# Assignment\_2\_Thrisha Rajkumar

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# Statistical Computing and Empirical Methods, TB1, 2024

#Assignment 2

# 1. Data Wrangling

### 1.1 (Q1) Select and filter

#View the first few rows of the data

(Q1). Use a combination of the select() and filter() functions to generate a data frame called "hSF" which is a sub-table of the original Hawks data frame, such that 1. Your data frame should include the columns: a) "Wing" b) "Weight" c) "Tail" 2. Your data frame should contain a row for every hawk such that: a) They belong to the species of Red-Tailed hawks b) They have weight at least 1kg. 3. Use the pipe operator "%>%" to simplify your code.

# 1.1 (Q2)

head(Hawks)

```
# Load required packages
library(Stat2Data)
library(tidyverse)
## -- Attaching core tidyverse packages ----- tidyverse 2.0.0 --
## v dplyr
              1.1.4
                        v readr
                                    2.1.5
## v forcats
              1.0.0
                        v stringr
                                    1.5.1
## v ggplot2
              3.5.1
                        v tibble
                                    3.2.1
## v lubridate 1.9.3
                        v tidyr
                                    1.3.1
## v purrr
              1.0.2
## -- Conflicts -----
                                            ## x dplyr::filter() masks stats::filter()
## x dplyr::lag()
                    masks stats::lag()
## i Use the conflicted package (<a href="http://conflicted.r-lib.org/">http://conflicted.r-lib.org/</a>) to force all conflicts to become error
library(dplyr)
library(knitr)
#Load the Hawks data frame
data(Hawks)
```

```
Month Day Year CaptureTime ReleaseTime BandNumber Species Age Sex Wing
## 1
        9 19 1992
                          13:30
                                              877-76317
                                                              RT
                                                                   Ι
                                                                          385
## 2
         9 22 1992
                          10:30
                                              877-76318
                                                              RT
                                                                          376
                                                                   Ι
## 3
         9 23 1992
                          12:45
                                              877-76319
                                                              RT
                                                                          381
                                                                   Ι
## 4
         9 23 1992
                          10:50
                                               745-49508
                                                              CH
                                                                   Ι
                                                                       F
                                                                          265
         9 27 1992
## 5
                          11:15
                                              1253-98801
                                                              SS
                                                                 I
                                                                       F 205
         9 28 1992
                          11:25
                                             1207-55910
                                                              RT
                                                                          412
    Weight Culmen Hallux Tail StandardTail Tarsus WingPitFat KeelFat Crop
##
## 1
        920
              25.7
                     30.1 219
                                                NA
                                                            NA
                                                                    NA
                                                                         NA
## 2
        930
                     NA 221
                                         NA
                                                NA
                                                            NA
                                                                    NA
                                                                         NA
              NA
## 3
       990
              26.7
                     31.3 235
                                         NA
                                                NA
                                                            NA
                                                                    NA
                                                                         NA
## 4
        470
                     23.5 220
                                         NA
                                                            NA
                                                                         NA
            18.7
                                                NA
                                                                    NA
## 5
       170
             12.5
                                         NA
                                                NA
                                                            NA
                                                                         NA
                     14.3
                           157
                                                                    NA
## 6
       1090
              28.5
                     32.2 230
                                         NA
                                                NA
                                                            NA
                                                                    NA
                                                                         NA
# Get the column headers
column headers <- colnames(Hawks)</pre>
# Print the column headers
print(column_headers)
## [1] "Month"
                       "Day"
                                      "Year"
                                                      "CaptureTime"
                                                                     "ReleaseTime"
                                                      "Sex"
## [6] "BandNumber"
                       "Species"
                                       "Age"
                                                                     "Wing"
## [11] "Weight"
                       "Culmen"
                                      "Hallux"
                                                      "Tail"
                                                                     "StandardTail"
## [16] "Tarsus"
                       "WingPitFat"
                                      "KeelFat"
                                                      "Crop"
# Get unique species
unique_species <- unique(Hawks$Species)</pre>
# Print the unique species
print(unique_species)
## [1] RT CH SS
## Levels: CH RT SS
ncol(Hawks)
## [1] 19
# Filter and select relevant columns
hSF <- Hawks %>%
 filter(Species == "RT", Weight >= 1000) %>%
  select(Wing, Weight, Tail)
# Print the filtered data
print(hSF)
##
       Wing Weight Tail
## 1
       412.0
               1090 230
       412.0
## 2
              1210 210
```

```
## 3
       405.0
                 1120
                       238
## 4
       393.0
                 1010
                       222
                 1010
## 5
       371.0
                       217
       390.0
                       213
## 6
                 1120
## 7
       416.0
                 1170
                       243
## 8
       436.0
                 1390
                       232
## 9
       418.0
                 1150
                       238
                 1010
       396.0
                       227
## 10
## 11
       399.0
                 1070
                       222
## 12
       416.0
                 1190
                       237
## 13
       392.0
                 1330
                       213
       399.0
## 14
                 1100
                       190
       401.0
                 1190
## 15
                       245
## 16
       427.0
                 1490
                       246
## 17
       395.0
                 1040
                       207
## 18
       396.0
                 1030
                       200
## 19
       391.0
                 1300
                       215
## 20
       413.0
                 1500
                       219
## 21
       371.0
                 1080
                       198
       385.0
## 22
                 1320
                       207
## 23
       378.0
                 1490
                       204
## 24
       416.0
                 1500
                       205
       384.0
## 25
                 1060
                       230
## 26
       382.0
                 1140
                       227
## 27
       390.0
                       208
                 1030
## 28
       390.0
                 1000
                       231
##
  29
       393.0
                 1050
                       222
##
   30
       378.0
                 1040
                       225
       398.0
##
  31
                 1110
                       225
       412.0
## 32
                 1300
                       233
       422.0
## 33
                 1120
                       233
## 34
       394.0
                 1270
                       245
##
   35
       410.0
                       229
                 1255
##
  36
       241.0
                 1320
                       235
## 37
       408.0
                 1320
                       221
## 38
        37.2
                 1180
                       210
## 39
       396.0
                 1250
                       225
## 40
       416.0
                 1300
                       235
## 41
       390.0
                 1080
                       209
## 42
       391.0
                 1130
                       212
## 43
       387.0
                 1160
                       209
## 44
       420.0
                 1345
                       250
## 45
       435.0
                 1385
                       235
       400.0
                       222
##
  46
                 1210
## 47
       398.0
                 1455
                       236
       395.0
## 48
                 1180
                       210
       410.0
                       239
## 49
                 1500
## 50
       369.0
                 1025
                       228
## 51
       415.0
                 1360
                       233
## 52
       412.0
                 1255
                       236
## 53
       375.0
                 1065
                       216
## 54
       385.0
                       233
                 1125
## 55
       422.0
                 1340
                       248
## 56
       391.0
                       221
                 1050
```

```
## 57 410.0
                1210
                       227
## 58
       381.0
                1000
                       219
## 59
       416.0
                1390
                       225
       406.0
                       238
## 60
                1275
## 61
       418.0
                1180
                       235
## 62
       412.0
                1210
                       222
## 63
       405.0
                1085
                       238
       428.0
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## 64
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## 65
       381.0
                1010
                       210
## 66
       420.0
                       241
                1210
## 67
       395.0
                1170
                       238
       406.0
## 68
                1350
                       235
       414.0
                1370
## 69
                       240
## 70
       423.0
                1310
                       234
## 71
       365.0
                1035
                       232
## 72
       391.0
                1125
                       220
## 73
       392.0
                1140
                       214
## 74
       410.0
                1210
                       238
## 75
       422.0
                1205
                       238
## 76
       385.0
                1045
                       229
## 77
       363.0
                1090
                       202
## 78
       450.0
                1190
                       226
       385.0
## 79
                1110
                       122
## 80
       384.0
                1075
                       208
## 81
       363.0
                       204
                1070
## 82
       409.0
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                       229
## 83
       390.0
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                       205
## 84
       420.0
                1125
                       244
       381.0
## 85
                1100
                       225
       408.0
## 86
                1360
                       239
## 87
       398.0
                1095
                       209
## 88
       394.0
                1075
                       211
## 89
       394.0
                1140
                       216
## 90
       416.0
                1240
                       250
## 91
       445.0
                1465
                       260
## 92
       388.0
                1105
                       217
## 93
       397.0
                1010
                       228
## 94
       384.0
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                       218
## 95
       379.0
                1060
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## 96
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## 97
       386.0
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                1010
## 98
       397.0
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## 99
       382.0
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                       225
## 100 417.0
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                       231
## 101 403.0
                1360
                       235
## 102 401.0
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## 103 377.0
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                       200
## 104 432.0
                1670
                       216
## 105 390.0
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## 106 381.0
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## 107 403.0
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## 108 390.0
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## 109 386.0
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## 110 402.0
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## 111 374.0
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## 112 370.0
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## 113 398.0
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## 114 375.0
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## 115 409.0
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## 116 415.0
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## 117 381.0
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## 118 398.0
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## 119 412.0
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## 120 411.0
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## 121 415.0
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## 122 383.0
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## 123 390.0
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## 124 365.0
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## 125 345.0
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## 126 400.0
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## 127 380.0
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## 130 409.0
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## 131 411.0
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## 135 412.0
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## 137 410.0
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## 138 398.0
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## 139 425.0
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## 141 387.0
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## 142 420.0
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## 143 405.0
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## 165 362.0
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## 166 378.0
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## 173 400.0
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## 178 478.0
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                1310
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## 278 390.0
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## 281 402.0
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## 284 378.0
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## 312 372.0
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## 313 389.0
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## 314 404.0
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## 316 368.0
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## 318 277.0
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## 319 435.0
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## 320 393.0
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## 327 385.0
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## 328 400.0
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## 333 375.0
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## 334 401.0
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## 335 407.0
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## 345 372.0
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## 349 405.0
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## 354 365.0
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## 355 374.0
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## 356 376.0
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## 357 369.0
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## 358 391.0
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## 360 366.0
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## 361 385.0
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## 362 400.0
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                      234
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                      226
## 364 396.0
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                      228
## 365 391.0
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                      221
## 366 393.0
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                      223
## 367 397.0
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                      232
## 368 384.0
                1010
                      230
## 369 396.0
                1240
                      237
## 370 428.0
                1290
                      253
## 371 364.0
                1150
                      196
## 372 400.0
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                      227
## 373 398.0
                1205
                      227
## 374 385.0
                      238
                1040
## 375 372.0
                1010
                      197
## 376 400.0
                1285
                      230
## 377 403.0
                1350
                      140
## 378 375.0
                1010
                      218
## 379 385.0
                1370
                      233
## 380 392.0
                1250
                      222
```

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## 381 415.0
                1285
                      242
## 382 380.0
                      205
                1005
## 383 411.0
                1220
                      236
## 384 393.0
                      233
                1265
## 385 370.0
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                      211
## 386 371.0
                1160
                      218
## 387 400.0
                1585
                      241
## 388 382.0
                1140
                      218
## 389 371.0
                1115
                      208
## 390 370.0
                1145
                      212
## 391 392.0
                1030
                      196
## 392 387.0
                      212
                1065
## 393 400.0
                1050
                      237
## 394 370.0
                1000
                      201
## 395 360.0
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                1325
## 396 402.0
                1350
                      219
## 397 380.0
                1525
                      224
## 398 199.0
                      222
                1290
```

# 1.1 (Q2)

# How many variables does the data frame hSF have?

- -> In my assignment to understand the column headers in the Hawks data frame -> Get the column headers column\_headers <- colnames(Hawks) which provides the column names i.e. variables in a vector form.
- -> ncol(Hawks) will give you the number of columns or variables in the dataframe.

```
ncol(Hawks)
```

## [1] 19

# What would you say to communicate this information to a Machine Learning Practitioner?

- $-\!>$  To communicate the above Information to a Machine Learning Practitoner I would mention the features or variables in use which is Species name =Red Tailed Hawks, Weight of the hawks above 1000 g / 1 kg, wing and Tail.
- -> The importance of the number of variables for model complexity.
- ->The choices of the variables taken for the model performance.

# How many examples does the data frame hSF have? How many observations? How many cases?

```
num_observations <- nrow(hSF)
print(num_observations)</pre>
```

## [1] 398

```
\#\#1.2 (Q1) The arrange function ....
```

```
sorted_hSF <- hSF %>%
    arrange(Wing)

head(sorted_hSF, 5)

## Wing Weight Tail
## 1 37.2 1180 210
```

## 3 199.0 1290 222 ## 4 241.0 1320 235 ## 5 262.0 1020 200

## 2 111.0

##1.3 (Q1) Join and rename functions

1340 226

(Q1). Use data.frame() to create a data frame that is called hawkSpeciesNameCodes and is the same as the above data frame (i.e., containing the correspondence between codes and the full species names).

```
species_code <- c("CH", "RT", "SS")
species_name_full <- c("Cooper's", "Red-tailed", "Sharp-shinned")
hawkSpeciesNameCodes <- data.frame(species_code, species_name_full)
print(hawkSpeciesNameCodes)</pre>
```

##1.3 (Q2)

(Q2). Use a combination of the functions left\_join(), the rename() and the select() functions to create a new data frame called "hawksFullName" which is the same as the "Hawks" data frame except that the Species column contains the full names rather than the two-letter codes.

```
install.packages("dplyr")
```

## Warning: package 'dplyr' is in use and will not be installed

```
library(dplyr)

hawksFullName <- Hawks %>%
  left_join(hawkSpeciesNameCodes, by = c("Species" = "species_code")) %>%
  select(Species = species_name_full, Wing, Weight)

print(hawksFullName)
```

```
## Species Wing Weight
## 1 Red-tailed 385.0 920
## 2 Red-tailed 376.0 930
```

```
## 3
          Red-tailed 381.0
                                990
## 4
            Cooper's 265.0
                                470
## 5
       Sharp-shinned 205.0
                                170
## 6
          Red-tailed 412.0
                               1090
## 7
          Red-tailed 370.0
                                960
## 8
          Red-tailed 375.0
                                855
## 9
          Red-tailed 412.0
                               1210
## 10
          Red-tailed 405.0
                               1120
## 11
          Red-tailed 393.0
                               1010
## 12
          Red-tailed 371.0
                               1010
## 13
          Red-tailed 390.0
                               1120
## 14
          Red-tailed 393.0
                                 NA
## 15
          Red-tailed 416.0
                               1170
## 16
          Red-tailed 436.0
                               1390
## 17
          Red-tailed 418.0
                               1150
## 18
          Red-tailed 381.0
                                950
## 19
          Red-tailed 378.0
                                910
## 20
          Red-tailed 396.0
                               1010
## 21
          Red-tailed 399.0
                               1070
## 22
          Red-tailed 416.0
                               1190
## 23
          Red-tailed 415.0
                                101
## 24
          Red-tailed 392.0
                               1330
## 25
          Red-tailed 380.0
                                990
## 26
       Sharp-shinned 173.0
                                100
## 27
          Red-tailed 399.0
                               1100
## 28
          Red-tailed 401.0
                               1190
## 29
       Sharp-shinned 205.0
                                180
## 30
          Red-tailed 427.0
                               1490
## 31
          Red-tailed 395.0
                               1040
## 32
          Red-tailed 362.0
                                820
## 33
          Red-tailed 396.0
                               1030
## 34
          Red-tailed 391.0
                               1300
## 35
          Red-tailed 413.0
                               1500
## 36
          Red-tailed 371.0
                               1080
## 37
          Red-tailed 385.0
                               1320
## 38
          Red-tailed 378.0
                               1490
## 39
          Red-tailed 416.0
                               1500
## 40
       Sharp-shinned 193.0
                                100
## 41
       Sharp-shinned 171.0
                                 88
## 42
            Cooper's 233.0
                                324
## 43
          Red-tailed 384.0
                               1060
## 44
          Red-tailed 382.0
                               1140
## 45
          Red-tailed 390.0
                               1030
## 46
          Red-tailed 390.0
                               1000
## 47
          Red-tailed 393.0
                               1050
## 48
          Red-tailed 378.0
                               1040
## 49
          Red-tailed 398.0
                               1110
## 50
                               1300
          Red-tailed 412.0
## 51
          Red-tailed 400.0
                                980
## 52
          Red-tailed 422.0
                               1120
## 53
       Sharp-shinned 202.0
                                134
## 54
          Red-tailed 394.0
                               1270
## 55
          Red-tailed 369.0
                                880
## 56
            Cooper's 252.0
                                340
```

```
## 57
            Cooper's 240.0
                                340
## 58
                               1255
          Red-tailed 410.0
          Red-tailed 241.0
## 59
                               1320
## 60
          Red-tailed 408.0
                               1320
## 61
          Red-tailed 37.2
                               1180
## 62
          Red-tailed 380.0
                                760
## 63
          Red-tailed 396.0
                               1250
## 64
          Red-tailed 326.0
                                 NA
## 65
       Sharp-shinned 158.0
                                100
## 66
          Red-tailed 416.0
                               1300
## 67
            Cooper's 271.0
                                475
## 68
                                100
       Sharp-shinned 176.0
##
  69
       Sharp-shinned 194.0
                                 NA
## 70
          Red-tailed 390.0
                               1080
## 71
          Red-tailed 391.0
                               1130
## 72
          Red-tailed 387.0
                               1160
## 73
          Red-tailed 420.0
                               1345
## 74
          Red-tailed 435.0
                               1385
## 75
          Red-tailed 400.0
                               1210
## 76
          Red-tailed 398.0
                               1455
## 77
          Red-tailed 395.0
                               1180
## 78
          Red-tailed 410.0
                               1500
## 79
          Red-tailed 369.0
                               1025
## 80
          Red-tailed 372.0
                                930
## 81
          Red-tailed 415.0
                               1360
## 82
          Red-tailed 412.0
                               1255
## 83
       Sharp-shinned 205.0
                                194
##
   84
       Sharp-shinned 204.0
                                159
## 85
                                 90
       Sharp-shinned 170.0
## 86
            Cooper's 233.0
                                340
## 87
          Red-tailed 375.0
                               1065
## 88
          Red-tailed 385.0
                               1125
## 89
          Red-tailed 422.0
                               1340
## 90
                               1050
          Red-tailed 391.0
## 91
          Red-tailed 410.0
                               1210
## 92
          Red-tailed 385.0
                                980
## 93
          Red-tailed 381.0
                               1000
## 94
          Red-tailed 416.0
                               1390
## 95
          Red-tailed 406.0
                               1275
## 96
          Red-tailed 418.0
                               1180
## 97
          Red-tailed 412.0
                               1210
## 98
       Sharp-shinned 208.0
                                168
##
  99
       Sharp-shinned 208.0
                                146
  100 Sharp-shinned 175.0
                                108
## 101 Sharp-shinned 166.0
                                 94
## 102
          Red-tailed 405.0
                               1085
## 103 Sharp-shinned 198.0
                                188
## 104 Sharp-shinned 200.0
                                154
## 105
          Red-tailed 380.0
                                810
## 106
          Red-tailed 381.0
                                905
                                 94
## 107 Sharp-shinned 169.0
## 108
            Cooper's 260.0
                                420
## 109
          Red-tailed 428.0
                               1240
## 110
          Red-tailed 400.0
                                990
```

```
## 111
            Cooper's 265.0
                               365
## 112 Sharp-shinned 177.0
                                 91
## 113
          Red-tailed 381.0
                              1010
## 114
          Red-tailed 403.0
                               980
## 115
          Red-tailed 382.0
                               860
## 116
          Red-tailed 382.0
                               970
## 117
          Red-tailed 399.0
                               980
## 118
          Red-tailed 380.0
                               985
## 119
          Red-tailed 375.0
                               990
## 120
          Red-tailed 420.0
                              1210
## 121 Sharp-shinned 170.0
                                 89
                                 97
## 122
       Sharp-shinned 177.0
## 123
          Red-tailed 375.0
                               990
## 124
          Red-tailed 395.0
                              1170
## 125 Sharp-shinned 170.0
                                 93
## 126
          Red-tailed 406.0
                              1350
## 127
          Red-tailed 414.0
                              1370
## 128 Sharp-shinned 173.0
                               103
## 129 Sharp-shinned 209.0
                               151
## 130 Sharp-shinned 177.0
                               101
## 131
          Red-tailed 388.0
                               985
## 132
          Red-tailed 277.0
                               940
## 133
          Red-tailed 423.0
                              1310
## 134
          Red-tailed 365.0
                              1035
## 135
          Red-tailed 383.0
                               965
## 136
          Red-tailed 391.0
                              1125
## 137
          Red-tailed 389.0
                               975
## 138
          Red-tailed 392.0
                              1140
## 139
          Red-tailed 375.0
                               950
## 140
          Red-tailed 410.0
                              1210
## 141 Sharp-shinned 179.0
                                 95
## 142 Sharp-shinned 172.0
                                 89
                               195
## 143 Sharp-shinned 202.0
                                 93
## 144 Sharp-shinned 174.0
## 145
          Red-tailed 422.0
                              1205
## 146
          Red-tailed 385.0
                              1045
## 147
          Red-tailed 363.0
                              1090
## 148
          Red-tailed 450.0
                              1190
## 149
          Red-tailed 380.0
                               960
## 150
          Red-tailed 385.0
                               955
## 151
          Red-tailed 385.0
                              1110
## 152 Sharp-shinned 171.0
                               100
          Red-tailed 380.0
## 153
                               900
## 154
          Red-tailed 384.0
                              1075
## 155
          Red-tailed 373.0
                               980
## 156
                               940
          Red-tailed 381.0
## 157
          Red-tailed 363.0
                              1070
## 158
          Red-tailed 409.0
                              1120
## 159
          Red-tailed 390.0
                              1060
## 160 Sharp-shinned 204.0
                               168
## 161 Sharp-shinned 197.0
                               211
## 162
          Red-tailed 420.0
                              1125
## 163
          Red-tailed 381.0
                              1100
## 164
          Red-tailed 408.0
                              1360
```

```
## 165
          Red-tailed 388.0
                               995
## 166
                              1095
          Red-tailed 398.0
## 167 Sharp-shinned 209.0
                               196
## 168 Sharp-shinned 209.0
                               176
## 169
          Red-tailed 394.0
                              1075
## 170 Sharp-shinned 204.0
                               180
## 171 Sharp-shinned 204.0
                               164
## 172 Sharp-shinned 209.0
                               158
## 173
          Red-tailed 394.0
                              1140
## 174
          Red-tailed 416.0
                              1240
## 175
          Red-tailed 445.0
                              1465
## 176 Sharp-shinned 209.0
                               169
## 177
          Red-tailed 388.0
                              1105
## 178
          Red-tailed 397.0
                              1010
## 179
          Red-tailed 384.0
                              1075
## 180
          Red-tailed 379.0
                              1060
## 181
          Red-tailed 393.0
                              1015
## 182
          Red-tailed 386.0
                              1100
## 183
          Red-tailed 397.0
                              1010
## 184
          Red-tailed 382.0
                              1000
## 185
          Red-tailed 386.0
                               980
## 186
          Red-tailed 417.0
                              1240
## 187
          Red-tailed 403.0
                              1360
## 188
            Cooper's 239.0
                               183
## 189
          Red-tailed 401.0
                              1405
## 190
          Red-tailed 377.0
                              1055
## 191
          Red-tailed 432.0
                              1670
## 192
          Red-tailed 390.0
                              1250
## 193
                              1030
          Red-tailed 381.0
## 194
          Red-tailed 403.0
                              1040
## 195 Sharp-shinned 213.0
                               190
## 196 Sharp-shinned 172.0
                               105
## 197
          Red-tailed 390.0
                              1090
                               190
## 198 Sharp-shinned 204.0
## 199
          Red-tailed 386.0
                              1050
## 200
          Red-tailed 402.0
                              1110
## 201 Sharp-shinned 201.0
                               206
## 202 Sharp-shinned 202.0
                               195
## 203
          Red-tailed 374.0
                              1010
## 204
          Red-tailed 358.0
                               880
## 205
          Red-tailed 370.0
                              1060
## 206
          Red-tailed 390.0
                               920
## 207
          Red-tailed 398.0
                              1195
## 208
          Red-tailed 360.0
                               890
## 209
          Red-tailed 355.0
                               900
## 210
          Red-tailed 375.0
                              1110
## 211 Sharp-shinned 200.0
                               160
                               105
## 212 Sharp-shinned 179.0
## 213 Sharp-shinned 175.0
                                99
## 214 Sharp-shinned 203.0
                               165
## 215 Sharp-shinned 205.0
                               100
## 216 Sharp-shinned 213.0
                               125
## 217
          Red-tailed 409.0
                              1100
## 218 Sharp-shinned 202.0
                                NA
```

```
## 219 Sharp-shinned 195.0
                               155
## 220
          Red-tailed 415.0
                              1285
## 221
            Cooper's 236.0
                               390
## 222
          Red-tailed 363.0
                               920
## 223
          Red-tailed 381.0
                              1025
## 224
          Red-tailed 350.0
                               940
## 225
          Red-tailed 398.0
                              1240
## 226
          Red-tailed 412.0
                              1160
## 227 Sharp-shinned 203.0
                               150
## 228 Sharp-shinned 201.0
                               130
## 229
          Red-tailed 411.0
                              1240
## 230
          Red-tailed 373.0
                               930
## 231 Sharp-shinned 178.0
                                90
## 232
          Red-tailed 415.0
                              1240
## 233
          Red-tailed 383.0
                              1030
## 234
            Cooper's 223.0
                               550
## 235
          Red-tailed 390.0
                              1250
## 236
          Red-tailed 390.0
                               999
## 237
          Red-tailed 365.0
                              1120
## 238
          Red-tailed 345.0
                              1000
## 239
            Cooper's 273.0
                               530
## 240
          Red-tailed 400.0
                              1040
## 241
          Red-tailed 380.0
                              1150
## 242
          Red-tailed 330.0
                              1000
## 243
          Red-tailed 410.0
                              1360
## 244
          Red-tailed 313.0
                               930
## 245
          Red-tailed 384.0
                               980
## 246
          Red-tailed 409.0
                              1260
## 247
          Red-tailed 390.0
                               900
## 248
          Red-tailed 411.0
                              1300
## 249
            Cooper's 259.0
                               470
## 250
          Red-tailed 380.0
                              1040
## 251
          Red-tailed 370.0
                               950
## 252
          Red-tailed 415.0
                              1320
## 253 Sharp-shinned 215.0
                               180
          Red-tailed 410.0
## 254
                              1280
## 255
          Red-tailed 412.0
                              1310
## 256
          Red-tailed 384.0
                               910
## 257
          Red-tailed 404.0
                              1220
## 258
          Red-tailed 375.0
                               920
## 259
          Red-tailed 410.0
                              1135
## 260
          Red-tailed 384.0
                               940
## 261
          Red-tailed 385.0
                               920
## 262
          Red-tailed 398.0
                              1280
## 263
            Cooper's
                               480
## 264
          Red-tailed 425.0
                              1220
## 265
          Red-tailed 401.0
                              1000
## 266
          Red-tailed 387.0
                              1120
## 267
          Red-tailed 376.0
                               925
## 268 Sharp-shinned 171.0
                                 90
## 269
          Red-tailed 420.0
                              1280
## 270
          Red-tailed 385.0
                               985
## 271
          Red-tailed 405.0
                              1350
## 272
          Red-tailed 350.0
                               730
```

```
## 273
          Red-tailed 388.0
                               890
## 274
                              1020
          Red-tailed 398.0
## 275
          Red-tailed 410.0
                              1000
## 276 Sharp-shinned 202.0
                               150
## 277
       Sharp-shinned 204.0
                               180
## 278
          Red-tailed 382.0
                              1020
## 279
          Red-tailed 111.0
                              1340
## 280
          Red-tailed 396.0
                              1300
## 281
          Red-tailed 363.0
                              1015
## 282
          Red-tailed 360.0
                               900
## 283
          Red-tailed 390.0
                              1000
## 284 Sharp-shinned 195.0
                               150
  285
          Red-tailed 390.0
                              1050
## 286
          Red-tailed 380.0
                               950
## 287
            Cooper's 225.0
                               350
## 288
            Cooper's 247.0
                               375
## 289
          Red-tailed 415.0
                              1175
## 290
          Red-tailed 354.0
                               980
## 291
          Red-tailed 417.0
                              1260
## 292
          Red-tailed 379.0
                              1050
## 293
          Red-tailed 412.0
                              1330
## 294
          Red-tailed 377.0
                               980
## 295
          Red-tailed 372.0
                               920
## 296
          Red-tailed 420.0
                              1540
## 297
          Red-tailed 368.0
                              1060
## 298
          Red-tailed 406.0
                              1420
## 299
            Cooper's 225.0
                               400
  300
          Red-tailed 392.0
                              1142
## 301 Sharp-shinned 191.0
                               157
## 302
       Sharp-shinned 161.0
                                98
## 303
          Red-tailed 365.0
                               813
  304 Sharp-shinned 156.0
                                94
   305 Sharp-shinned 191.0
                               155
  306 Sharp-shinned 198.0
                               181
   307 Sharp-shinned 160.0
                                92
## 308 Sharp-shinned 164.0
                                98
## 309
          Red-tailed 352.0
                              1024
## 310
            Cooper's 265.0
                               464
## 311 Sharp-shinned 163.0
                               100
## 312 Sharp-shinned 193.0
                               186
## 313 Sharp-shinned 190.0
                               164
## 314 Sharp-shinned 165.0
                                98
## 315 Sharp-shinned 162.0
                                NA
## 316 Sharp-shinned 198.0
                               158
## 317
          Red-tailed 370.0
                              1023
## 318
          Red-tailed 230.0
                               340
## 319
            Cooper's 194.0
                               163
## 320 Sharp-shinned 157.0
                                92
## 321 Sharp-shinned 260.0
                               550
## 322
            Cooper's 377.0
                              1119
## 323
                               917
          Red-tailed 364.0
## 324
          Red-tailed 367.0
                               946
## 325
          Red-tailed 233.0
                               337
## 326
            Cooper's 336.0
                               742
```

```
## 327 Sharp-shinned 259.0
                               439
## 328
            Cooper's 199.0
                               187
## 329 Sharp-shinned 165.0
                                92
## 330 Sharp-shinned 370.0
                              1094
##
  331
          Red-tailed 362.0
                               832
## 332
          Red-tailed 368.0
                              1244
## 333
          Red-tailed 368.0
                              1244
## 334
          Red-tailed 367.0
                               974
## 335
          Red-tailed 400.0
                              1289
## 336
          Red-tailed 379.0
                               999
## 337
          Red-tailed 360.0
                               994
## 338
                              1004
          Red-tailed 362.0
##
  339
          Red-tailed 378.0
                              1097
  340 Sharp-shinned 201.0
                               185
## 341 Sharp-shinned 193.0
                               181
## 342
          Red-tailed 372.0
                              1092
## 343
                              1049
          Red-tailed 369.0
## 344
          Red-tailed 354.0
                               998
                              1091
## 345
          Red-tailed 367.0
## 346
          Red-tailed 349.0
                               987
## 347
          Red-tailed 375.0
                              1151
## 348
          Red-tailed 343.0
                               971
## 349
          Red-tailed 360.0
                              1159
## 350
          Red-tailed 395.0
                              1239
## 351
          Red-tailed 400.0
                              1216
## 352 Sharp-shinned 170.0
                               177
## 353
            Cooper's 268.0
                               469
  354
##
          Red-tailed 369.0
                              1039
## 355
          Red-tailed 365.0
                              1079
## 356
          Red-tailed 375.0
                              1159
## 357 Sharp-shinned 230.0
                               162
## 358
            Cooper's 213.0
                               339
## 359
          Red-tailed 382.0
                              1403
                               175
## 360 Sharp-shinned 168.0
## 361
          Red-tailed 478.0
                              1473
##
  362
          Red-tailed 469.0
                              1001
## 363 Sharp-shinned 172.0
                                 98
## 364
          Red-tailed 373.0
                               914
## 365
          Red-tailed 386.0
                              1059
## 366
          Red-tailed 384.0
                              1147
## 367
          Red-tailed 351.0
                               949
## 368 Sharp-shinned 168.0
                               102
          Red-tailed 363.0
   369
                               973
##
  370 Sharp-shinned 199.0
                               196
            Cooper's 235.0
## 371
                               327
## 372
          Red-tailed 374.0
                               1064
## 373
          Red-tailed 369.0
                               749
## 374 Sharp-shinned 202.0
                               201
## 375 Sharp-shinned 202.0
                               180
## 376
          Red-tailed 374.0
                              1213
## 377
          Red-tailed 389.0
                              1200
## 378
          Red-tailed 379.0
                              1040
## 379 Sharp-shinned 170.0
                               113
## 380
          Red-tailed 373.0
                               919
```

```
## 381
          Red-tailed 403.0
                              1487
## 382
          Red-tailed 400.0
                              1254
## 383 Sharp-shinned 205.0
                               156
## 384
          Red-tailed 405.0
                              1189
  385
          Red-tailed 382.0
                               979
## 386 Sharp-shinned 165.0
                                 96
## 387
          Red-tailed 407.0
                              1339
## 388
          Red-tailed 403.0
                              1199
## 389
          Red-tailed 406.0
                              1344
## 390
          Red-tailed 381.0
                               969
## 391
          Red-tailed 395.0
                              1039
## 392
                              1054
          Red-tailed 410.0
##
  393
            Cooper's 243.0
                               334
   394 Sharp-shinned 170.0
                               101
  395 Sharp-shinned 210.0
                               181
## 396
          Red-tailed 425.0
                              1449
## 397
                               985
          Red-tailed 402.0
## 398
          Red-tailed 480.0
                              1598
## 399
          Red-tailed 412.0
                               992
## 400
            Cooper's 260.0
                               569
## 401
          Red-tailed 381.0
                              1354
## 402
          Red-tailed 392.0
                               969
## 403
          Red-tailed 382.0
                              1080
## 404 Sharp-shinned 196.0
                               185
## 405
          Red-tailed 391.0
                              1097
## 406
          Red-tailed 397.0
                              1439
## 407
          Red-tailed 361.0
                                NA
## 408
          Red-tailed 376.0
                              1019
## 409
                              1239
          Red-tailed 410.0
## 410
          Red-tailed 381.0
                              1399
## 411
          Red-tailed 396.0
                              1169
## 412
          Red-tailed 383.0
                              1164
## 413 Sharp-shinned 203.0
                               176
## 414
          Red-tailed 271.0
                                NA
## 415
            Cooper's 255.0
                               429
## 416 Sharp-shinned 196.0
                               160
## 417 Sharp-shinned 168.0
                               105
## 418
          Red-tailed 392.0
                               890
## 419 Sharp-shinned 163.0
                               100
## 420 Sharp-shinned 205.0
                                NA
## 421
          Red-tailed 395.0
                              1215
## 422 Sharp-shinned 163.0
                               100
## 423
          Red-tailed 370.0
                              1030
## 424
            Cooper's 225.0
                               340
## 425 Sharp-shinned 197.0
                               165
## 426
          Red-tailed 376.0
                               985
## 427
                               975
          Red-tailed 372.0
## 428
                              1165
          Red-tailed 391.0
## 429 Sharp-shinned 165.0
                                95
## 430
          Red-tailed 364.0
                              1044
## 431 Sharp-shinned 202.0
                               180
## 432 Sharp-shinned 163.0
                               125
## 433
          Red-tailed 381.0
                              1080
## 434
          Red-tailed 375.0
                               890
```

```
## 435
          Red-tailed 392.0
                              1105
## 436
                              1310
          Red-tailed 398.0
## 437
          Red-tailed 371.0
                               895
## 438
          Red-tailed 367.0
                              1060
## 439 Sharp-shinned 193.0
                               165
## 440
          Red-tailed 375.0
                               960
## 441
          Red-tailed 408.0
                              1270
## 442
            Cooper's 252.0
                               540
## 443
            Cooper's 237.0
                               365
## 444
          Red-tailed 399.0
                              1170
## 445
            Cooper's 268.0
                               590
## 446
                              1225
          Red-tailed 388.0
## 447 Sharp-shinned 200.0
                               175
## 448 Sharp-shinned 192.0
                               170
## 449 Sharp-shinned 167.0
                               100
## 450
          Red-tailed 370.0
                              1060
## 451 Sharp-shinned 162.0
                               110
## 452
          Red-tailed 363.0
                               850
## 453
          Red-tailed 371.0
                              1145
## 454 Sharp-shinned 199.0
                               165
## 455
          Red-tailed 386.0
                              1090
## 456
          Red-tailed 400.0
                              1345
          Red-tailed 390.0
                              1050
## 457
## 458 Sharp-shinned 199.0
                               200
## 459
            Cooper's 254.0
                               505
## 460
          Red-tailed 382.0
                              1040
## 461 Sharp-shinned 196.0
                               155
## 462
          Red-tailed 354.0
                               965
## 463
          Red-tailed 403.0
                              1105
## 464 Sharp-shinned 194.0
                               155
## 465
          Red-tailed 375.0
                              1045
## 466
          Red-tailed 369.0
                               900
## 467
          Red-tailed 368.0
                              1015
## 468
          Red-tailed 393.0
                              1385
## 469
          Red-tailed 370.0
                              1165
## 470
                               905
          Red-tailed 378.0
## 471
          Red-tailed 369.0
                               985
## 472
          Red-tailed 364.0
                               800
## 473
          Red-tailed 387.0
                               995
## 474
          Red-tailed 410.0
                              1460
## 475 Sharp-shinned 194.0
                               165
## 476
          Red-tailed 410.0
                              1585
## 477 Sharp-shinned 194.0
                               185
## 478
          Red-tailed 373.0
                               920
## 479
          Red-tailed 390.0
                              1460
## 480
                              1315
          Red-tailed 404.0
## 481
          Red-tailed 375.0
                              1019
## 482
          Red-tailed 395.0
                              1215
## 483
          Red-tailed 379.0
                              1005
## 484
          Red-tailed 382.0
                              1350
## 485
          Red-tailed 404.0
                              1145
## 486 Sharp-shinned 205.0
                               175
## 487
          Red-tailed 377.0
                              1010
## 488 Sharp-shinned 184.0
                               155
```

```
## 489
          Red-tailed 390.0
                              1120
## 490
          Red-tailed 404.0
                               280
## 491
          Red-tailed 351.0
                               775
## 492
          Red-tailed 394.0
                              1210
## 493 Sharp-shinned 175.0
                               107
## 494
          Red-tailed 372.0
                               909
## 495 Sharp-shinned 169.0
                                 95
## 496
          Red-tailed 376.0
                              1145
## 497
          Red-tailed 399.0
                              1150
## 498
                              1560
          Red-tailed 399.0
## 499
          Red-tailed 357.0
                               940
## 500 Sharp-shinned 195.0
                               145
## 501
          Red-tailed 400.0
                              1089
## 502
          Red-tailed 382.0
                              1194
## 503 Sharp-shinned 161.0
                                 95
## 504
          Red-tailed 390.0
                              1015
## 505
          Red-tailed 379.0
                               975
## 506
          Red-tailed 376.0
                              1035
## 507
          Red-tailed 331.0
                              1055
## 508
          Red-tailed 371.0
                               945
## 509 Sharp-shinned 160.0
                                90
## 510
          Red-tailed 361.0
                               785
## 511
            Cooper's 268.0
                                56
## 512
          Red-tailed 394.0
                              1220
## 513 Sharp-shinned 191.0
                               175
## 514
          Red-tailed 358.0
                              1025
## 515
          Red-tailed 380.0
                              1000
## 516 Sharp-shinned 188.0
                               175
## 517
          Red-tailed 397.0
                              1210
## 518 Sharp-shinned 196.0
                               180
## 519 Sharp-shinned 190.0
                               175
## 520 Sharp-shinned 161.0
                                 90
## 521
            Cooper's 145.0
                               320
## 522
          Red-tailed 392.0
                              1185
## 523 Sharp-shinned 190.0
                                NA
## 524
                               350
            Cooper's 230.0
## 525
          Red-tailed 406.0
                                NA
## 526
          Red-tailed 363.0
                               950
## 527 Sharp-shinned 162.0
                                 85
## 528
          Red-tailed 374.0
                               895
## 529
          Red-tailed 412.0
                              1150
## 530
            Cooper's 272.0
                               565
## 531
          Red-tailed 388.0
                               950
## 532
          Red-tailed 371.0
                               990
## 533
            Cooper's 225.0
                               325
## 534
          Red-tailed 391.0
                              1120
## 535 Sharp-shinned 163.0
                                 90
## 536
                              1120
          Red-tailed 410.0
## 537
          Red-tailed 368.0
                               820
## 538
          Red-tailed 410.0
                              1200
## 539
          Red-tailed 425.0
                              1120
## 540
          Red-tailed 368.0
                              1120
## 541
          Red-tailed 392.0
                              1220
## 542 Sharp-shinned 250.0
                               470
```

```
## 543
          Red-tailed 418.0
                              1310
## 544
          Red-tailed 370.0
                               920
## 545 Sharp-shinned 170.0
                               100
## 546 Sharp-shinned 203.0
                               180
## 547
          Red-tailed 370.0
                               955
## 548
          Red-tailed 408.0
                              1225
## 549
          Red-tailed 393.0
                              1400
## 550
          Red-tailed 365.0
                              1025
## 551
          Red-tailed 405.0
                              1140
## 552
          Red-tailed 381.0
                              1080
## 553
          Red-tailed 390.0
                              1135
## 554
          Red-tailed 384.0
                              1430
## 555
          Red-tailed 390.0
                              1050
## 556
          Red-tailed 381.0
                               920
## 557 Sharp-shinned 194.0
                               175
## 558
          Red-tailed 371.0
                               905
## 559
          Red-tailed 356.0
                               915
## 560
          Red-tailed 377.0
                               910
## 561
          Red-tailed 362.0
                               950
## 562 Sharp-shinned 165.0
                               105
## 563 Sharp-shinned 155.0
                               100
## 564
          Red-tailed 400.0
                              1225
## 565 Sharp-shinned 161.0
                               105
## 566
          Red-tailed 387.0
                              1090
                              1350
## 567
          Red-tailed 409.0
## 568 Sharp-shinned 195.0
                               200
## 569 Sharp-shinned 196.0
                               200
## 570 Sharp-shinned 169.0
                               100
## 571
          Red-tailed 385.0
                               940
## 572
          Red-tailed 377.0
                               920
## 573
          Red-tailed 400.0
                              1190
## 574 Sharp-shinned 164.0
                                 90
## 575 Sharp-shinned 165.0
                                 95
## 576
          Red-tailed 363.0
                               915
## 577
            Cooper's 266.0
                               535
## 578
          Red-tailed 403.0
                              1310
## 579
          Red-tailed 375.0
                               950
## 580 Sharp-shinned 166.0
                               120
## 581
          Red-tailed 373.0
                               975
## 582
          Red-tailed 373.0
                               990
## 583 Sharp-shinned 170.0
                               110
## 584
          Red-tailed 373.0
                              1270
## 585
          Red-tailed 354.0
                               960
## 586
                              1290
          Red-tailed 394.0
## 587 Sharp-shinned 165.0
                                 95
## 588
            Cooper's 261.0
                               505
## 589
          Red-tailed 377.0
                              1045
## 590
                              1090
          Red-tailed 364.0
          Red-tailed 385.0
## 591
                              1180
## 592
          Red-tailed 390.0
                              1195
## 593
          Red-tailed 383.0
                              1055
## 594
          Red-tailed 365.0
                              1025
## 595 Sharp-shinned 201.0
                               175
## 596
          Red-tailed 376.0
                               940
```

```
## 597
          Red-tailed 402.0
                              1335
## 598
          Red-tailed 394.0
                              1085
            Cooper's 223.0
## 599
                               375
## 600
          Red-tailed 378.0
                              1090
## 601
          Red-tailed 369.0
                               995
## 602
          Red-tailed 378.0
                              1090
## 603
          Red-tailed 262.0
                              1020
## 604 Sharp-shinned 165.0
                               100
## 605
          Red-tailed 379.0
                              1000
## 606
          Red-tailed 380.0
                              1085
## 607
          Red-tailed 380.0
                              1045
## 608
          Red-tailed 393.0
                              1060
## 609
          Red-tailed 389.0
                              1135
## 610
          Red-tailed 395.0
                              1210
## 611 Sharp-shinned 207.0
                               185
## 612
          Red-tailed 367.0
                               965
## 613
                               905
          Red-tailed 368.0
## 614
          Red-tailed 377.0
                              1125
          Red-tailed 374.0
## 615
                               950
## 616
          Red-tailed 385.0
                              1195
## 617
          Red-tailed 363.0
                              1005
## 618 Sharp-shinned 164.0
                                95
## 619 Sharp-shinned 199.0
                               175
## 620
          Red-tailed 378.0
                               995
## 621 Sharp-shinned 200.0
                               165
## 622
          Red-tailed 405.0
                              1170
## 623 Sharp-shinned 194.0
                               145
                               175
## 624 Sharp-shinned 190.0
## 625 Sharp-shinned 195.0
                               170
## 626 Sharp-shinned 161.0
                               105
## 627 Sharp-shinned 168.0
                                95
## 628 Sharp-shinned 160.0
                                95
## 629
          Red-tailed 392.0
                              1085
## 630 Sharp-shinned 201.0
                               165
## 631
          Red-tailed 395.0
                              1115
## 632
          Red-tailed 390.0
                              1195
## 633 Sharp-shinned 161.0
                               100
## 634
            Cooper's 230.0
                               365
## 635 Sharp-shinned 192.0
                               170
## 636 Sharp-shinned 156.0
                                90
  637 Sharp-shinned 158.0
                                95
  638 Sharp-shinned 167.0
                               115
## 639 Sharp-shinned 199.0
                               155
## 640
            Cooper's 320.0
                               229
## 641
          Red-tailed 389.0
                              1165
## 642
            Cooper's 227.0
                               155
## 643
            Cooper's 230.0
                               340
## 644 Sharp-shinned 199.0
                               180
## 645
          Red-tailed 378.0
                               235
## 646 Sharp-shinned 164.0
                               105
## 647
          Red-tailed 385.0
                              1085
## 648
          Red-tailed 369.0
                               970
## 649
          Red-tailed 378.0
                              1030
## 650
          Red-tailed 373.0
                              1015
```

```
## 651
          Red-tailed 419.0
                              1030
## 652
          Red-tailed 414.0
                              1215
## 653 Sharp-shinned 203.0
                               185
## 654 Sharp-shinned 162.0
                                95
  655 Sharp-shinned 165.0
                                95
   656 Sharp-shinned 161.0
                                95
## 657 Sharp-shinned 161.0
                               110
          Red-tailed 404.0
## 658
                              1170
## 659 Sharp-shinned 165.0
                               105
                               175
## 660 Sharp-shinned 199.0
  661 Sharp-shinned 171.0
                                95
  662 Sharp-shinned 213.0
                               190
   663 Sharp-shinned 169.0
                               105
## 664
          Red-tailed 379.0
                              1435
## 665
          Red-tailed 407.0
                              1235
## 666
          Red-tailed 376.0
                              1055
## 667
          Red-tailed 380.0
                              1105
## 668 Sharp-shinned 172.0
                               120
## 669
          Red-tailed 408.0
                              1390
## 670 Sharp-shinned 167.0
                               105
## 671
          Red-tailed 374.0
                              1080
## 672
          Red-tailed 376.0
                               910
## 673
            Cooper's 258.0
                               490
## 674
          Red-tailed 370.0
                               845
## 675 Sharp-shinned 202.0
                               175
## 676
          Red-tailed 363.0
                               865
## 677 Sharp-shinned 200.0
                               180
## 678
          Red-tailed 372.0
                              2030
## 679
          Red-tailed 378.0
                               960
## 680
            Cooper's 254.0
                               515
          Red-tailed 361.0
## 681
                               970
## 682
          Red-tailed 389.0
                              1265
## 683
          Red-tailed 404.0
                              1300
                               170
## 684 Sharp-shinned 195.0
   685
       Sharp-shinned 203.0
                               191
            Cooper's 253.0
                               540
##
  686
  687 Sharp-shinned 199.0
                               185
## 688 Sharp-shinned 194.0
                               165
  689 Sharp-shinned 196.0
                               205
## 690 Sharp-shinned 168.0
                                95
## 691
          Red-tailed 395.0
                              1170
## 692
          Red-tailed 368.0
                              1075
## 693 Sharp-shinned 210.0
                               200
## 694
          Red-tailed 401.0
                               965
## 695
          Red-tailed 355.0
                               785
## 696
          Red-tailed 377.0
                              1095
## 697 Sharp-shinned 197.0
                               160
## 698 Sharp-shinned 171.0
                                95
## 699
          Red-tailed 277.0
                              1500
## 700
          Red-tailed 435.0
                              1595
## 701
          Red-tailed 372.0
                               985
## 702
            Cooper's 260.0
                               565
## 703
          Red-tailed 390.0
                               240
## 704
          Red-tailed 359.0
                               875
```

```
## 705
          Red-tailed 393.0
                              1225
## 706
          Red-tailed 402.0
                              1255
## 707
          Red-tailed 362.0
                               950
## 708
            Cooper's 227.0
                               330
## 709
          Red-tailed 359.0
                               895
## 710 Sharp-shinned 155.0
                               100
            Cooper's 261.0
## 711
                               455
## 712 Sharp-shinned 160.0
                                90
## 713 Sharp-shinned 166.0
                               105
## 714
          Red-tailed 362.0
                              1305
## 715
          Red-tailed 375.0
                               975
## 716
          Red-tailed 388.0
                              1185
## 717
          Red-tailed 392.0
                              1030
## 718
          Red-tailed 412.0
                              1215
## 719
          Red-tailed 371.0
                              1075
## 720
            Cooper's 255.0
                               960
## 721
                               795
          Red-tailed 391.0
## 722 Sharp-shinned 190.0
                               180
## 723
          Red-tailed 385.0
                              1225
## 724 Sharp-shinned 166.0
                               110
## 725 Sharp-shinned 197.0
                               165
## 726
          Red-tailed 370.0
                               920
## 727 Sharp-shinned 190.0
                               170
## 728
          Red-tailed 400.0
                              1315
## 729 Sharp-shinned 162.0
                               100
## 730 Sharp-shinned 194.0
                               155
## 731
          Red-tailed 395.0
                              1155
## 732
          Red-tailed 403.0
                              1160
## 733
          Red-tailed 386.0
                               955
## 734 Sharp-shinned 193.0
                               190
## 735 Sharp-shinned 203.0
                               180
## 736 Sharp-shinned 159.0
                                90
  737 Sharp-shinned 163.0
                               100
                               190
## 738 Sharp-shinned 191.0
## 739
       Sharp-shinned 166.0
                               105
## 740
          Red-tailed 361.0
                              1030
## 741
            Cooper's 260.0
                               565
## 742
            Cooper's 234.0
                               380
## 743
          Red-tailed 377.0
                              1040
## 744
          Red-tailed 361.0
                               910
## 745
          Red-tailed 375.0
                              1005
## 746 Sharp-shinned 193.0
                               180
  747 Sharp-shinned 196.0
                               175
  748 Sharp-shinned 159.0
                               105
## 749 Sharp-shinned 192.0
                               190
## 750
          Red-tailed 401.0
                              1250
## 751 Sharp-shinned 198.0
                               185
## 752
          Red-tailed 407.0
                              1275
## 753 Sharp-shinned 162.0
                               105
  754
            Cooper's 169.0
                               640
  755 Sharp-shinned 168.0
                               100
## 756 Sharp-shinned 168.0
                               105
## 757
            Cooper's 223.0
                               390
## 758
          Red-tailed 375.0
                              1030
```

```
## 759
          Red-tailed 406.0
                              1290
## 760
          Red-tailed 386.0
                              1020
## 761 Sharp-shinned 160.0
                               105
## 762
          Red-tailed 378.0
                              1055
##
  763 Sharp-shinned 193.0
                               185
## 764
          Red-tailed 379.0
                              1010
## 765
          Red-tailed 364.0
                              1015
## 766
          Red-tailed 350.0
                              1115
## 767
          Red-tailed 380.0
                              1320
## 768
          Red-tailed 395.0
                              1180
## 769
          Red-tailed 372.0
                              1145
## 770
          Red-tailed 376.0
                               995
## 771
          Red-tailed 367.0
                              1045
## 772
            Cooper's 230.0
                               340
## 773
          Red-tailed 366.0
                               935
## 774
          Red-tailed 386.0
                              1065
## 775
          Red-tailed 375.0
                              1110
## 776 Sharp-shinned 199.0
                               190
## 777
          Red-tailed 405.0
                              1255
## 778
          Red-tailed 371.0
                              1015
## 779 Sharp-shinned 193.0
                               190
## 780
          Red-tailed 350.0
                               960
## 781 Sharp-shinned 193.0
                               200
## 782
          Red-tailed 368.0
                              1090
## 783 Sharp-shinned 185.0
                               170
## 784
          Red-tailed 372.0
                              1085
## 785
          Red-tailed 396.0
                              1065
## 786
          Red-tailed 365.0
                              1125
## 787
          Red-tailed 374.0
                              1050
## 788
            Cooper's 253.0
                               525
## 789
          Red-tailed 372.0
                               980
## 790
          Red-tailed 376.0
                              1180
## 791 Sharp-shinned 166.0
                               115
## 792
          Red-tailed 369.0
                              1095
## 793
          Red-tailed 391.0
                              1330
## 794 Sharp-shinned 193.0
                               185
## 795
          Red-tailed 397.0
                              1100
## 796
          Red-tailed 366.0
                              1115
## 797
          Red-tailed 385.0
                              1400
## 798
          Red-tailed 400.0
                              1175
## 799 Sharp-shinned 198.0
                               185
## 800
          Red-tailed 365.0
                               970
  801 Sharp-shinned 194.0
                               170
   802 Sharp-shinned 195.0
                               175
## 803 Sharp-shinned 200.0
                               165
## 804
          Red-tailed 384.0
                              1260
## 805 Sharp-shinned 159.0
                               110
  806 Sharp-shinned 161.0
                               100
## 807 Sharp-shinned 191.0
                               190
## 808
            Cooper's 216.0
                               305
##
  809
            Cooper's 256.0
                               510
## 810 Sharp-shinned 161.0
                                 90
## 811
          Red-tailed 396.0
                              1195
## 812
          Red-tailed 391.0
                              1035
```

```
## 813 Sharp-shinned 160.0
                                90
## 814 Sharp-shinned 183.0
                               160
## 815 Sharp-shinned 156.0
                                85
## 816
          Red-tailed 393.0
                              1155
## 817
          Red-tailed 397.0
                              1260
## 818 Sharp-shinned 159.0
                                95
            Cooper's 255.0
                               440
## 819
## 820 Sharp-shinned 197.0
                               175
## 821
            Cooper's 218.0
                               295
## 822 Sharp-shinned 196.0
                               160
## 823
          Red-tailed 384.0
                              1010
## 824
          Red-tailed 385.0
                               990
## 825 Sharp-shinned 181.0
                               150
            Cooper's 225.0
## 826
                               300
## 827
          Red-tailed 379.0
                               945
## 828 Sharp-shinned 201.0
                               188
                                95
## 829
      Sharp-shinned 161.0
## 830
            Cooper's 230.0
                               330
## 831 Sharp-shinned 159.0
                                95
## 832
          Red-tailed 367.0
                               970
## 833
          Red-tailed 396.0
                              1240
## 834 Sharp-shinned 157.0
                               105
## 835 Sharp-shinned 191.0
                               165
## 836 Sharp-shinned 191.0
                               170
## 837 Sharp-shinned 168.0
                               100
## 838
          Red-tailed 428.0
                              1290
## 839
            Cooper's 252.0
                               470
## 840 Sharp-shinned 192.0
                               175
## 841 Sharp-shinned 195.0
                               195
## 842
            Cooper's 233.0
                               335
          Red-tailed 364.0
## 843
                              1150
## 844
          Red-tailed 370.0
                               960
## 845
          Red-tailed 400.0
                              1130
## 846
                               925
          Red-tailed 375.0
## 847
          Red-tailed 398.0
                              1205
## 848
          Red-tailed 385.0
                              1040
## 849 Sharp-shinned 156.0
                               100
## 850
            Cooper's 230.0
                               335
## 851
            Cooper's 220.0
                               335
## 852 Sharp-shinned 163.0
                               105
## 853
          Red-tailed 376.0
                               860
## 854
          Red-tailed 360.0
                               935
## 855
          Red-tailed 372.0
                              1010
## 856
          Red-tailed 400.0
                              1285
## 857 Sharp-shinned 194.0
                               210
## 858
          Red-tailed 370.0
                               830
## 859 Sharp-shinned 160.0
                                95
## 860
          Red-tailed 403.0
                              1350
## 861
          Red-tailed 375.0
                              1010
## 862
          Red-tailed 373.0
                               960
## 863
          Red-tailed 385.0
                              1370
## 864 Sharp-shinned 143.0
                               170
## 865 Sharp-shinned 162.0
                               100
## 866
          Red-tailed 392.0
                              1250
```

```
## 867
          Red-tailed 372.0
                               915
## 868
          Red-tailed 375.0
                               850
## 869
          Red-tailed 415.0
                              1285
                                95
## 870 Sharp-shinned 161.0
## 871
          Red-tailed 380.0
                              1005
## 872
          Red-tailed 411.0
                              1220
## 873 Sharp-shinned 161.0
                                 95
## 874 Sharp-shinned 165.0
                               100
## 875
          Red-tailed 393.0
                              1265
## 876
          Red-tailed 370.0
                              1020
## 877
            Cooper's 230.0
                               360
## 878
          Red-tailed 365.0
                               895
## 879
          Red-tailed 371.0
                              1160
## 880
          Red-tailed 400.0
                              1585
## 881
          Red-tailed 382.0
                              1140
## 882
          Red-tailed 371.0
                              1115
## 883
          Red-tailed 370.0
                              1145
  884 Sharp-shinned 194.0
                               185
## 885
          Red-tailed 347.0
                               990
##
  886
          Red-tailed 363.0
                               945
##
  887 Sharp-shinned 195.0
                               165
## 888
          Red-tailed 392.0
                               1030
## 889
            Cooper's 220.0
                               320
## 890 Sharp-shinned 193.0
                               105
## 891
          Red-tailed 365.0
                               990
## 892
          Red-tailed 387.0
                              1065
## 893
          Red-tailed 400.0
                              1050
## 894
          Red-tailed 362.0
                               840
  895 Sharp-shinned 198.0
                               190
  896 Sharp-shinned 190.0
                               200
  897
       Sharp-shinned 197.0
                               185
## 898
          Red-tailed 370.0
                              1000
##
  899 Sharp-shinned 200.0
                               185
                              1325
## 900
          Red-tailed 360.0
## 901
          Red-tailed 366.0
                               945
## 902
          Red-tailed 402.0
                              1350
## 903
          Red-tailed 366.0
                               805
## 904
          Red-tailed 380.0
                              1525
## 905 Sharp-shinned 190.0
                               175
## 906
          Red-tailed 360.0
                               790
## 907
          Red-tailed 369.0
                               860
## 908
          Red-tailed 199.0
                              1290
```

### 1.3 (Q3)

Q3). Use a combination of the head() and select() functions to print out the top seven rows of the columns "Species", "Wing" and "Weight" of the data frame called "hawksFullName". Do this without modifying the data frame you just created.

```
head(hawksFullName, 7)
```

```
## Species Wing Weight
## 1 Red-tailed 385 920
```

```
## 2
        Red-tailed
                     376
                             930
## 3
        Red-tailed
                     381
                             990
## 4
          Cooper's
                     265
                             470
## 5 Sharp-shinned
                     205
                             170
## 6
        Red-tailed
                     412
                            1090
## 7
        Red-tailed
                     370
                             960
```

#Does it matter what type of join function you use here? In what situations would it make a difference?

- -> Yes, Left join is the suitable join function for this dataframe creation. As Left join keeps all the rows from the lft data frame which is Hawks and addss the matching rows from the right data frame (hawksSpeciesNameCodes). If there is no match NA will be inserted for the missing values from the right data frame.
- -> Left join is idea as we require all the Rows from the Hawks dataframe.

#### 1.4 The mutate function

(Q1). Use the mutate(), select() and arrange() functions to create a new data frame called "hawksWithBMI" which has the same number of rows as the original Hawks data frame but only two columns - one with their Species and one with their "bird BMI". Also, arrange the rows in descending order of "bird BMI".

```
# Create the hawksWithBMI data frame
hawksWithBMI <- Hawks %>%
mutate(bird_BMI = Weight*1000 / (Wing^2)) %>% # Calculate bird BMI
select(Species, bird_BMI) %>% # Select only Species and bird BMI
arrange(desc(bird_BMI)) # Arrange rows in descending order of bird BMI
# View the new dataframe
head(hawksWithBMI, 8)
```

```
##
     Species bird BMI
## 1
          RT 852.69973
## 2
          RT 108.75741
## 3
          RT
              32.57493
## 4
          RT
              22.72688
## 5
          CH
             22.40818
## 6
          RT
              19.54932
## 7
          CH
               15.21998
## 8
          RT
              14.85927
```

##1.5 (Q1) Summarize and group-by functions

(Q1). In combination with the summarize() and the group\_by functions, create a summary table, broken down by Hawk species, which contains the following summary quantities: 1. The number of rows (num\_rows); 2. The average wing span in centimeters (mn\_wing); 3. The median wing span in centimeters (nd\_wing); 4. The trimmed average wing span in centimeters with trim=0.1, i.e., the mean of the numbers after the 10% largest and the 10% smallest values being removed (t\_mn\_wing); 5. The biggest ratio between wing span and tail length (b\_wt\_ratio). Hint: type ?summarize to see a list of useful functions (mean, sum, etc) that can be used to compute the summary quantities.

```
hawksSummary <- Hawks %>%
group_by(Species) %>%
```

```
## # A tibble: 3 x 6
     Species num_rows mn_wing nd_wing t_mn_wing b_wt_ratio
##
     <fct>
                 <int>
                          <dbl>
                                   <dbl>
                                              <dbl>
                                                          <dbl>
## 1 CH
                    70
                           244.
                                     240
                                               243.
                                                           1.67
## 2 RT
                   577
                           383.
                                     384
                                               385.
                                                           3.16
## 3 SS
                   261
                           185.
                                     191
                                               184.
                                                           1.67
```

##1.5 (Q2) Summarize and group-by functions (Q2). Next create a summary table of the following form: Your summary table will show the number of missing values, broken down by species, for the columns Wing, Weight, Culmen, Hallux, Tail, StandardTail, Tarsus, and Crop. You can complete this task by combining the select(), group\_by(), summarize(), across(), everything(), sum() and is.na() functions.

```
missingValuesSummary <- Hawks %>%
group_by(Species) %>%
summarize(across(c(Wing, Weight, Culmen, Hallux, Tail, StandardTail, Tarsus, Crop), ~sum(is.na(.))))
#A formula that sums the number of NA values in each specified column.

# View the summary table
print(missingValuesSummary)
```

```
## # A tibble: 3 x 9
     Species Wing Weight Culmen Hallux Tail StandardTail Tarsus
##
##
              <int>
                     <int>
                             <int>
                                    <int> <int>
                                                         <int>
                                                                 <int> <int>
## 1 CH
                          0
                                 0
                                         0
                                                                    62
                  1
                                               0
                                                             19
                                                                           21
## 2 RT
                  0
                          5
                                 4
                                         3
                                               0
                                                           250
                                                                   538
                                                                          254
## 3 SS
                  0
                          5
                                 3
                                         3
                                               0
                                                             68
                                                                   233
                                                                           68
```

##2. Random experiments, events and sample spaces, and the set theory

##2.1 Random experiments, events and sample spaces (Q1) Firstly, write down the definition of a random experiment, event and sample space. This question aims to help you recall the basic concepts before completing the subsequent tasks.

->

Random experiments - It is a procedure which is either real or imaginary which has a well-defined set of possible outcomes and could (atleast in principle) be repeated arbitarily many times.

Event - An event is a set of possible outcomes of an experiments Sample Spaces - It is a set of all possible outcomes of interest of an experiment.

(Q2) Consider a random experiment of rolling a dice twice. Give an example of what is an event in this random experiment. Also, can you write down the sample space as a set? What is the total number of different events in this experiment? Is the empty set considered as an event?

Event A: Getting at least one "6" when rolling the die twice.

Sample space is all the possible outcomes  $S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,3)\}$ 

2<sup>n</sup> Total number of different events=2<sup>36</sup>

Yes an empty set is considered as an event.

##2.2 Set theory Remember that a set is just a collection of objects. All that matters for the identity of a set is the objects it contains. In particular, the elements within the set are unordered, so for example the set  $\{1, 2, 3\}$  is exactly the same as the set  $\{3, 2, 1\}$ . In addition, since sets are just collections of objects, each object can only be either included or excluded and multiplicities do not change the nature of the set. In particular, the set  $\{1, 2, 2, 2, 3, 3\}$  is exactly the same as the set  $A = \{1, 2, 3\}$ . In general there is no concept of "position" within a set, unlike a vector or matrix.

(Q1) Set operations: Let the sets A, B, C be defined by A :=  $\{1, 2, 3\}$ , B :=  $\{2, 4, 6\}$ , C :=  $\{4, 5, 6\}$ . 1. What are the unions and ? ->  $\{1,2,3,4,6\}$  and  $\{1,2,3,4,5,6\}$ 

- 2. What are the intersections and  $? -> \{2\}$  and  $\{null\}$
- 3. What are the complements and  $? \rightarrow \{1,3\}$  and  $\{1,2,3\}$
- 4. Are and disjoint? Are and disjoint? -> NO and YES
- 5. Are and disjoint? -> YES
- 6. Write down an arbitrary partition of  $\{1,2,3,4,5,6\}$  consisting of two sets. Also, write down another partition of  $\{1,2,3,4,5,6\}$  consisting of three sets. -> P1= $\{\{1,2,3\},\{4,5,6\}\}$  P2= $\{\{1,2\},\{3,4\},\{5,6\}\}$
- (Q2) Complements, subsets and De Morgan's laws 1. Can you give an expression for  $(A^{c)}c$  without using the notion of a complement?

$$A^c=w\in\Omega:w\notin A$$

2. What is  $\hat{c}$ ?

$$(A^c)^c = w \in \Omega : w \not\in Ac = w \in \Omega : w \in A = A$$

3. (Subsets) Show that if , then Let

$$w \in B^c \implies w \notin B$$

$$w \notin A \implies w \in A^c$$

Therefore,

$$B^c \subset A^c$$

4. (De Morgan's laws) Show that ( ) =  $\phantom{a}$  . Let's suppose we have a sequence of events 1, 2, , Can you write out an expression for ( =1 )?

$$\left(\bigcap_{k=1}^{K} A_k\right)^c = \bigcup_{k=1}^{K} A_k^c$$

5. (De Morgan's laws) Show that ( ) =

$$\left(\bigcup_{k=1}^K A_k\right)^c = \bigcap_{k=1}^K A_k^c$$

Let's suppose we have a sequence of events  $1, 2, \dots$  Can you write out an expression for (=1)?

$$\left(\bigcup_{k=1}^K A_k\right)^c = \bigcap_{k=1}^K A_k^c$$

(Q3) Cardinality and the set of all subsets: Suppose that  $=\{1, 2, , \}$  contains elements for some natural number . Here has cardinality K. Let be a set of all subsets of , i.e.,  $:=\{\mid \}$ . Note that here is a set. Give a formula for the cardinality of in terms of .

$$|E| = 2^{K}$$

- (Q4) Disjointness and partitions. Suppose we have a sample space, and events 1, 2, 3, 4 are subsets of . 1. Can you think of a set which is disjoint from every other set? That is, find a set such that = for all .
- $\rightarrow$  The empty set = is disjoint from every other set, since = A B= for all  $\Omega$ .
  - 2. Define events 1:=1, 2=2 1, 3=3 (1 2), 4=4 (1 2 3). Show that 1, 2, 3, 4 form a partition of 1 2 3 4.

->

Disjoint and Union proves

- (Q5) Indicator function. Suppose we have a sample space , and the event  $\,$  is a subset of . Let  $\,$  be the indicator function of  $\,$ .
  - 1. Write down the indicator function of (use in your formula).

$$1_A^c = 1_\Omega - 1_A$$

2. Can you find a set whose indicator function is +?

$$B = A^{c}$$
 
$$1_{B} = 1_{A}^{c}$$
 
$$1_{A}^{c} + 1_{A} = 1_{\Omega}$$

3. Recall that = and = max( , ) = + - for any and . Combining this with the conclusion from Question (Q5) 1, use indicator functions to prove ( ) = (De Morgan's laws).

$$\mathbb{1}_{(A\cap B)^c}=\mathbb{1}_{\Omega}-\mathbb{1}_{A\cap B}=1-\mathbb{1}_A\cdot\mathbb{1}_B$$

Using the union of complements, we have:

$$\mathbb{1}_{A^c \sqcup B^c} = \max(\mathbb{1}_{A^c}, \mathbb{1}_{B^c}) = (1 - \mathbb{1}_A) + (1 - \mathbb{1}_B) - (1 - \mathbb{1}_A)(1 - \mathbb{1}_B)$$

This simplifies to:

$$\mathbb{1}_{A^c \sqcup B^c} = 1 - \mathbb{1}_A \cdot \mathbb{1}_B$$

Hence, we have verified that:

$$(A \cap B)^c = A^c \cup B^c$$

- (Q6) Uncountable infinities (this is an optional extra). This is a challenging optional extra. You may want to return to this question once you have completed all other questions. Show that the set of numbers := [0,1] is uncountably infinite.
- ##3. Probability theory
- ##3.1 Rules of probability
- (Q1) Construct a probability function based on the Rules of probability Consider a sample space  $= \{ , , \}$  and a set of events  $= \{ \}$  (i.e., consists of all subsets of ). Based on the rules of probability, find a probability function :  $\to [0,1]$  that satisfies ( $\{ , \}$ ) = 0.6 and ( $\{ , \}$ ) = 0.5. In your example, you need to define a function called . The function maps each event in to a number. Make sure that your function satisfies the three rules, but you don't need to write down the proof (that it satisfies the three rules).

Let 
$$P(\{a\}) = x$$
,  $P(\{b\}) = y$ ,  $P(\{c\}) = z$ 

$$x+y+z = 1$$
,  $x+y = 0.6$ ,  $y+z = 0.5$ 

- (Q2) Verify that the following probability space satisfies the rules of probability. Consider a setting in which the sample space  $= \{0,1\}$ , and  $= \{ \} = \{,\{0\},\{1\},\{0,1\}\}$ . For a fixed [0,1], define a function  $: \to [0,1]$  by () = 0, ( $\{0\}$ ) = 1 -, ( $\{1\}$ ) =, ( $\{0,1\}$ ) = 1. Show that the probability space (, , ) satisfies the three rules of probability.
- $P(\ ) = 0 \ P(\{0\}) = 1$   $q \ P(\{1\}) = q \ P(\{0,1\}) = 1$
- $P(S) = P() + P(\{0\}) + P(\{1\}) + P(\{0,1\}) = 1$  Hence all the rules of probability satisfied

### 3.2 Deriving new properties from the rules of probability

(Q1) Union of a finite sequence of disjoint events.

for an infinite sequence of pairwise disjoint events 1, 2, . Show that for a finite sequence of disjoint events 1, 2, , for any integer bigger than 1, the below equality holds as a consequence of Rule 3:

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$$

(Q2) Probability of a complement. Prove that if is a sample space, is an event and is is its complement, then we have ( ) = 1 - ( ).

$$P(S^c) = P(\Omega \setminus S) = 1 - P(S)$$

(Q3) The union bound In Rule 3, for pairwise disjoint events 1, 2, , we have Give an example of a probability space and a sequence of sets 1, 2,

$$P(S_1 \cup S_2) < P(S_1) + P(S_2)$$

(Q4) Probability of union and intersection of events. Show that for events and , we have ( ) = ( ) + ( ) - ( )

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$