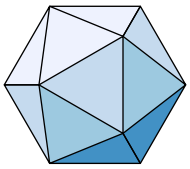
Proofs without words I

Exercises in METAPOST

Toby Thurston

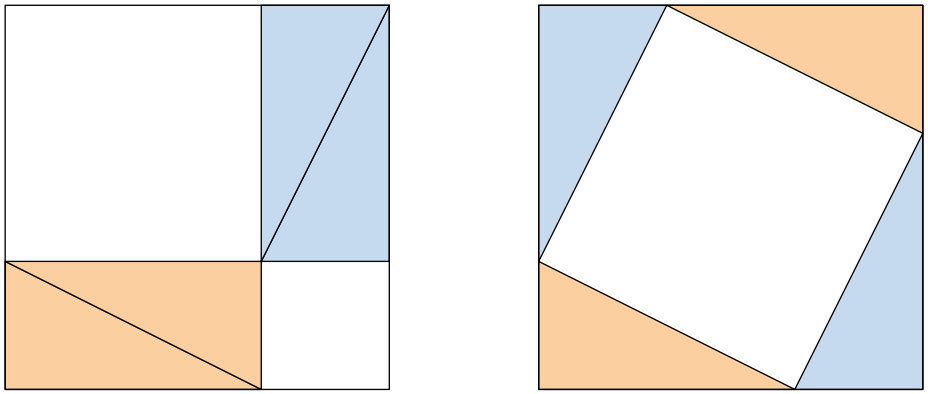
March 2021 —



Contents

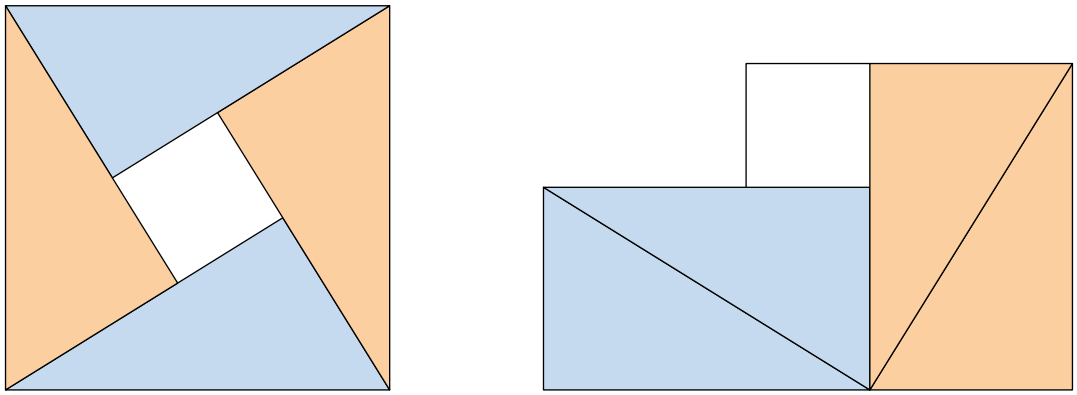
Geometry and Algebra

The Pythagorean theorem I



— adapted from the *Chou pei san ching*

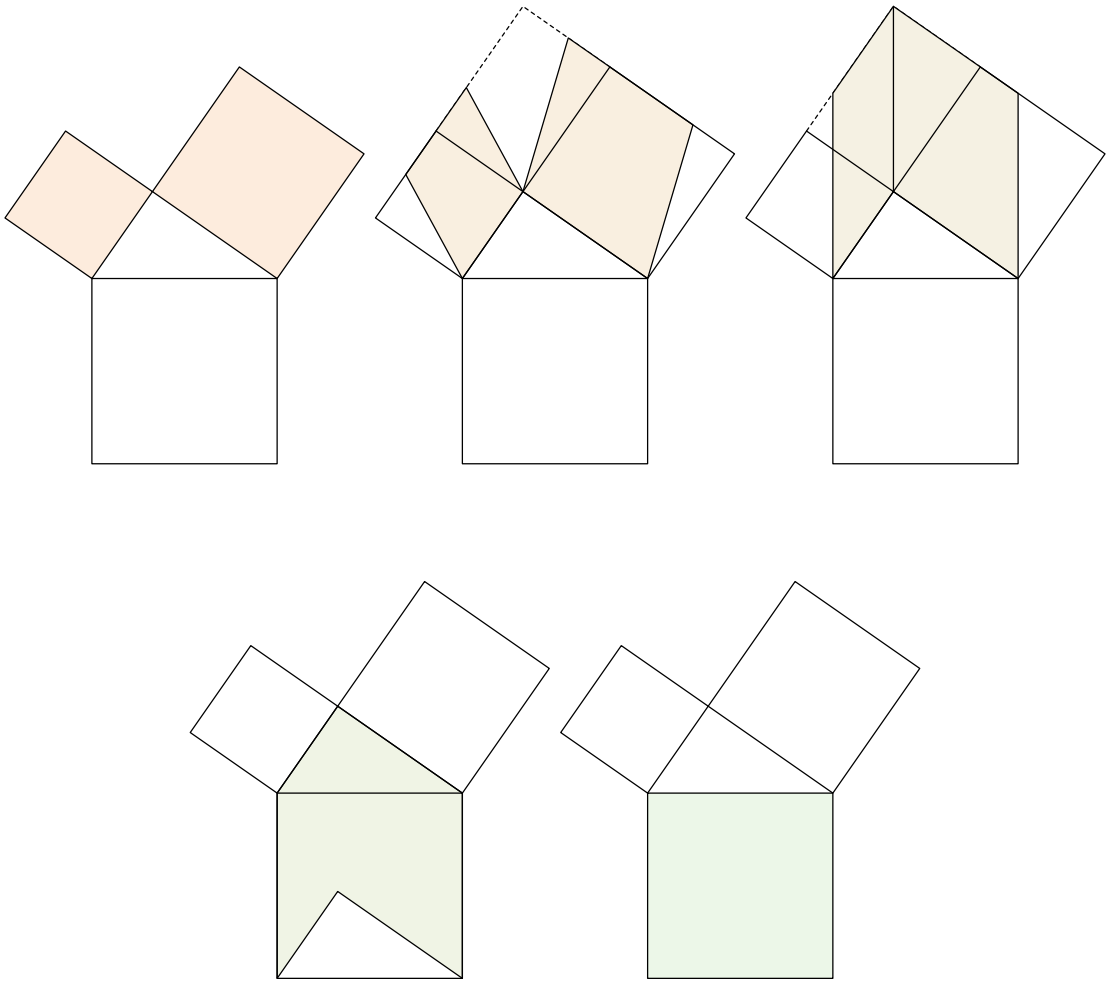
The Pythagorean theorem II



Behold!

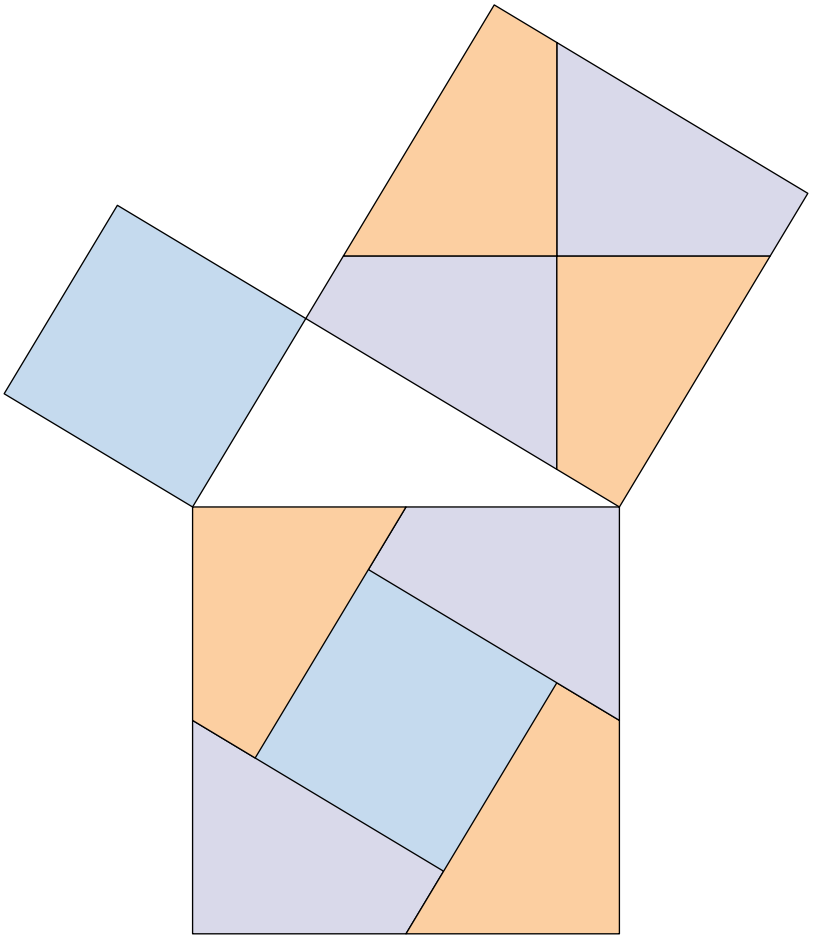
— Bhāskara (12th century)

The Pythagorean theorem III



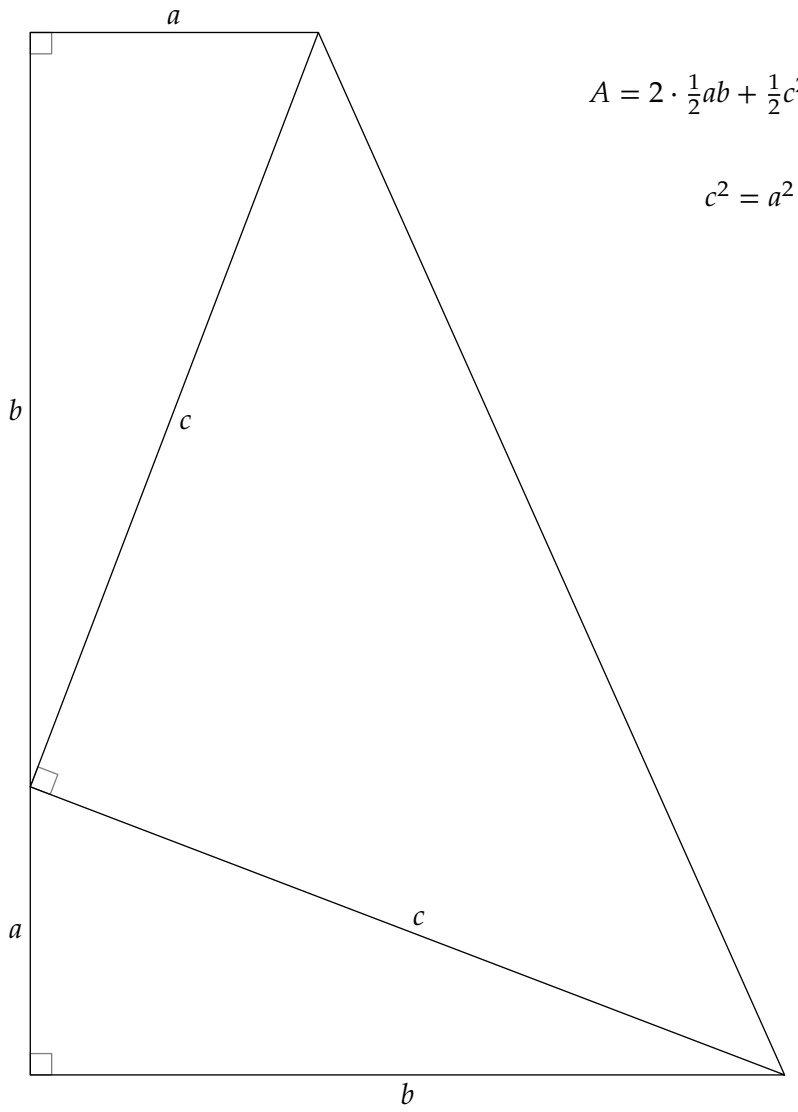
— based on Euclid's proof

The Pythagorean theorem IV



— H. E. Dudeney (1917)

The Pythagorean theorem V



$$A = 2 \cdot \tfrac{1}{2}ab + \tfrac{1}{2}c^2 = \tfrac{1}{2}\,(a+b)^2$$

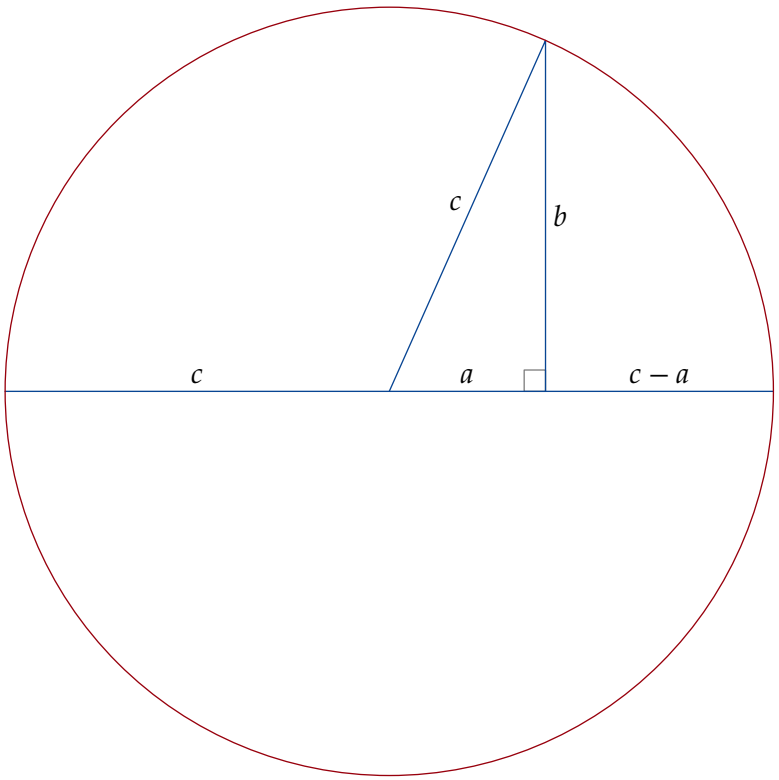
$$c^2 = a^2 + b^2$$

— James A. Garfield (1876)

The Pythagorean theorem VI

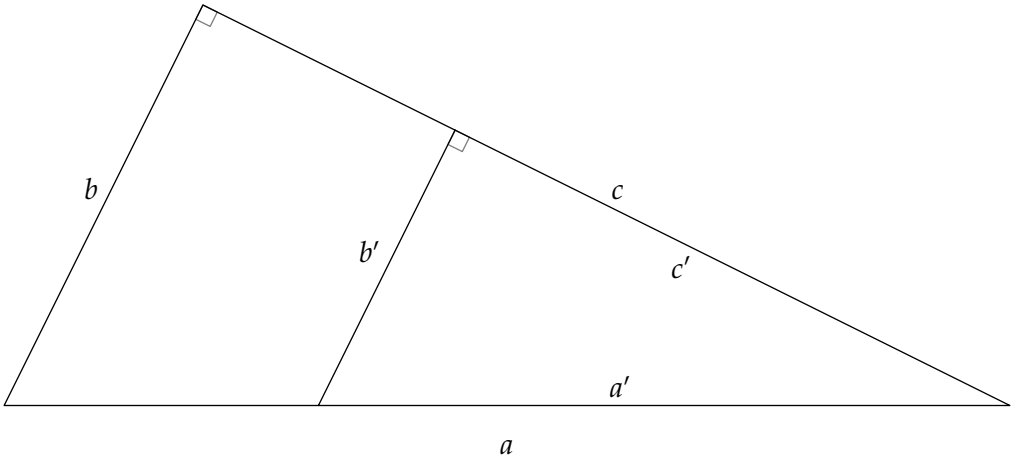
$$\frac{c+a}{b} = \frac{b}{c-a}$$

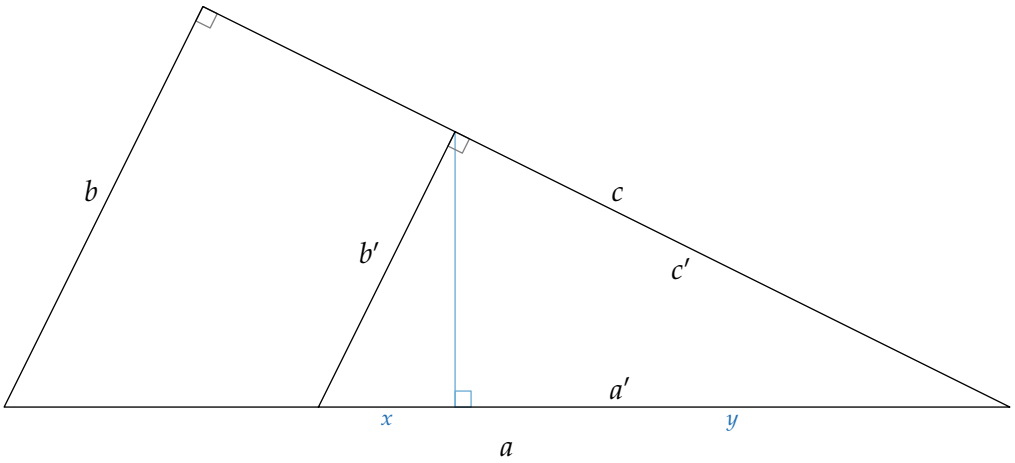
$$a^2 + b^2 = c^2$$



— Michael Hardy

A Pythagorean theorem: $aa^{'} = bb^{'} + cc^{'}$





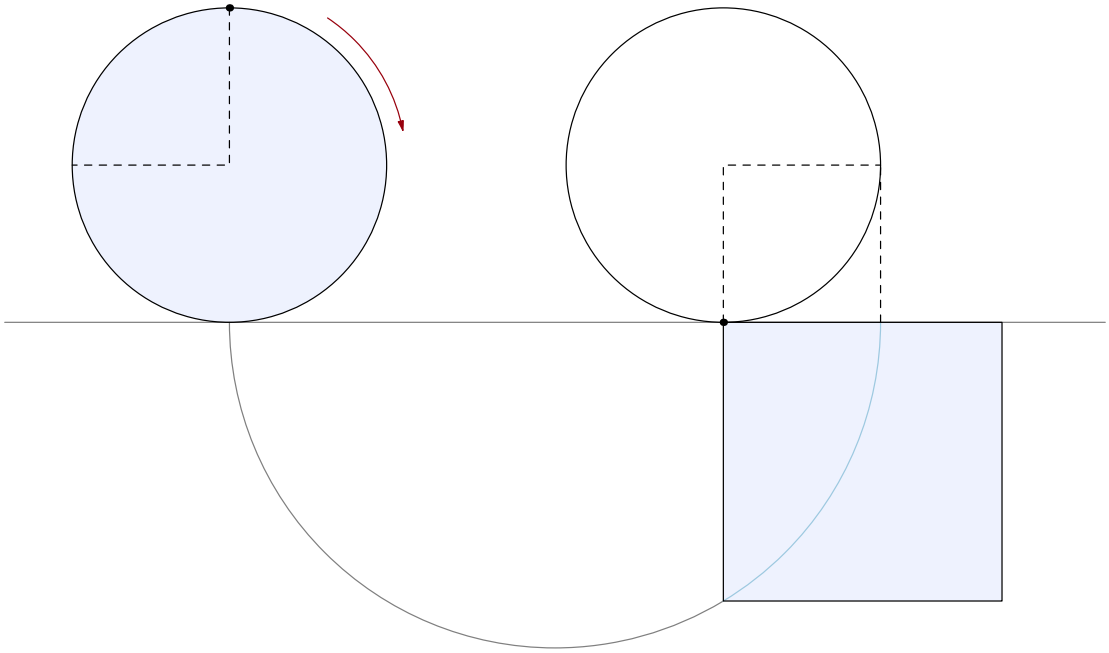
$$\frac{x}{b'} = \frac{b}{a} \implies \frac{x}{b} = \frac{b'}{a} \implies ax = bb';$$

$$\frac{y}{c'} = \frac{c}{a} \implies \frac{y}{c} = \frac{c'}{a} \implies ay = cc';$$

$$\therefore \quad aa' = a\,(x + y) = bb' + cc'.$$

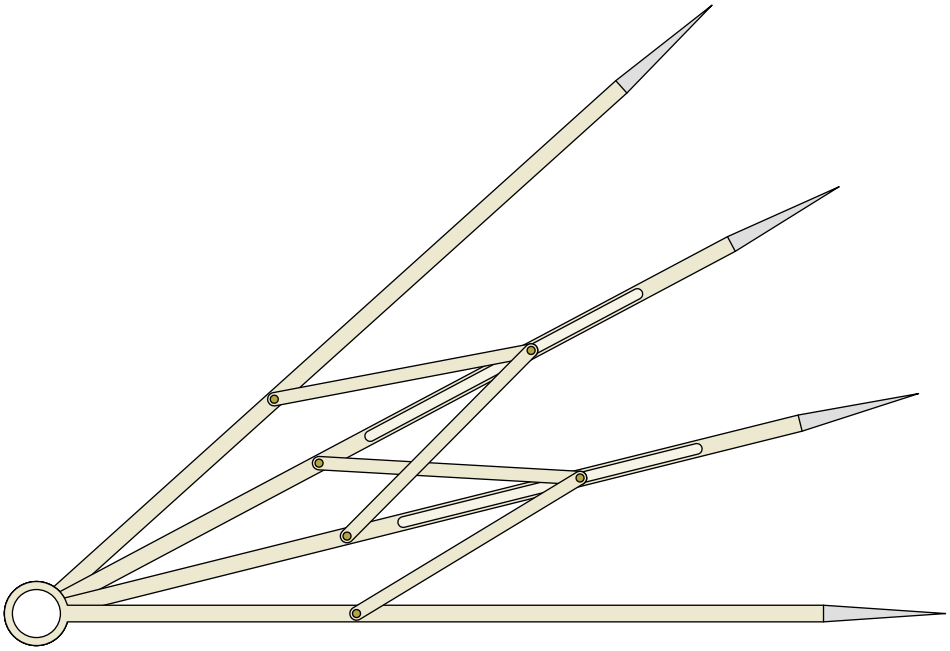
— Enzo R. Gentile

The rolling circle squares itself



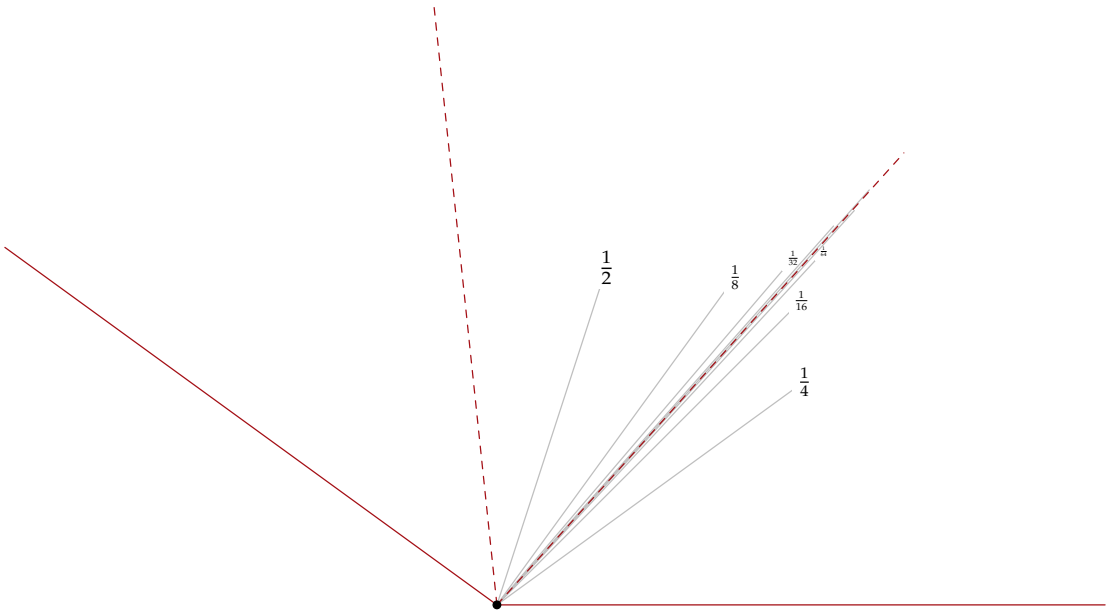
— Thomas Elsner

On trisecting an angle



— Rufus Isaacs

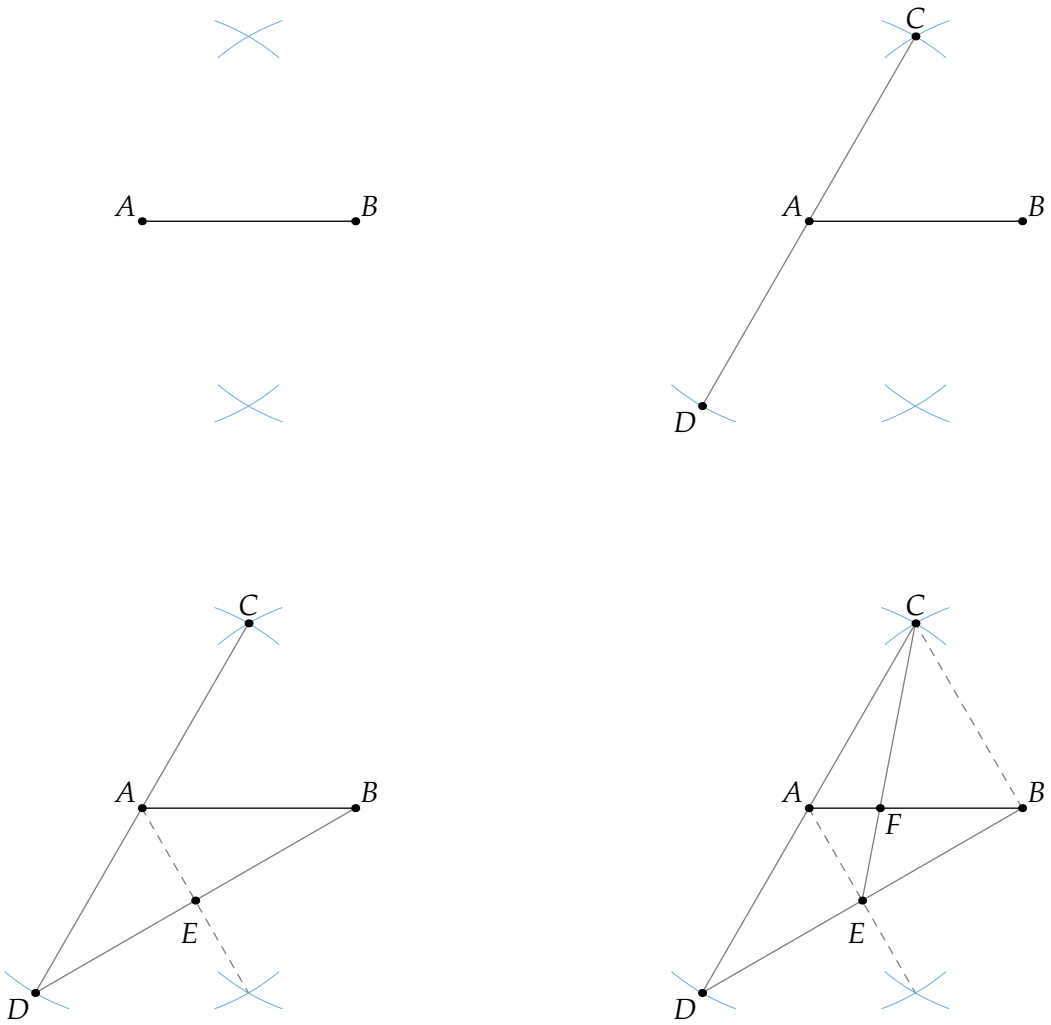
Trisection in an infinite number of steps



$$\frac{1}{3} = \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \cdots$$

— Eric Kincanon

Trisection of a line segment



$$\overline{AF} = \frac{1}{3} \cdot \overline{AB}$$

— Scott Cobel