

Optimal Multi-Agent Map Coverage Path Planning Algorithm

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Abstract—Multi-agent path planning is one of the important research directions of multi-agent cooperative control, among which multi-agent covering path planning (mCPP) is a hot topic at present. Nowadays, many researchers have put forward relevant algorithms, but in some specific cases, problems such as low success rate of planning and low computational efficiency have appeared. In this paper, a hybrid algorithm based on the combination of the initial position partitioning algorithm and Two-pass algorithm is proposed for map pre-scanning and map full coverage path planning. Before the coverage path planning, this algorithm firstly scans the complex environment, calculates an evaluation coefficient, then divides the map according to the initial position of multiple agents, and finally carries out path planning in each area to complete the map's full coverage path planning. Compared with the traditional method, the designed algorithm can prescan the map without dividing the region through multiple iterations, which has higher computing efficiency and solves the shortcoming of low success rate of the traditional algorithm under special circumstances. It ensures the security, integrity and robustness of covering path planning without collision of multiple agents.

Keywords—mCpp, path planning, multi-agent, map coverage algorithm

I. INTRODUCTION

With the gradual introduction of mobile robots into the public's vision, path planning technology has been widely studied and discussed, among which the problem of covering path planning (CPP) is one of the focuses of current research. Multi-agent coverage path planning is to plan an optimal path for each agent individually when meeting some specific conditions, and the overall performance is also optimized. Multi-agent coverage path planning is widely used in regional monitoring, security patrol, target search and other fields.

This paper briefly summarizes the current multi-agent coverage path planning techniques, and introduces the algorithm which divides areas algorithm for optimal multi-robot coverage path planning (DARP). Aiming at the problems of low planning success rate and long operation time in the traditional DARP algorithm, a hybrid algorithm combining DARP algorithm and Two-pass algorithm is proposed. Firstly, the map is preliminarily evaluated by Two-pass algorithm, and the threshold value applicable to the initial location and map of multiple agents is calculated. Finally, a non-conflicting path is assigned to each agent by DARP algorithm, which can cover the whole map. Through experimental simulation, it is proved

that the improved algorithm is effective and fast, which provides a new idea for coverage path planning of multi-intelligent mobile robots.

II. MULTI-AGENT COVERAGE PATH PLANNING TECHNIQUE

Multi-agent coverage path planning refers to: given the initial positions of multiple agents, a path is assigned to each agent according to obstacles in the map, and all positions in the target environment are covered. According to different situations, the paper tries to satisfy the characteristics of complete coverage area, few repeated paths and high efficiency. But in general, as the coverage increases, the duplicate path also increases. Therefore, it is the focus and difficulty of current research to improve map coverage and reduce repeated paths.

At present, different algorithms are proposed for different optimization objectives at home and abroad. A multi-agent spanning tree coverage (MSTC) algorithm based on approximate unit decomposition is proposed, which ensures the coverage of the planned path and ensures the collision without obstacles, thus improving the robustness and rapidity of the system[1]. Some scholars point out that in map coverage path planning for multiple agents, the path should be well planned for each agent, and each path combined can cover the whole map and make the calculation time the fastest[2]. A collaborative optimal coverage path planning (CCPP) method was proposed, which simultaneously collaboratively optimizes uav path and iteratively optimizes coverage path using particle swarm optimization (PSO) framework on the premise of ensuring safety and detection requirements[3]. A multi-robot system that traverses the known environment was studied[4]. The literature first generates a minimum loop path, then traverses each cell in the map to find the best new initial location for each agent to minimize the maximum distance between them and the initial location. However, this method is equivalent to splitting two tasks, which limits the performance of the algorithm. By dividing the map and assigning each part to each agent, some researchers ensure that all paths can cover the whole map and there is no conflict or backtracking between paths. Some scholars propose a CACP framework in which multiple agents can transform its shape into three different forms to ensure that the map is covered with a predetermined size. Finally, using TSP as a model, evolutionary method, genetic algorithm (GA) and ant colony algorithm (ACO) are adopted to solve the optimization process of full coverage trajectory[5]. The author proposes an coverage algorithm based on ergodic problem, which is based on the representation of

radial basis function of ergodic problem and the solution of the potential field steady-state heat equation appropriately designed, and realizes the centralized feedback control of multi-agent system[6]. In the multi-agent region partitioning problem [7], it all relies on Lloyd's algorithm. These methods are applicable to the mCPP problem, and most of them solve the region partitioning problem of the initial location of agents. But applying these algorithms directly to the mCPP problem may yield locally optimal results. Although the agent's area may be evenly distributed, problems such as backtracking and path conflicts may be excluded. Literature [11] makes intermittent exploration and builds maps of unknown maps, using the multi-robot coverage strategy based on Voronoi partition, using Manhattan distance measurement to solve the coverage problem, using the frontier-based exploration strategy to carry out exploration mapping. A path planning algorithm based on the initial position of multiple agents to divide the region was proposed[8]. Within the known terrain range, the algorithm subdivides the region according to the number and initial position of agents. The minimum spanning tree algorithm is then used to generate the exact path for each agent dedicated region. The algorithm guarantees that each path can be divided equally without conflict, and each path can be combined to cover the whole map through constraints. This method provides a new way of thinking for coverage path planning, and experiments show that the algorithm is effective and fast.

Based on the introduction and analysis of the traditional DARP algorithm and its disadvantages, such as low success rate and low operational efficiency in special maps and initial locations, a hybrid algorithm combined with Two-pass algorithm is proposed in this paper. The improved algorithm first evaluates the map through Two-pass algorithm, sets a threshold that conforms to the specific environment, and then uses DARP algorithm to divide the complex environment. After successful partition, each region is covered by the minimum spanning tree algorithm, such as Kruskal algorithm and Prim algorithm, to generate the closed path of the region. Through experimental simulation analysis, the optimal DARP algorithm can improve the success rate of path planning and significantly improve the computing efficiency.

III. DIVIDE AREAS BASED ON ROBOTS INITIAL POSITIONS

Traditional multi-agent coverage path planning can be regarded as an optimization problem:

$$\begin{aligned} & \underset{L}{\text{minimize}} \max_{i \in \{1, \dots, n_r\}} |L_i| \\ & \text{subject to } L_1 \cup L_2 \cup \dots \cup L_{n_r} \supseteq \mathcal{L} \end{aligned} \quad (1)$$

where L_1, L_2, \dots, L_{n_r} denotes the path set of the n_r agents; \mathcal{L} denotes a walkable position on a map. There are different requirements for different situations. The path set in this paper needs to meet the following five conditions:

1. $L_i \cap L_j = \emptyset, \forall i, j \in 1, \dots, n_r, i \neq j$;
2. $L_1 \cup L_2 \cup \dots \cup L_{n_r} = \mathcal{L}$;
3. $|L_1| \approx |L_2| \dots \approx |L_{n_r}|$;

4. L_i is connected $\forall i \in 1, \dots, n_r$;

5. $\chi_i(t_0) \in L_i$

The first condition states that any two paths must not intersect, that is, duplicate paths cannot occur. This ensures that the sum of path lengths of multiple agents is the shortest. The second condition means that all planned paths can cover all walking points in the map, that is, there will be no untraversed grid points. The third condition indicates that the planned multi-agent path should be kept equal as far as possible, so that the energy of each agent can be fully utilized and the resources can be evenly distributed. The fourth condition indicates that the path set of each agent L_i should be compact and form a whole without jumping lattice points. The fifth condition indicates that the initial location of each agent $\chi_i(t_0)$ should be included in its corresponding path L_i . As long as any algorithm can meet the above five conditions, it can combine with STC algorithm to calculate the corresponding path.

DARP algorithm is a kind of coverage path planning algorithm which can solve the above problems and keep the best state. The algorithm divides map into n_r subregions. For each agent i , there is an assessment matrix E_i that describes the distance from each walkable point to the initial position $\chi_i(t_0)$ of the agent i (such as Euclidean distance). Each time, the optimal solution is obtained by iterative allocation matrix A .

$$A_{x,y} = \underset{i \in \{1, \dots, n_r\}}{\text{argmin}} E_{i|x,y}, \forall (x, y) \in \mathcal{L} \quad (2)$$

For the above optimization problem, some scholars proved that there is a fair partition for any polygon and any number of partitions[9]. This literature is an extension of the above problem, with additional conditions attached, so the above optimization problem cannot always have an optimal solution, and it depends heavily on the initial position of each agent.

The partition path of each agent can be calculated directly from the allocation matrix A :

$$\begin{aligned} L_i &= \{(x, y) \in \mathcal{L} : A(x, y) = i\} \\ &\forall i \in \{1, \dots, n_r\} \end{aligned} \quad (3)$$

In addition, the length of each partition path is the number of elements owned by the set L_i .

$$k_i = |L_i|, \forall i \in \{1, \dots, n_r\} \quad (4)$$

Through the above allocation strategy, no matter what the assessment matrix E_i of an agent is, the obtained planning path can always meet the first, the second and the fifth conditions. In short, the DARP algorithm is an iterative process in which each iteration modifies the evaluation matrix of each agent until the third and the fourth conditions are met to exit the iteration cycle.

A. Distribute Map Space Equally

At first, an agent's evaluation matrix E_i consisted only of the distance from each point to its initial position.

$$E_{i|x,y} = d(\chi_i(t_0), [x, y]^T), \forall i \in \{1, \dots, n_r\} \quad (5)$$

where $d(\cdot)$ denotes the chosen Euclidean distance function. Euclidean distance can be expressed as (6).

$$h(n) = \sqrt{(x_i - x)^2 + (y_i - y)^2} \quad (6)$$

where (x_i, y_i) denotes agent initial position coordinates; (x, y) denotes the walkable points position coordinates on a map.

The DARP algorithm modifies the evaluation matrix E_i each iteration by introducing correction factors m_i .

$$E_i = m_i E_i \quad (7)$$

Therefore, the third condition can be equivalent to minimization:

$$J_i = \frac{1}{2} (k_i - f)^2 \quad (8)$$

Where k_i denotes the length of an agent's path; f denotes ideal global average path length: $f = \frac{|L|}{n_r}$.

The correction factor m_i was updated by using the standard gradient descent method:

$$\begin{aligned} m_i &= m_i - \eta \frac{\partial J_i}{\partial m_i} = m_i - \eta (k_i - f) \frac{\partial k_i}{\partial m_i} \\ &= m_i + c (k_i - f) \end{aligned} \quad (9)$$

where c denotes positive variable parameters.

It can be seen from Formula (9) that the change of k_i with respect to m_i is always negative. When k_i bigger than f , m_i gets bigger, and then the elements of the evaluation matrix E_i get bigger, and then k_i gets smaller. After iteration cycle optimization, exit iteration cycle when the third condition is met.

B. Build Continuous Areas

While the above procedure guarantees that each agent can be assigned a path of equal length, there is no guarantee that each agent's path will be continuous during iteration. In order to solve this problem, the DARP algorithm corrects the discontinuity of the allocated path by introducing the reward and punishment matrix C_i .

$$\begin{aligned} C_{i|x,y} &= \min \left(\left\| [x, y] - r \right\| \right) \\ &- \min \left(\left\| [x, y] - q \right\| \right) \forall r \in R_i, q \in Q_i \end{aligned} \quad (10)$$

where R_i denotes the region assigned to the agent i and in which its initial location $\chi_i(t_0)$ is. Q_i denotes the region assigned to the agent i but in which its initial location $\chi_i(t_0)$ is not. In other words, C_i is constructed by rewarding the region around the subregion where the agent i was initially located and punishing other regions that are not connected to that subregion. In this way, a continuous region is built step by step. If the region assigned to an agent belongs to a continuous region, C_i is set to a matrix with all 1 elements.

C. Shortcoming

From the above introduction, it can be seen that in the iteration process of DARP algorithm, suboptimal results will be completely discarded and will not be output as results. In addition, the performance of DARP algorithm is unlikely to be affected by the number of agents and the number of obstacles, but the initial position of agents may lead to the failure of coverage path planning. Ideally, the planned path length for each agent is approximately equal to the global average path length, that is, the number of walkable cells in the map divided by the number of agents. However, in some special cases, the planned path of an agent should not and cannot be equal to the global average path length.

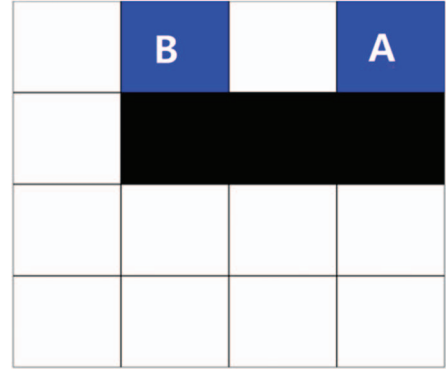


Fig.1: The location of an agent is restricted to a local area

As shown in Fig.1, the black squares represent obstacles, the white squares represent walkable points, and the blue squares represent agents. When agent A is surrounded by agent B and obstacles, the path planned by agent A cannot reach the global average path length, which leads to the failure of path planning.

Therefore, it is very important to calculate the maximum difference of the length of the agent planning path in the map before iterative calculation. In this paper, the Two-pass algorithm is combined with DARP algorithm. First, the Two-pass algorithm is used to evaluate the complex environment before covering path planning, calculate the maximum path length difference that is in line with the current environment and the initial location of the agent, and then DARP algorithm is used to carry out map segmentation. This can not only improve the success rate of the algorithm, but also improve the efficiency of the operation.

IV. TWO-PASS ALGORITHM

In image processing, connected region generally refers to the image region composed of pixels with the same pixel and adjacent position. The connected region analysis is to find out all the connected domains in the image and mark them. At present, connected region analysis is widely used in image recognition and processing, and the processed image is a binarized image.

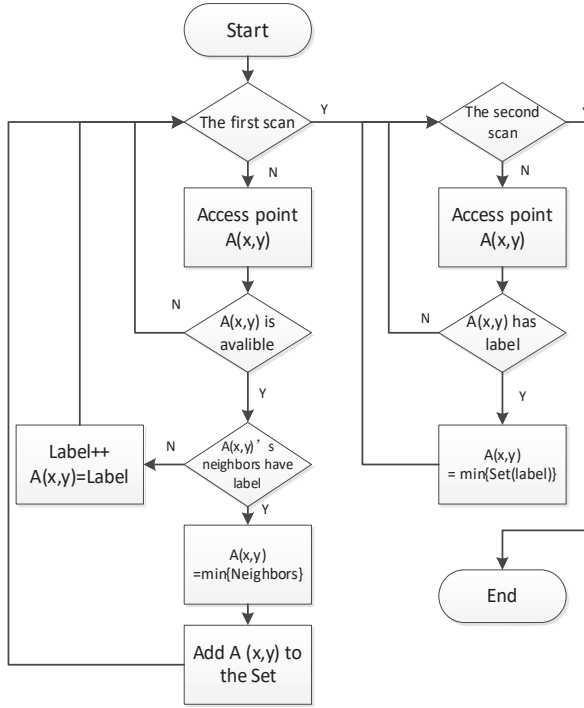


Fig.2: Two-pass algorithm

Two-pass algorithm means that by scanning the image twice, all the connected domains in the image can be found and labeled. The first scan assigns a label to each pixel. In the scanning process, if a pixel has a label in its neighborhood, the label of that pixel is the same as the minimum label of its neighborhood. If there is no label in its neighborhood, a new label is assigned to it. After the first scan, there may be multiple tags in the pixels of the same connected region, so it is necessary to combine these pixels belonging to the same connected domain but with different tags, and record that they belong to the same set. The second scan is to group the pixels in the same set into the same connected domain and assign them to the same label, which is usually the minimum value of the label in the set.

Therefore, the problem of determining whether an agent is surrounded can be simplified as follows: For an agent, other agents can also be regarded as obstacles, and there are several connected areas in the map at this time. If there is only one connected region, it proves that the agent is not surrounded. If there are more than one connected region, the length of the region where the agent's initial position is located is judged. If the length is greater than or equal to the global average path length, the agent is considered not surrounded. If the length is less than the global average path length, the agent is considered to be surrounded. Therefore, the key to solve the problem is to analyze the connected region of the map. In this paper, Two-pass algorithm is used to analyze the connected region.

As shown in Fig.3(a), for agent A, other agents are regarded as obstacles. Fig.3(b) is the label of the map after agent A has been scanned twice. It can be seen that after agent B is regarded as an obstacle, the map is divided into two connected domains. The connected domain length 2 where the agent A is located is less than the global average path length 6,

so it is considered that the agent A belongs to the case of being surrounded. In this case, the third optimization condition of DARP algorithm cannot be satisfied, so the third condition is relaxed to $|L_1| - |L_2| \geq 4$.

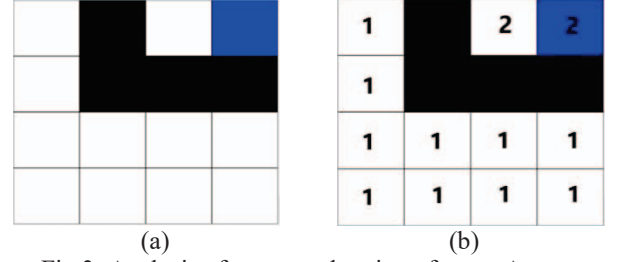


Fig.3: Analysis of connected region of agent A

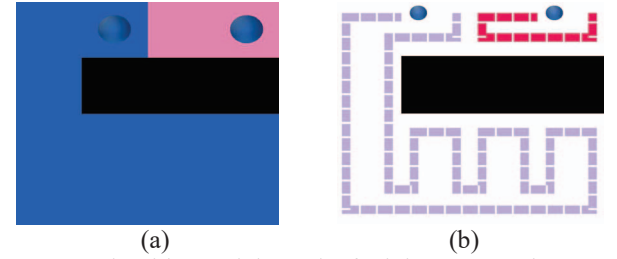


Fig.4: DARP algorithm and the path of minimum spanning tree

By relaxing the third constraint condition, DARP algorithm will not fall into the problem of discarding suboptimal results, which will lead to map partitioning failure. It can be seen from Fig.4(a) that the hybrid algorithm after the fusion of Two-pass algorithm and DARP algorithm can divide the map into blue and pink regions well. Then, Kruskal algorithm is used to construct a minimum spanning tree for the grid points in the divided sub-regions, forming the final closed path. As shown in Fig.4(b), the hybrid algorithm can well handle the problem that the DARP algorithm fails to partition the map and thus leads to the failure of coverage path planning under special circumstances.

V. SIMULATION RESULTS

The coverage path planning algorithm adopted in this paper is simulated on Eclipse. The hardware environment is Intel(R) Core(TM) I5-4210 CPU @1.70ghz processor, 8GB memory, Graphics card of Intel(R) HD Graphics Graphics Family. The software environment is Eclipse Java 2019. In this paper, two experiments are designed to verify the effectiveness and rapidity of the improved DARP algorithm: 1. Comparing the success rate of path planning between THE DARP algorithm and the improved DARP algorithm in maps with different Numbers of agents and obstacles, so as to prove the effectiveness of the improved DARP algorithm; 2. The operation time of DARP algorithm and the improved DARP algorithm under the condition of successful planning is compared to prove that the improved DARP algorithm improves the planning efficiency.

A. Verify Algorithm Success Rate

First of all, this paper creates maps of 10*10, 50*50, and 100*100 sizes, and places different Numbers of obstacles randomly in the map. In this experiment, the success rate of path planning of DARP algorithm and the optimal DARP

algorithm is discussed under the conditions of two agents and eight agents. In each case, 1000 experimental calculations were carried out, and the maximum iteration number of the algorithm in each experiment was 8000 times.

TABLE I. SUCCESS RATE ANALYSIS

Agents Grid Size	DARP		Optimal DARP	
	2	8	2	8
10*10	98%	96.5%	100%	100%
50*50	93.2%	90.1%	100%	99.9%
100*100	85.6%	80.3%	99.7%	99.4%

It can be seen from Table I that the success rate of the traditional DARP algorithm will decrease with the increase of the number of agents and the size of the map. The optimal DARP algorithm has a high success rate in the case of three maps with different sizes and different number of agents. However, with the increase of the map, the amount of computation will also increase, so there will be an operation timeout, which will lead to the failure of coverage path planning. At the same time, it can also be seen that under the environment map of the same size, the increase of the number of agents will also lead to the increase of calculation times, leading to the failure of path planning.

As can be seen from Fig.5, the optimal DARP algorithm can divide the map into several regions according to the initial location of agents, and each agent makes path planning in its own region to ensure that each path will not have conflicts and will not cover some grid points repeatedly. Therefore, it can be concluded that the improved DARP algorithm can obviously solve the problem of path planning failure when an agent is surrounded by other agents or obstacles, and the increase of the number of agents and the increase of map size will reduce the success rate of path planning.

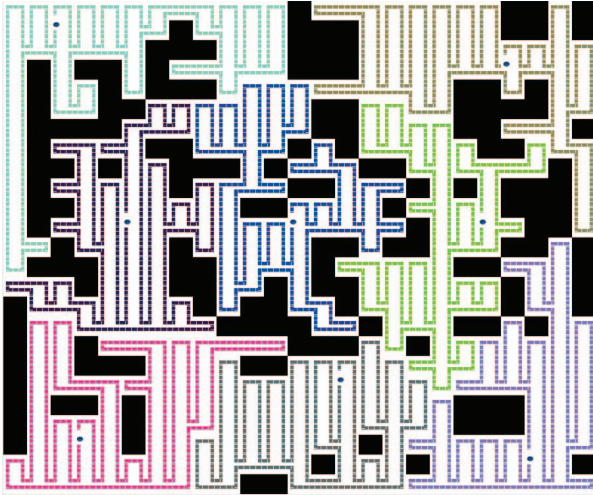


Fig.5: Coverage paths of eight agents under the 50*50 map

B. Verify Computing Efficiency

Experiment 1 shows that the optimal DARP algorithm can successfully calculate the planned path. But in order to better adapt to the complex environment, the algorithm also put forward a higher requirement for the speed of operation. In this

paper, the operation speed of the two algorithms is compared under the condition that the two algorithms can run successfully. In this experiment, 1000 experiments were conducted for each case, and the maximum iteration number of the algorithm in each experiment was 8000 times, and the operation speed was recorded, so as to prove that the optimal DARP algorithm was superior to the traditional DARP algorithm in terms of operational efficiency.

TABLE II. EFFICIENCY ANALYSIS

Agents Grid Size	DARP			Optimal DARP		
	2	4	8	2	4	8
10*10	0.0134s	0.0823s	0.1361s	0.0067s	0.0158s	0.0638s
50*50	0.1204s	0.1824s	1.5422s	0.0738s	0.1067s	0.5997s
100*100	0.2541s	3.1531s	6.6172s	0.1039s	2.1691s	4.3371s

As can be seen from Table II, under the condition of the same size map and the same number of agents, the improved DARP algorithm has a significant improvement in operation time.

As can be seen from Figure 6, when the size of local map and the number of agents increase, the operation time of both algorithms will increase, but the improved DARP algorithm can significantly improve the operation efficiency. In the case of larger map size and more agents, the improved DARP algorithm has better computing efficiency than the traditional DARP algorithm.

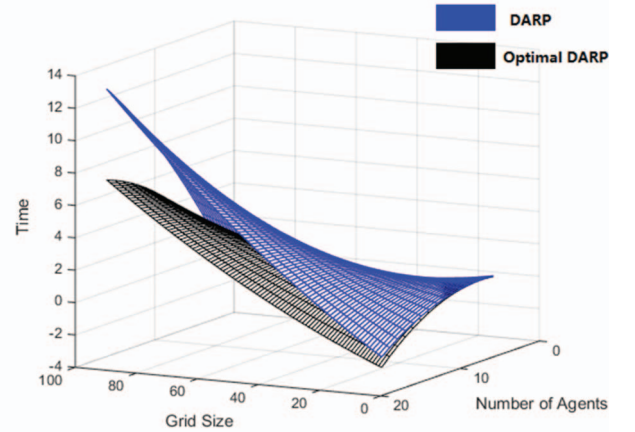


Fig.6: Comparison of operation time

According to the analysis, the traditional DARP algorithm needs to keep cycling to get the threshold suitable for complex maps, which greatly reduces the efficiency of the algorithm. The optimal DARP algorithm only needs to evaluate the map in advance to obtain the threshold value of the map. By introducing Two-pass algorithm, the operation time of covering path planning is greatly reduced.

VI. CONCLUSIONS

This paper briefly introduces the application scenarios and research directions of current coverage path planning, briefly

introduces the commonly used coverage path algorithm, and then introduces the DARP algorithm and its advantages and disadvantages in detail. As the traditional DARP algorithm may discard the optimal solution in the iterative operation process, the success rate of the planned path is low and the operation time is long. To solve this problem, a hybrid algorithm combining DARP algorithm and two-pass algorithm is proposed in this paper. Through the assessment of the complex environment, the threshold value conforming to the complex environment is set in advance. Finally, DARP algorithm is used to segment the map so as to realize the map coverage path planning. By evaluating the map in advance, the improved algorithm can obtain the threshold in line with the map and then carry out the planning operation, which can effectively avoid the occurrence of abandoning the optimal solution. Therefore, the improved algorithm has a better success rate and operational efficiency. Through experimental simulation analysis, it can be found that the improved DARP algorithm can well solve the failure of coverage path planning due to the agent being contained. And through experiments, it can be found that the improved DARP algorithm does not need to calculate the environmental threshold again and again, so its operational efficiency is significantly improved. However, it can also be found that the improved DARP algorithm has no improvement in operation time under certain circumstances due to the addition of new operation process, but the worst operation time is similar to the traditional DARP algorithm. In general, the improved DARP algorithm ensures the success rate of path planning in special environments and improves the operational efficiency to some extent. Therefore, it can be determined that the improved DARP algorithm can better adapt to the research of coverage path planning in complex environments.

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