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Multi-Robot Path Planning for Sweep Coverage On 2-D Area

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***Abstract*—In this paper, the sweep coverage on a 2-D area is studied by utilizing multi-robot. Based on spatial resolution requirements and sensor sensing capabilities, the target area can be discretized into multiple grids to be covered. The sweep coverage of an area can be transferred into a multi-robot coverage path planning problem (mCPP). In order to solve this problem with less optimization computational cost, a novel partitioning method is firstly proposed to divide the target area into sub-areas, and an improved spanning tree coverage algorithm is then proposed to solve the path planning of a single robot covering its sub-area. Finally, the simulation results are provided to verify the effectiveness of the proposed method.**

***Index Terms*—Complete area coverage, multi-robot, cooperative coverage, path planning**

I. Introduction

Multi-robot is widely used in various fields such as formation collaboration [1], industrial dispatch [2], SLAM [3], rescue [4] and sweep coverage [5]. Among them, sweep coverage is an important problem, whose objective is to achieve the environmental surveillance via planning the patrol paths of multi-robot in the target area. The main approach for sweep coverage follows three steps [6]. The first step is modelling the area to be covered. Specifically, the area is discretized into many grids, and the grids not occupied by obstacles are the target grids for coverage. The second step is to model the robot. Specifically, based on the sensing capabilities and kinematic constraints of the robot sensor, the path movement constraints and the robot's coverage model at specific grid on the path are constructed. The third step is to construct a cost function to describe the total cost of path planning for multi-robot to complete scanning coverage, and a suitable optimization algorithm is designed to solve the optimization problem.

Some relevant works for sweep coverage problem of multi-robot have been reported in [5], [7]–[9]. Specifically, the sweep coverage problem of field sensor network is studied in [5], and a gradient-ascent algorithm is proposed to optimize the path of the field sensors. In [7], an area to be searched is divided into multiple triangular grids, and a distributed control algorithm is proposed to drive a multi-robot team to visit the vertices of the triangles. In [8], an efficient algorithm, called multi-robot spanning tree coverage star, is proposed to solve the path planning problem and achieve the overall minimum time to complete the coverage task. An algorithm based on deep reinforcement learning is proposed to control the robots to perform coverage path planning task in [9]. Note that the aforementioned optimization approaches necessitate reducing path conflicts in multi-robot systems, including addressing coverage overlap and collision avoidance. However, a serious problem of these approaches is that the computational cost of optimization will increases significantly with the size enhancement of the area [10].

For the sweep coverage problem, a straightforward idea is to divide the target area into several sub-areas, then each robot is responsible only for covering its assigned sub-area. By this method, the optimization process is simplified and the overall coverage performance is also guaranteed. Many of the state-of-the-art area partitioning approaches for multi-robot systems are primarily relied on Lloyd's algorithm [11] or Voronoi partition [12]. However, the direct application of these methods to multi-robot coverage path planning problem may lead to sub-optimal results. This is because these methods can achieve the workload balance of robots, such as the area being equally divided, but do not consider the time/cost required for the robots to traverse through the sub-areas. Therefore, a straightforward method of multi-robot for sweep coverage is to follow area division with considering path cost. Some relevant works for coverage problem of multi-robot based on area partition have been reported in [13]–[15]. In these above works, their area partitioning methods rely on the spanning tree coverage algorithm (STC) [16] for path planning. As the STC algorithm treats a group of four grids (in a 2×2 structure) as a single spanning tree unit, the partitioning result based on this algorithm must be a multiple of four grids in a 2×2 structure to ensure that the path generated by the STC algorithm can traverse the entire sub-area. However, in many practical scenarios, due to the distribution of terrain or obstacles, the area may not meet the partitioning conditions

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based on the STC algorithm. Therefore, it is necessary to propose a novel framework to address sweep coverage for various types of areas.

This paper aims at utilizing a group of robots to cover a 2-D area. To reduce the computational optimization cost, a novel framework is constructed to address this problem. Specifically, the sweep coverage problem is firstly transformed into multi-robot coverage path planning. Secondly, a new partition method is proposed for area partitioning based on robot path exploration. Thirdly, followed by the idea of spanning tree algorithm, an improved path planning algorithm is developed to accomplish the path planning of individual robots.

The organization of the remaining parts of the paper is as follows: Section II introduces the problem statement including area model, the robot model, and the coverage cost function is constructed. Then Section III describes the coverage cost function and optimization algorithm. A simulation example is provided in Section IV to illustrate the effectiveness of the proposed method. At last, the paper is concluded in Section V.

Notations: Let $\| \cdot \|$ denote the Euclidean norm, and $\mathbb{N}^+$ represents a positive integer.

II. Problem Statement

A. Area Model

Assume that a bounded planar area Ω is to be covered, and it is discretized into a finite number $K \in \mathbb{N}^+$ of equal grids, the number of grids is determined by the required spatial resolution. The target area can be expressed as

$$\Omega = \{(x, y) \mid x \in [1, rows], y \in [1, cols]\} \tag{1}$$

where $rows$ and $cols$ are the number of rows and columns, the total number of girds is $K = rows \times cols$.

It is also assumed that some obstacles $\mathcal{O}$ occupying a total of $K^O \in \mathbb{N}^+$ grids are placed in the area Ω. Note that the area grids and the positions of obstacles are known a priori. Thus, denote $\Omega^{\mathcal{O}}$ to be the set of obstacles, expressed as

$$\Omega^{\mathcal{O}} = \{(x, y) \in \Omega : (x, y) \text{ is occupied}\} \tag{2}$$

Since robots cannot traverse obstacles, thus the grids need to be covered is reduced to the difference between Ω and $\Omega^{\mathcal{O}}$. Denote Ω^C to be the set of target coverage grids, then the number of coverage grids is $K^C = K - K^O$, and Ω^C can be expressed as

$$\Omega^C = \Omega \setminus \Omega^{\mathcal{O}} \tag{3}$$

where $\setminus$ represents the subtraction operation of the set. Furthermore, denote $\mathbf{g}_k = [x_k \ y_k]^{\mathrm{T}}$ to be the k^{th} coverage grid $(k = 1, \cdots, K^C)$ in the area. The definition of two adjacent grids is given as follows:

Definition 1. *For any two grids $\mathbf{g}_i$ and $\mathbf{g}_j$ in the area Ω, the two grids are considered adjacent if the condition is satisfied*

$$\| x_i - x_j \| + \| y_i - y_j \| = 1 \tag{4}$$

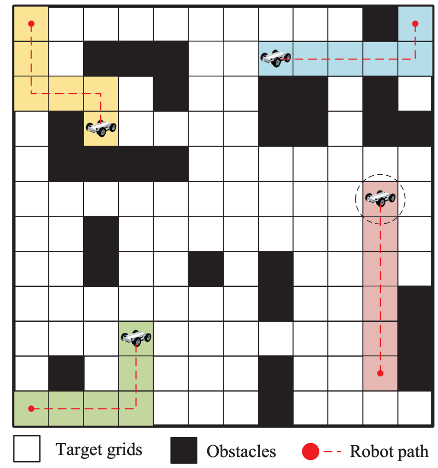


Fig. 1. Overview of sweep coverage for a 2-D area.

B. Robot Model

We assume that the robot is only located at the grid center and when the robot comes to the center of the grid, it denotes the current grid is covered in this paper, it means that the robot only covers the grid it currently occupies at each instant. It is also assumed that the robot can know its position and the time-stamp inside Ω, and it can travel from the current grid to any adjacent unblocked grid. Therefore, the robot path can be defined as follows:

Definition 2. *Denote $\mathbf{L}$ to be a valid robot path as a specific sequence of grids with a length of Q, expressed as*

$$\mathbf{L} = \{(x_1, y_1), \cdots, (x_Q, y_Q)\} \tag{5}$$

where the following constraints are hold
$(x_q, y_q) \in \Omega^C, \forall q = \{1, \ldots, Q\}$
$\| x_q - x_{q+1} \| + \| y_q - y_{q+1} \| \leq 1, \forall q = \{1, \ldots, Q-1\}$

C. Coverage Cost Function

Given a group of $N \in \mathbb{N}^+$ robots $\mathbf{R} = \{r_1, \cdots, r_N\}$ and a 2-D area Ω with the prescribed models. The sweep coverage problem can be formulated as optimizing the robots' paths $\{\mathbf{L}_1, \cdots, \mathbf{L}_N\}$ to traverse all the grids that need to be covered Ω^C. An example of sweep coverage scene is given as shown in Fig. 1.

The cost function of this problem is constructed as the time cost for the robots to complete the coverage task. Assumed that the robot move a grid per unit time, thus the cost function can be formulated as

$$\begin{aligned} &\underset{\mathbf{L}_1, \cdots, \mathbf{L}_N}{\arg\min}\, \mathcal{H} = \max\{\mid \mathbf{L}_1 \mid, \cdots, \mid \mathbf{L}_N \mid\}, \\ &s.t. \mathbf{L}_1 \cup \mathbf{L}_2 \cup \cdots \cup \mathbf{L}_N \subseteq \Omega^C. \end{aligned} \tag{6}$$

where $\mid \mathbf{L}_i \mid$ is the length of the path $\mathbf{L}_i$.

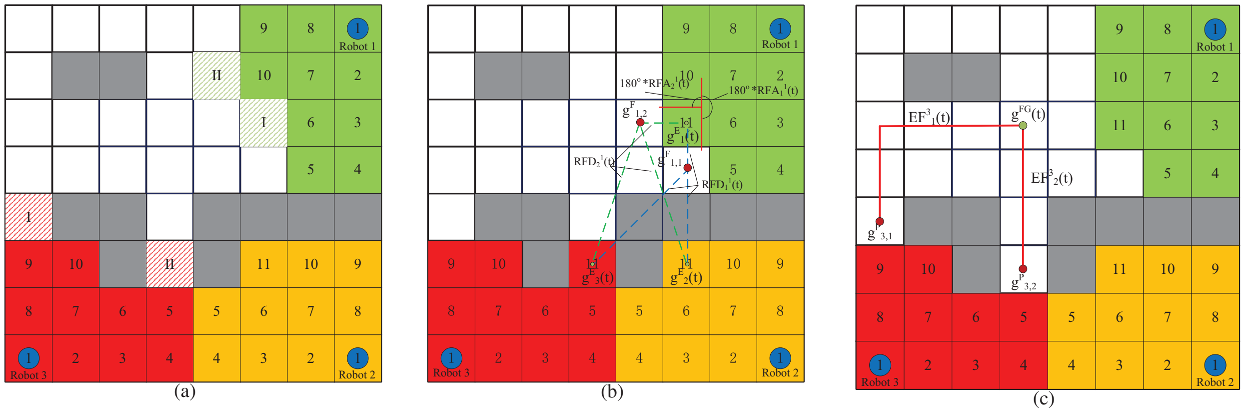


Fig. 2. (a)An example showing three states, FS(green), BS(orange) and PBS(red). (b)The FS strategy. (c)The PBS strategy.

III. Multi-robot Coverage Path Planning Optimization

In this section, a novel optimization is proposed to solve the problem (6). Specifically, a new partition method is proposed to divide the target area into multiple sub-areas for each robot, and an improved spanning tree coverage algorithm is used to solve the path planning of a single robot covering the sub-area.

A. Partition based on robot path exploration

Unlike the traditional partitioning methods based on Euclidean distance, a new partitioning method, called robot path exploration-based partition (RPEP), is proposed to divide the area for each robot with consideration of its path cost. By this approach, the divided sub-areas can be traversed by the robot with a lower-cost path.

The definition of robot path exploration-based partition is given as follows.

Definition 3. *For the area* Ω *and given* N *robots with their configuration* $\mathbf{R} = \{r_1, \cdots, r_N\}$, *the robot path exploration-based partition is the collection* $\mathbf{A} = \{\mathbf{A}_1, \cdots, \mathbf{A}_N\}$. *Denote* V_i *to be the number of grids within the sub-area* $\mathbf{A}_i$ *for the* i^{th} *robot. The sub-area* $\mathbf{A}_i$ *is formulated as*

$$\mathbf{A}_i = \{(x_{i,1}, y_{i,1}), \cdots, (x_{i,V_i}, y_{i,V_i})\},$$
$$\text{s.t. } (x_{i,v}, y_{i,v}) \in \Omega^C, \forall v = \{1, \cdots, V_i\}$$
$$\| x_{i,v} - x_{i,v+1} \| + \| y_{i,v} - y_{i,v+1} \| \leq 1, \forall v = \{1, \ldots, V_i - 1\}$$

The adjacent sub-areas do not share any common grid, i.e. $\mathbf{A}_i \cap \mathbf{A}_j = \emptyset$, for $i \neq j$ $(i, j \in \{1, \cdots, N\})$.

In RPEP, the start point of a robot's path exploration is selected as its initial position. The grids in the area Ω can be categorized into three types: free grids (FG), grids covered by the robots (CG), and grids occupied by obstacles (OG). Furthermore, during the path exploration process, robot may exist in three different states: free state (FS), blocking state (BS), and pseudo-blocking state (PBS), with their definitions given as follows.

Definition 4. *Free State(FS): A robot is in free state if there is a free grid adjacent to the grids where it is currently located.*

Definition 5. *Blocking State(BS): A robot is in blocking state if all the adjacent grids of its current location are grids covered by the robot (CG) or grids occupied by obstacles (OG).*

Definition 6. *Pseudo-Blocking State(PBS): A robot is in a pseudo-blocking state if all the adjacent grids of its current location are occupied by CG and OG, but there are still free grids (FG) adjacent to the grids traversed by the robot's exploration path.*

An example showing the three states is displayed in Fig. 2(a). For different states, robot will adopt corresponding strategies. Additionally, as the partition results based on path exploration may lead to imbalanced partitions, a partition redistribution operation is added to improve this results. The three strategies and the partition redistribution are introduced as follows.

1) Free State Strategy: For FS, at the exploration instant t, the i^{th} robot has $M \in \{1, 2, 3, 4\}$ candidate grids available for exploration. A reward function $\mathbf{RF}$ is constructed to evaluate the performance of each candidate grid, and the robot will select the grid with the highest reward value for its next step.

The reward function is composed of two components:$\mathbf{RFD}$ and $\mathbf{RFA}$, with k_1 and k_2 being the coefficients for $\mathbf{RFD}$ and $\mathbf{RFA}$, respectively. $\mathbf{RFD}$ represents the sum of the distance from the candidate grid to the current multi-robot. The larger $\mathbf{RFD}$ is, the smaller the probability of the robot falling into a blocking state. $\mathbf{RFA}$ represents the ratio of the angle between the candidate grid and the robot's current path to a straight angle. The larger $\mathbf{RFA}$ is, the smaller the need for the robot to make turns. Let t represent the timestamp of the grid exploration phase. Denote $G_i^F(t) = \{g_{i,1}^F, \cdots, g_{i,M}^F\}$ to be the set of candidate grids, and denote $g_i^E(t)$ to be the grid where the i^{th} robot is located at t during path exploration. Thus, the $\mathbf{RF}$ value of m^{th} candidate grid is calculated as follows.

$$\mathbf{RF}_m^i(t) = k_1 \mathbf{RFD}_m^i(t) + k_2 \mathbf{RFA}_m^i(t) \tag{7}$$

$$\mathbf{RFD}_m^i(t) = \sum_{i=1}^{N} \parallel g_{i,m}^F(t) - g_i^E(t) \parallel \tag{8}$$

$$\mathbf{RFA}_m^i(t) = \frac{\angle g_{i,m}^F g_i^E(t) g_i^E(t-1)}{\pi} \tag{9}$$

where $\angle$ represents the angle between the candidate grid, the current grid and the previous grid. An example of the exploration strategy of the FS is shown in Fig. 2(b).

2) Pseudo-Blocking State Strategy: For PBS, at the exploration instant t, the i^{th} robot has $U \in \mathbf{N}^+$ candidate grids available for exploration. An expansion function ($\mathbf{EF}$) is constructed to assess the expansiveness performance of the candidate grids. Specifically, g^{FG} represents the grid that contains the centroid of all remaining free grids, and the $\mathbf{EF}$ value is defined as the path length from the candidate grid to g^{FG}. The smaller the $\mathbf{EF}$ value, the more expansive the candidate grid is. Denote $G_i^P(t) = \{g_{i,1}^P, \cdots, g_{i,U}^P\}$ to be the set of candidate grids, and the $\mathbf{EF}$ of u^{th} candidate grid for the i^{th} robot at instant t is calculated as follows.

$$\mathbf{EF}_u^i(t) = A^*(g^{FG}(t), g_{i,u}^P), \tag{10}$$

where $A^*(\cdot)$ function is used to calculate the path length based on the A^* algorithm [17].

An example of the exploration strategy of the PBS is shown in Fig. 2(c), the red lines is the A^* path length.

3) Blocking State Strategy: At the exploration instant t, if the i^{th} robot is in Blocking state, its path exploration will be paused.

4) Partition Redistribution: Since the cost for robots to complete their tasks is proportional to the number of grids in their sub-area, thus the objective of partition redistribution is equalize the number of grids in each sub-area as much as possible, formulated as

$$V_1 \approx V_2 \approx \cdots \approx V_N \tag{11}$$

During the partition redistribution process, the sub-areas are first sorted in ascending order based on the number of grids, and then grids are allocated from adjacent sub-areas to those with fewer grids. The procedure for partition redistribution is demonstrated by Algorithm 1.

The flowchart of the RPEP is shown in Fig. 3.

Algorithm 1 Partition redistribution algorithm

Input: Original partition results $\mathbf{A}$, N, $\mathbf{R}$, Ω^c
Output: Partition results after redistribution $\mathbf{A}^D$

OE_i serves as the current exploration point sets in A_i
Calculate $f = (\sum_{i=1}^{N} V_i)/N$
Set k_i = min(V_i)
while $|f - k_i| >= 1$ **do**
    Find the neighbors of A_i as A_j
    Calculate $M_i = \lfloor |f - k_i| \rfloor$
    $PE_i = OE_i$
    Set PE_i as exploration point for A_i
    Exploration M_i grids from A_j base on $\mathbf{FS}$ or $\mathbf{PBS}$
    Update A and OE_i
    Set k_i = min(V_i)
end while
Set $A^D = A$
return $\mathbf{A}^D$

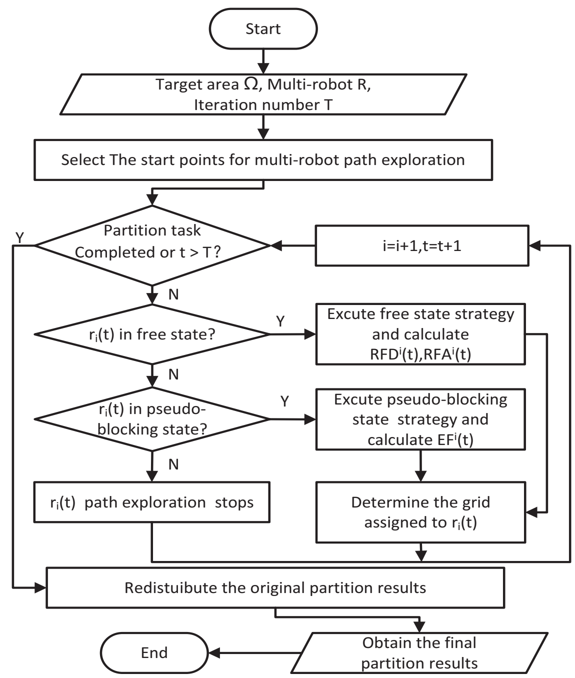
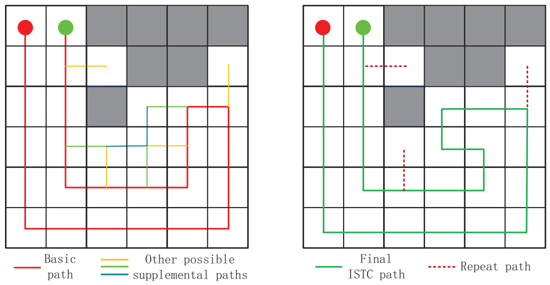


Fig. 3. The flowchart of robot path exploration-based partition (RPEP).

B. Single-robot coverage path planning

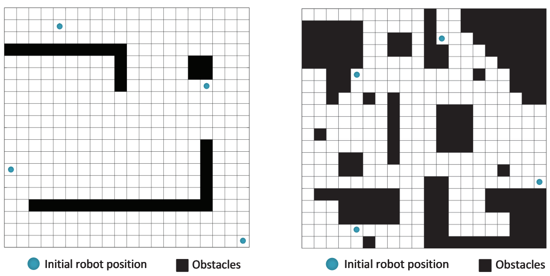
The Spanning Tree Coverage algorithm (STC) is widely used to solve the path planning of single robot for sweep coverage. The standard STC algorithm requires treating four grids (a 2×2 structure) as a large unit for constructing the spanning tree, meaning that the planned path can only guarantee traversal of a multiple of four grids. However, the target area and the sub-areas may not meet this criterion. The STC algorithm may possess no good performance in the coverage problem. Therefore, an improved spanning tree coverage (ISTC) algorithm is proposed in this paper to solve the path planning of each robot within the sub-area. An additional operation is incorporated to optimize the path planning of individual robots, enabling ISTC to complete the sweep coverage of multi-robot for various target area.

In ISTC, we also first treat the 2×2 structure grids as a large unit. Secondly, the basic path of single robot to traverse the large units in the sub-area is obtained based on the STC algorithm. Thirdly, find the uncovered grids in the sub-area, and some possible supplemental paths are generated from the nearest grid on the path to the uncovered grids. Finally, record all possible path sequences under the condition that the number



(a) (b)

Fig. 4. Path planning based on ISTC.



(a) (b)

Fig. 5. The target areas in simulation. (a) Area a. (b) Area b.

of uncovered grids is zero, a greedy algorithm is used to obtain the shortest supplemental paths. An example showing the basic path and the supplemental paths is displayed in Fig.4, and the supplementary path in the figure is a two-way return path.

It is assumed in this paper that there are N_s possible supplemental paths for each robot. Let $\mathbf{LS}_i = \{\mathbf{LS}_i^1, \cdots, \mathbf{LS}_i^{N_s}\}$ denote the set of all N_s combinations of supplemental paths for the i^{th} robot. Denote $\mathbf{LB}_i$ to be the basic path of the i^{th} robot by using the STC algorithm.

Consequently, the i^{th} robot path is formulated as

$$\mathbf{L}^i = \mathbf{LB}_i + \mathbf{LS}_i^* \tag{12}$$

where the set of supplemental paths $\mathbf{LS}_i^* \in \mathbf{LS}_i$ is selected such that the cost function defined in (6) is minimum.

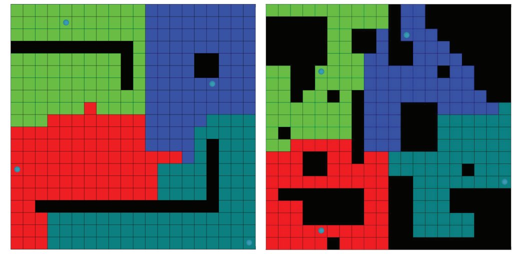
IV. Simulation

In this section, a square region is given as the target area to be completely covered, and the square area is discretized into $K = 1600$ grids, $rows = 40$, $cols = 40$. Some obstacles are placed randomly in the square region, and two areas (Area a and Area b) with different obstacle proportions are set 9.25% and 38.25%. The grids to be covered in Area a and Area b are $K^C = 1452$ and $K^C = 988$, respectively.

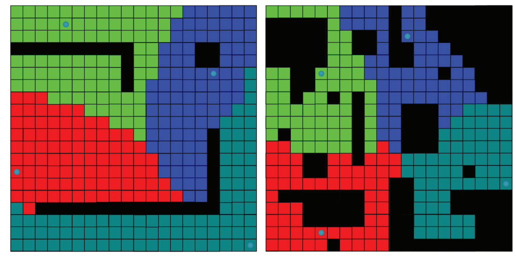
A group of $N = 4$ moveable robots is employed to complete the coverage task. The initial positions of robots are randomly generated, and the robot only covers a specific grid on that grid. Fig. 5 shows the area map and the initial coordination of multi-robot. The parameters in the simulation are set to $k_1 = k_2 = 0.5$, and the possible number of supplementary paths is set $N_s = 8$.

A fair comparison is given between Kapoutsis's method [15] and our method. The following aspects are considered as the performances of the multi-robot coverage path planning:

1) *Length of the Longest Path (LLP):* The length of the longest path among the robot paths generated, which

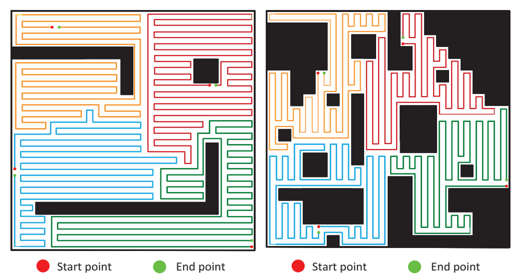


(a) Partition results of Area a in RPEP. (b) Partition results of Area b in RPEP.

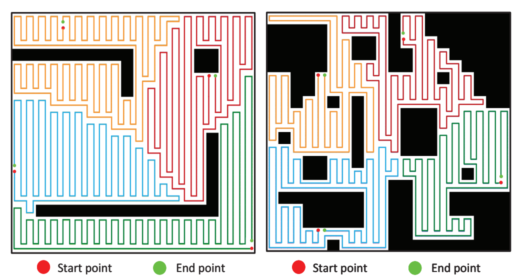


(c) Partition results of Area a in Kapouts's method [15]. (d) Partition results of Area b in Kapouts's method [15].

Fig. 6. Partition results in our method and Kapouts's method [15].



(a) Paths based on our proposed method in Area a. (b) Paths based on our proposed method in Area b.



(c) Paths based on Kapouts's method [15] in Area a. (d) Paths based on Kapouts's method [15] in Area b.

Fig. 7. Comparison of paths in our method and Kapouts's method [15].

TABLE I
ANALYSIS OF SIMULATION DATA

Area	Kapoutsis method		RPEP	
	LLP	TPT	LLP	TPT
a	515	304	469	212
b	364	237	359	176

represents the time to complete the coverage task, as the cost function defined in Equation 6.

2) *Total Path Turns (TPT):* The total number of turns within the generated robot paths, which represents the additional energy cost of the robot paths.

The simulation results are shown in Fig. 6, 7. Fig. 6 shows the multi-robot partitioning results, and Fig. 7 shows the multi-robot path results, from which it can be seen that the sweep coverage task of the areas can be well accomplished by our method. The various performances of the path planning results are shown in Table I.

Our proposed partitioning method generates subregions with a more equal number of grids and a smoother focused partition shape. In addition our method generates paths with shorter longest paths and significantly fewer turns. This highlights that our method performs better in achieving sweep coverage with higher efficiency and lower cost.

V. Conclusion

In this paper, a sweep coverage problem of multi-robot on 2-D area is studied. Based on the gridding of the target area, the area sweep coverage problem is formulated as multi-robot path planning, and a cost function is constructed to characterize the time cost of the robot completing the coverage task. To simplify the optimization process and guarantee the coverage performance, a novel partition method is proposed to partition the target area into several sub-areas. An improved spanning tree algorithm is then proposed to solve the coverage path planning of a single robot in its sub-area. At last, some simulation results are given to illustrate the effectiveness of the proposed method. But till now, the researches on multi-robot coverage search have not formed a set of unified and completed theories. The following aspects can be considered for further investigation:

1) The research on the coverage search algorithm proposed in the article focuses on theory, and there are certain dynamic limitations in the design of the algorithm for robots in specific task environments. Therefore, combining the pose and coverage search algorithm in the robot control system is a direction for subsequent research.

2) The article proposes that the algorithm is based on known mission environment conditions. However, unpredictable emergencies may occur in complex mission scenarios. The solution to emergencies is also one of the directions for further expansion of this research.

3) The multi-agent coverage path planning algorithm proposed in this article focuses on the 2-D environment. In order to be suitable for more task scenarios, further improvements can be made in the direction of 3-D environment coverage.

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