test^2) / (2 * 2 * δ1test)];
T = Pi / Sqrt[ΩR^2 + ΔR^2];
s = With[{
     tf = T,
     Num = 1,
     Decay = True
    }, NDSolve[Flatten[{Block[{
          Ω1 = 68 * 10^6, Ω2 = 19 * 10^6,
          Δ = 0,
          δ1 = 1.8 * 10^9, δ2 = -δ1,
          Γge = 0 * 38.0 * 10^6,
          Γre = 0 * 1 * 10^3
           }, OpticalBloch1Decay], InitialConditions[Num]}],
     Vars[Num], {t, 0, tf}, MaxSteps → Infinity]];
With[{
  tf = T,
  Num = 1
 }, Plot[{
   ρ[1, 1][t] /. s,
   ρ[3, 3][t] /. s
  }, {t, 0, tf},
  PlotRange → {0, 1}
 ]]
```


1.0
0.8
0.6
0.4
0.2
0
1.×10^−6
2.×10^−6
3.×10^−6
4.×10^−6

```
Ω1test = 68 * 10^6; Ω2test = 19 * 10^6; δ1test = 1.8 * 10^9;
ΩR = N[Ω1test * Ω2test / (2 * δ1test)];
ΔR = N[(Ω2test^2 - Ω1test^2) / (2 * 2 * δ1test)];
T2 = Pi / Sqrt[ΩR^2 + ΔR^2];
s2 = With[{
     tf = T2,
     Num = 2,
     Decay = True
    }, NDSolve[Flatten[{Block[{
          Ω1 = 68 * 10^6, Ω2 = 19 * 10^6,
          Δ = 0,
          δ1 = 1.8 * 10^9, δ2 = -δ1,
          Γge = 38.0 * 10^6,
          Γre = 1 * 10^3
           }, OpticalBloch2Decay], InitialConditions[Num]}],
     Vars[Num], {t, 0, tf}, MaxSteps → Infinity]];
With[{
   tf = T2
 }, Plot[{
    ρ[1, 1][t] /. s2
  }, {t, 0, tf},
  PlotRange → {0, 1}
 ]]
```



```
Ω1test = 68 * 10^6; Ω2test = 19 * 10^6; δ1test = 1.8 * 10^9;
ΩR = N[Ω1test * Ω2test / (2 * δ1test)];
ΔR = N[(Ω2test^2 - Ω1test^2) / (2 * 2 * δ1test)];
T3 = Pi / Sqrt[ΩR^2 + ΔR^2];
s3 = With[{
     tf = T3,
     Num = 3,
     Decay = True
    }, NDSolve[Flatten[{Block[{
          Ω1 = 68 * 10^6, Ω2 = 19 * 10^6,
          Δ = 0,
          δ1 = 1.8 * 10^9, δ2 = -δ1,
          Γge = 38.0 * 10^6,
          Γre = 1 * 10^3
           }, OpticalBloch3Decay], InitialConditions[Num]}],
     Vars[Num], {t, 0, tf}, MaxSteps → Infinity]];

With[{
  tf = T3
 }, Plot[{
   ρ[1, 1][t] /. s3
  }, {t, 0, tf},
  PlotRange → {0, 1}
 ]]
```



```
With[{
   tf = T3
 }, Plot[{
    ρ[1, 1][t] /. s,
    ρ[1, 1][t] /. s2,
    ρ[1, 1][t] /. s3
  }, {t, 0, tf},
  PlotRange → {0, 1}
 ]]
```