1.1

$a \rightarrow 0.7 \quad b \rightarrow 0.4 \quad c \rightarrow 0.2 \quad d \rightarrow 0.3$

1.2

$$S = \{ (i,j) \mid 1 \leq i,j \leq 6 \;,\; i,j \in \mathbb{Z} \}$$

$$P(A) = \frac{5}{36} \approx 0.14$$

1.3

$$\frac{1}{2^{n-1}}$$

1.4

1) $F_X(x) = p F_d(x) + (1-p) F_c(x)$

2) $f_X(x) = p f_d(x) + (1-p) f_c(x)$

3) $E[X] = p E[X_d] + (1-p) E[X_c]$

4) $Var(X) = p Var(X_d) + (1-p) Var(X_c)$

$$+ p(1-p)\left(E[X_d] - E[X_c]\right)^2$$

1.5

1.5 $Cov(Z, W) = 2$

1.6 He is wrong.

Actual probability: $1 - \left(\frac{9}{10}\right)^4 \approx 59\%$

1.7

X = no. of errors

$$P(X > 120) = \sum_{i=121}^{1000} {}^{1000}C_i \, (0.1)^i \, (0.9)^{1000-i} \approx 0.015$$

(01)

$n = 1000 \quad p = 0.1 \quad \mu = 100 \quad \sigma^2 = 90$

$X \approx N(100, 90)$

$X \geq 120.5 \longrightarrow Z \geq \frac{120.5 - 100}{\sqrt{90}} \geq 2.16$

$P(z \geq 2.16) \approx 0.015$

1.8

Sandwiches $= 64 + \sqrt{32}(1.645)$

$\longrightarrow 74$

1.9 1) 0 and 0 2) 1 and 1 3) ρ

1.10 $y = X \mid Y$

$E[y] = 1 + \frac{1}{3}(y - 2)$

$Var[y] = 4 - \frac{1}{3} = \frac{11}{3}$

$$y \sim N\left(1 + \frac{y-2}{3}, \frac{11}{3}\right)$$

1.11 $E(Z) = 0 \quad Var(Z) = 9 + 16 - 12 = 13$

$$Z \sim N(0, 13)$$

$$Corr(Z, X) = \frac{3 - 2}{\sqrt{13}} = \frac{1}{\sqrt{13}}$$

1.12 $E[X \mid Y = y \text{ and } Z = z] = \frac{11y}{14} + \frac{5z}{14}$

$$Var[X \mid Y = y \text{ and } Z = z] = \frac{29}{14}$$