Proofs without words I

Exercises in METAPOST

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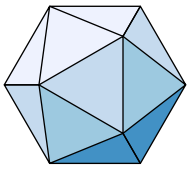
March 2021 —



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Geometry and Algebra

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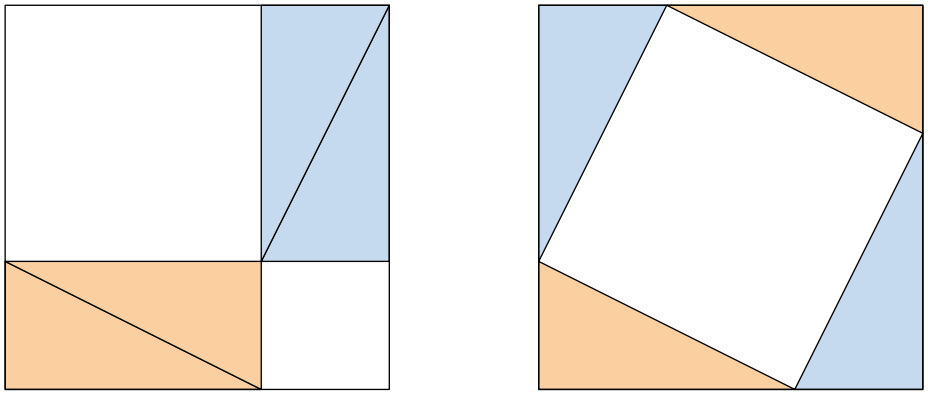
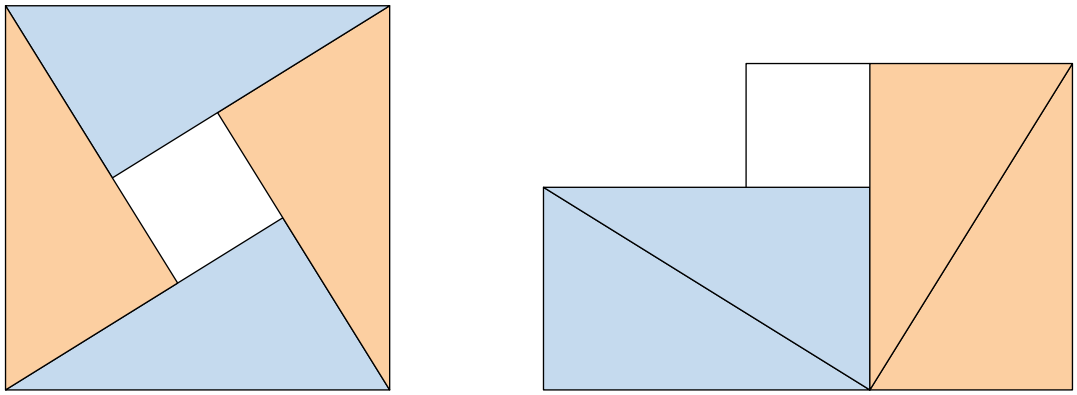
The Pythagorean theorem I



— adapted from the *Chou pei san ching*

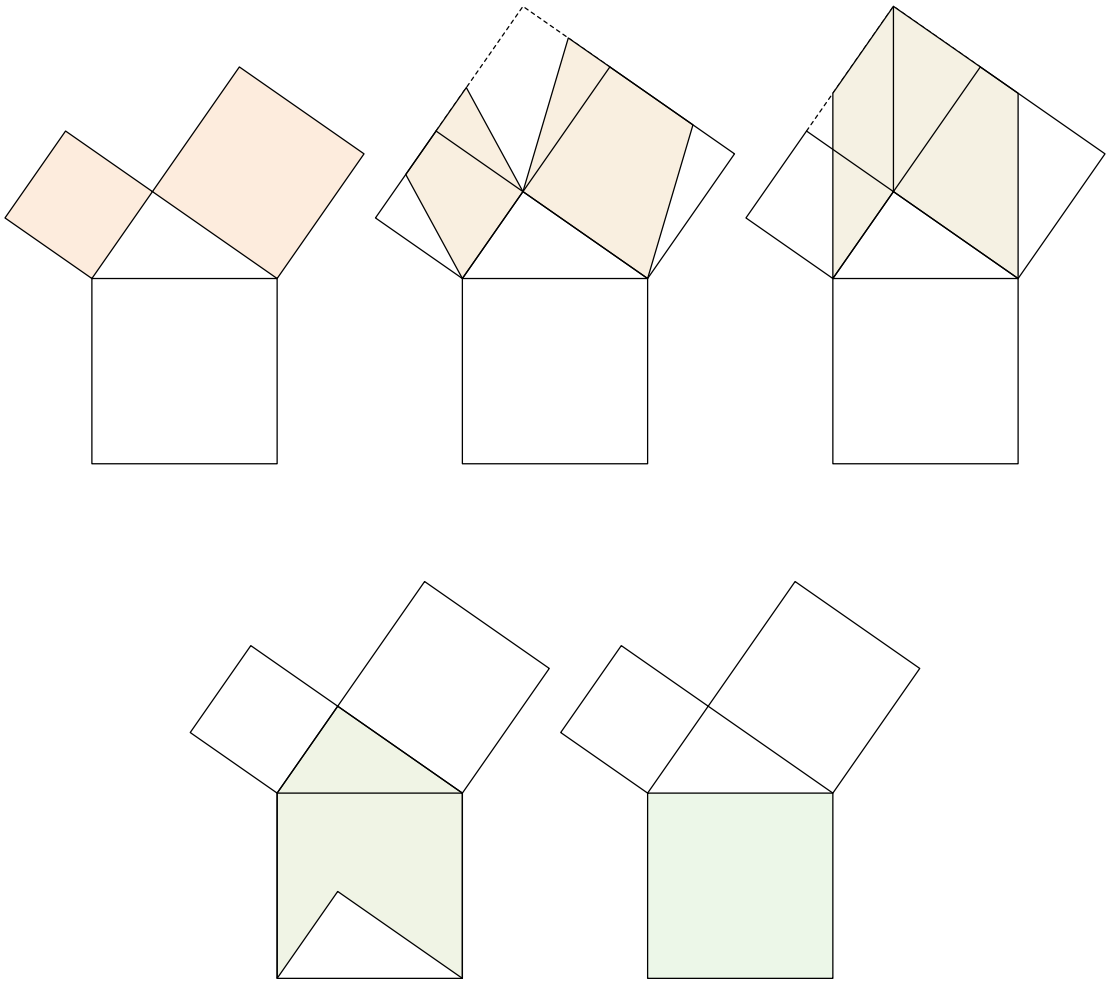
The Pythagorean theorem II



Behold!

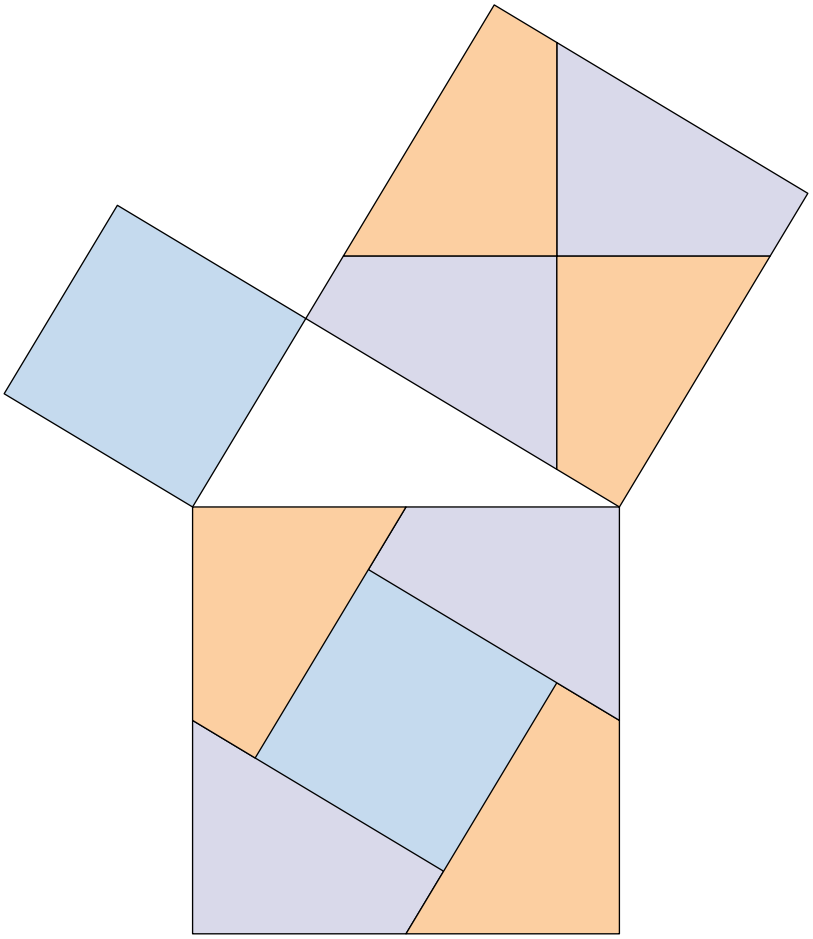
— Bhāskara (12th century)

The Pythagorean theorem III



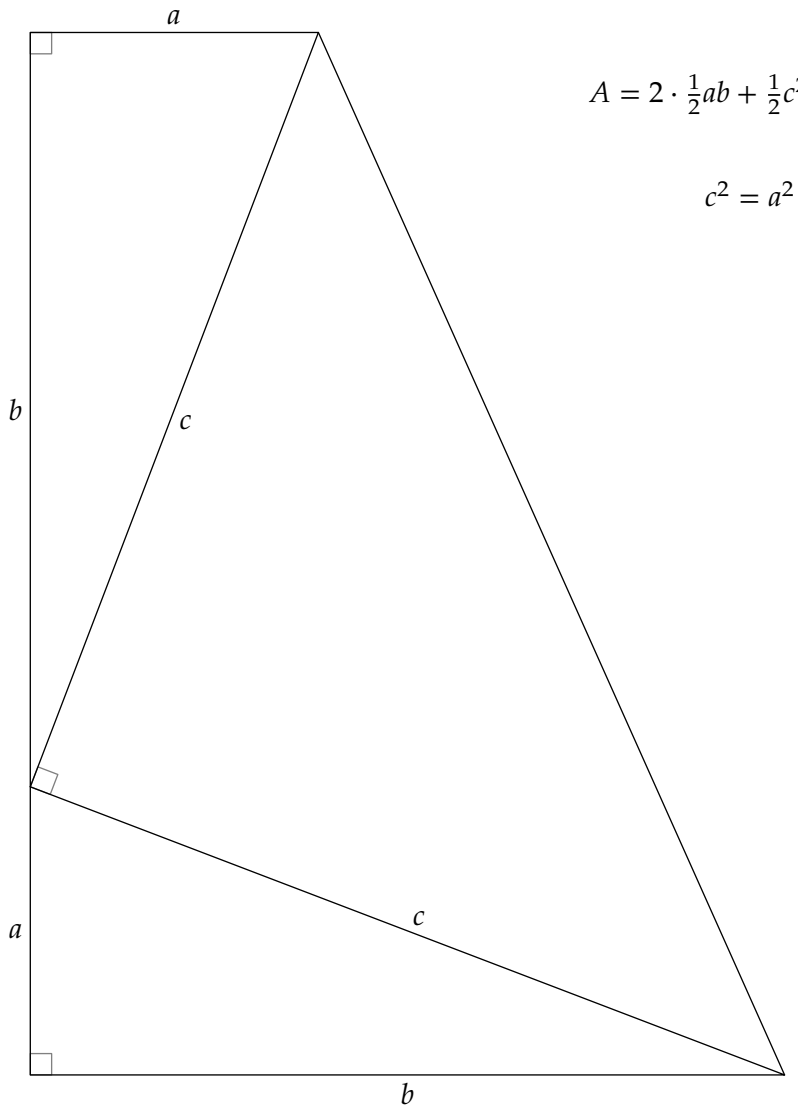
— based on Euclid's proof

The Pythagorean theorem IV



— H. E. Dudeney (1917)

The Pythagorean theorem V



$$A = 2 \cdot \tfrac{1}{2}ab + \tfrac{1}{2}c^2 = \tfrac{1}{2}\,(a+b)^2$$

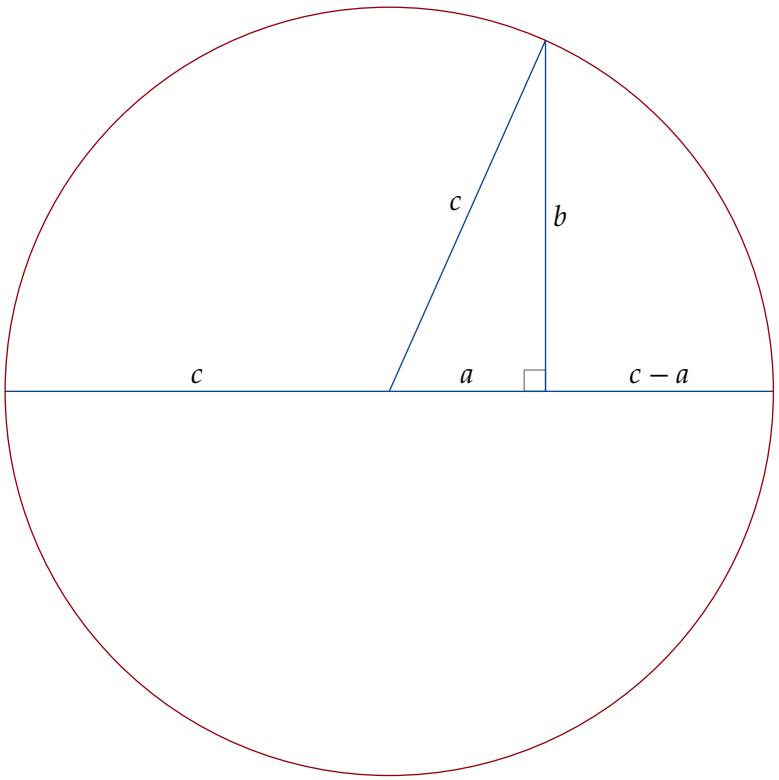
$$c^2 = a^2 + b^2$$

— James A. Garfield (1876)

The Pythagorean theorem VI

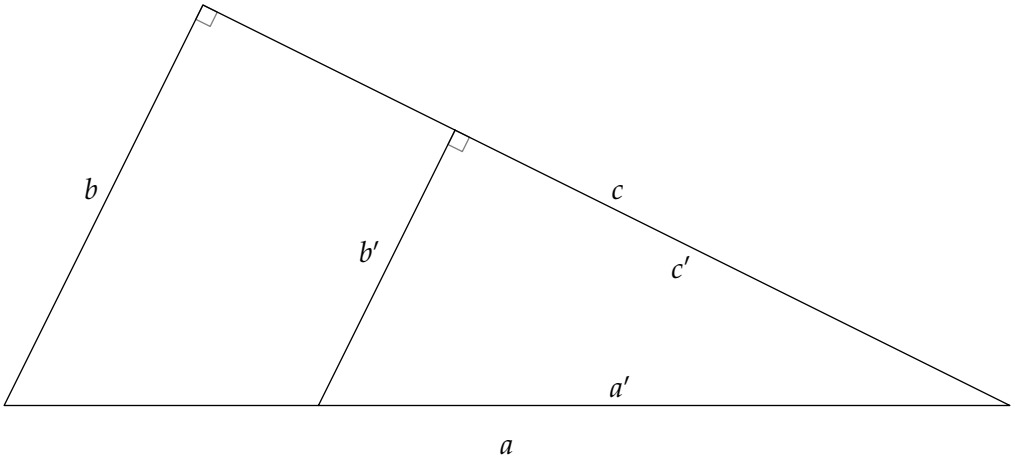
$$\frac{c+a}{b} = \frac{b}{c-a}$$

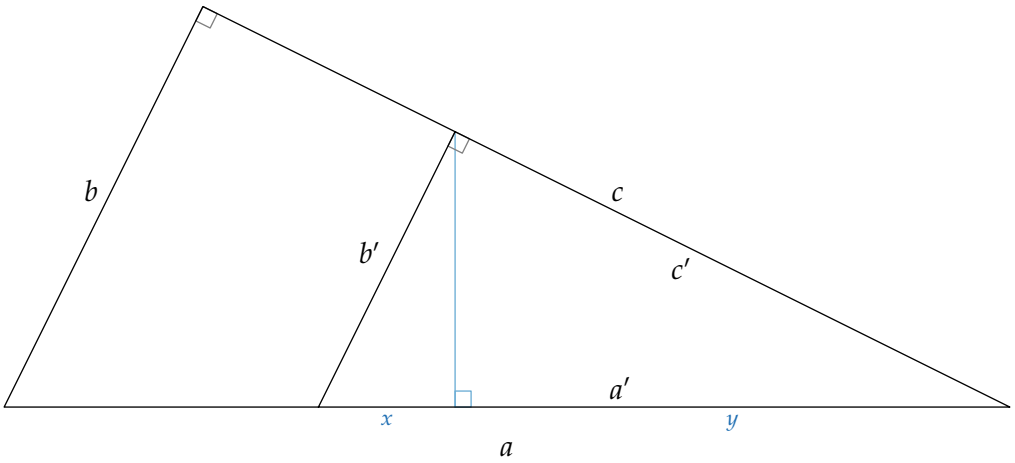
$$a^2 + b^2 = c^2$$



— Michael Hardy

A Pythagorean theorem: $aa^{'} = bb^{'} + cc^{'}$





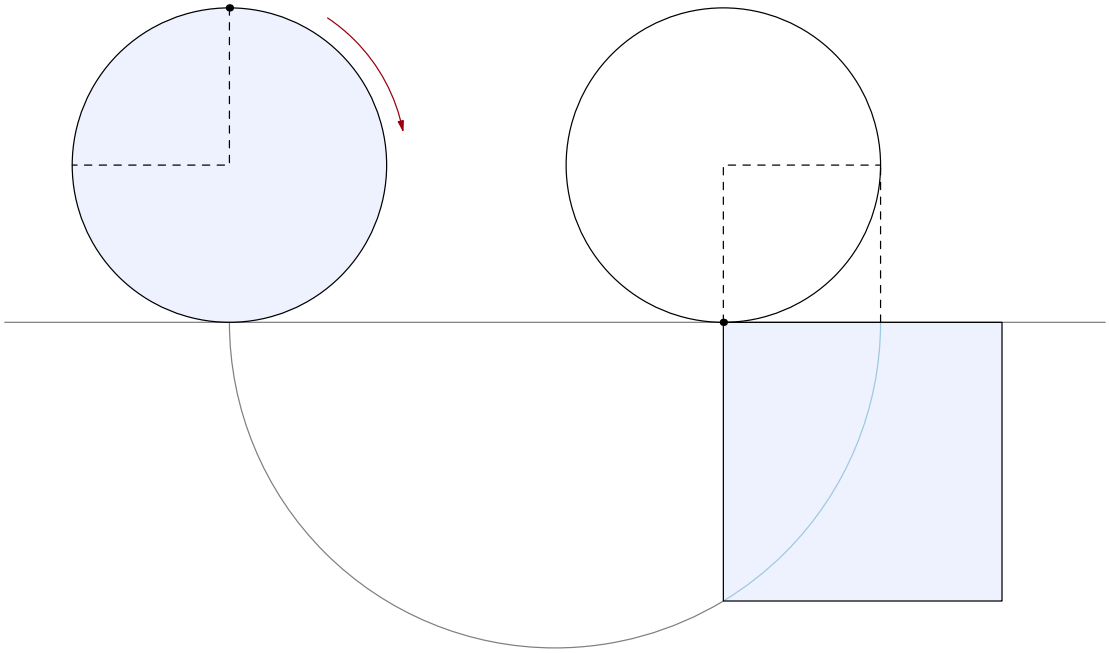
$$\frac{x}{b'} = \frac{b}{a} \implies \frac{x}{b} = \frac{b'}{a} \implies ax = bb';$$

$$\frac{y}{c'} = \frac{c}{a} \implies \frac{y}{c} = \frac{c'}{a} \implies ay = cc';$$

$$\therefore \quad aa' = a\,(x + y) = bb' + cc'.$$

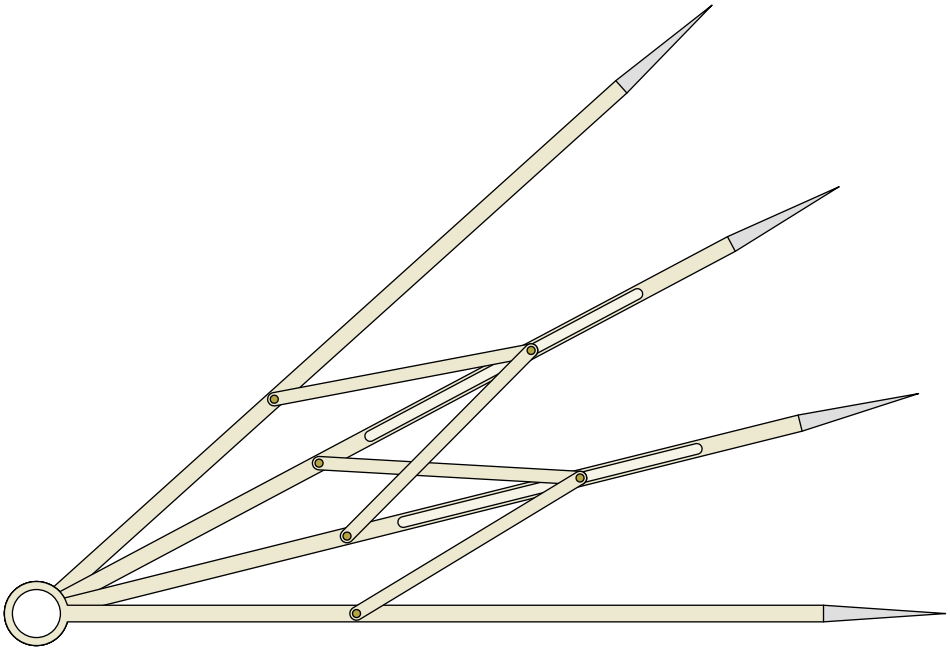
— Enzo R. Gentile

The rolling circle squares itself



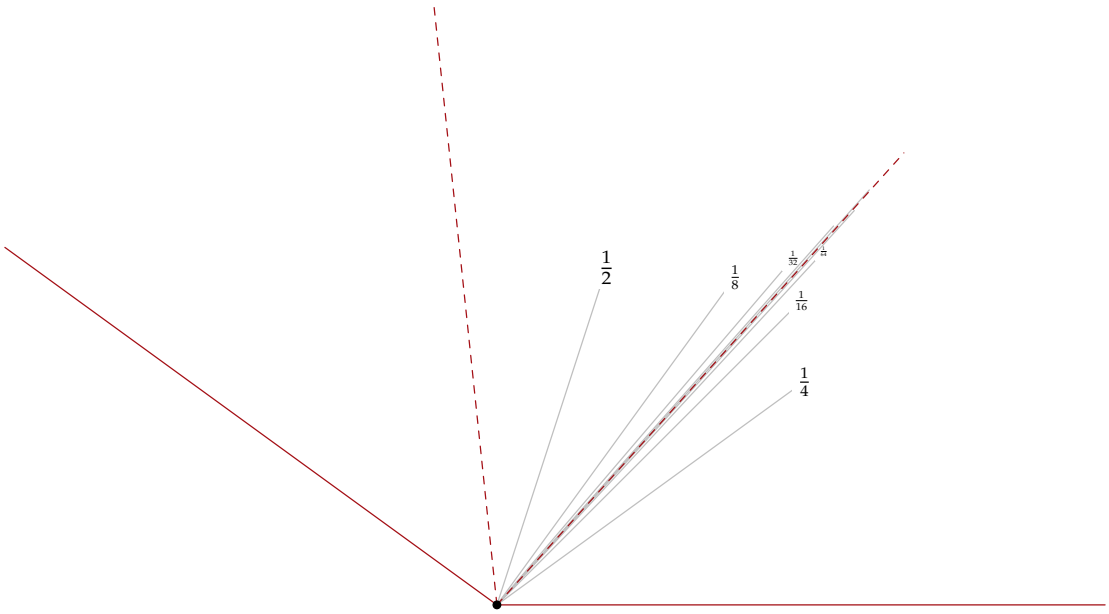
— Thomas Elsner

On trisecting an angle



— Rufus Isaacs

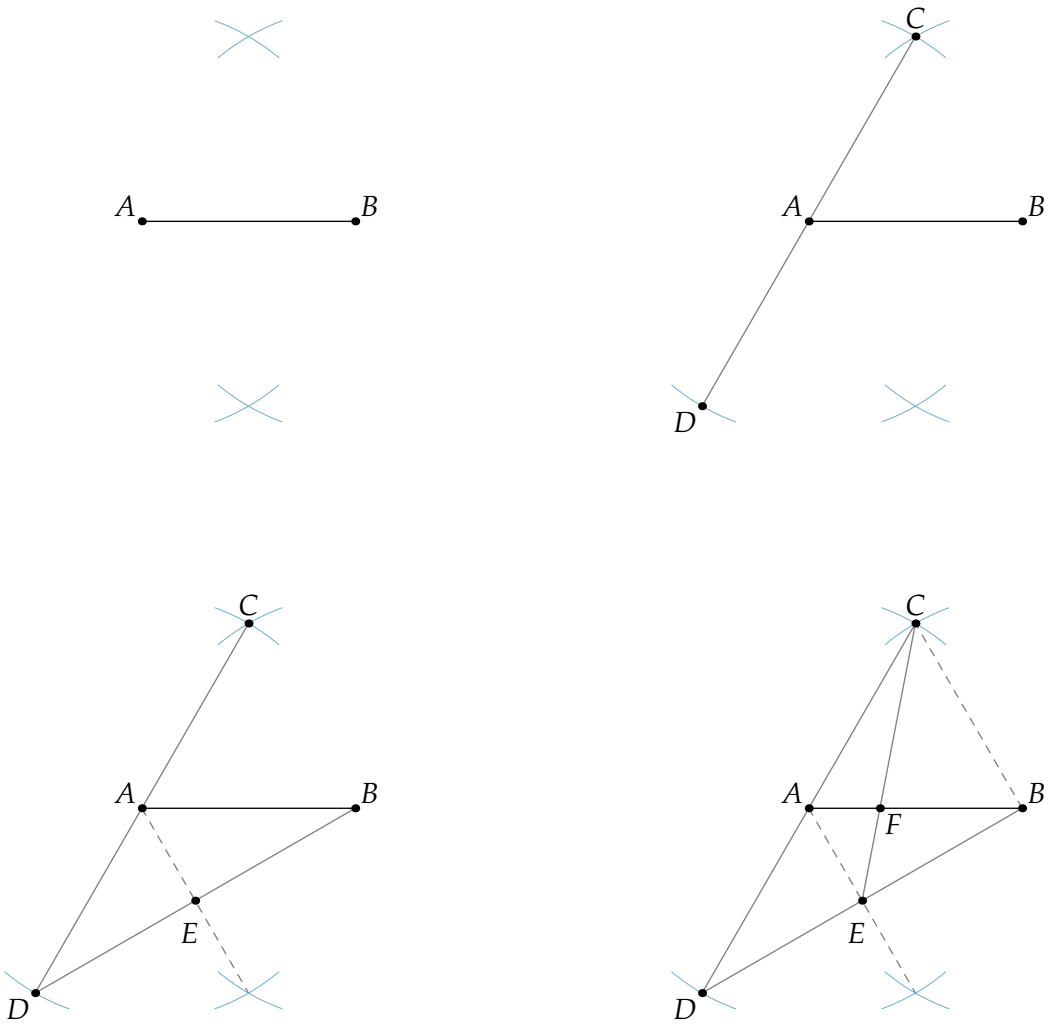
Trisection in an infinite number of steps



$$\frac{1}{3} = \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \cdots$$

— Eric Kincanon

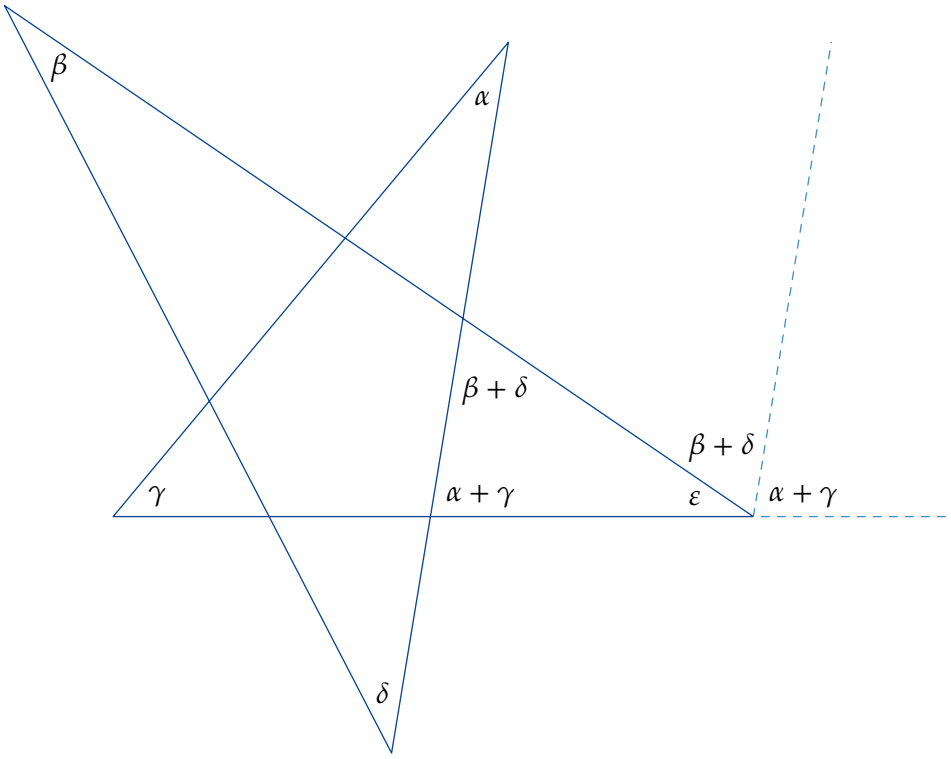
Trisection of a line segment



$$\overline{AF} = \frac{1}{3} \cdot \overline{AB}$$

— Scott Cobel

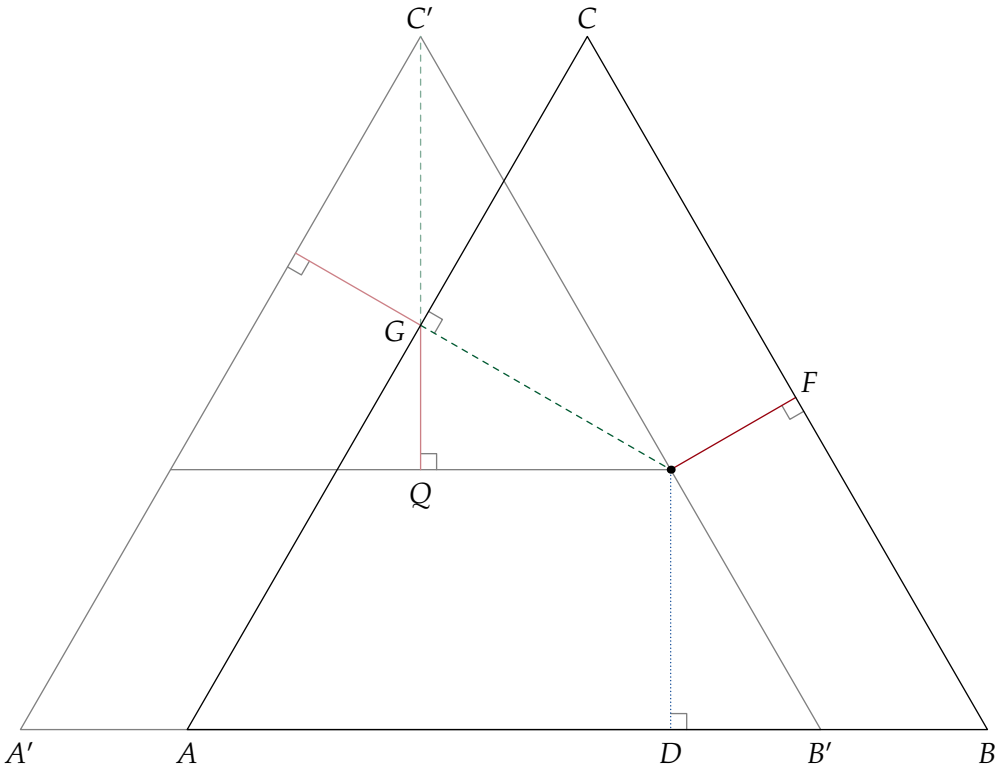
The vertex angles of a star sum to 180°



— Fouad Nakhli

Viviani's theorem I

The perpendiculars to the sides from a point on
the boundary or within an equilateral triangle
add up to the height of the triangle.

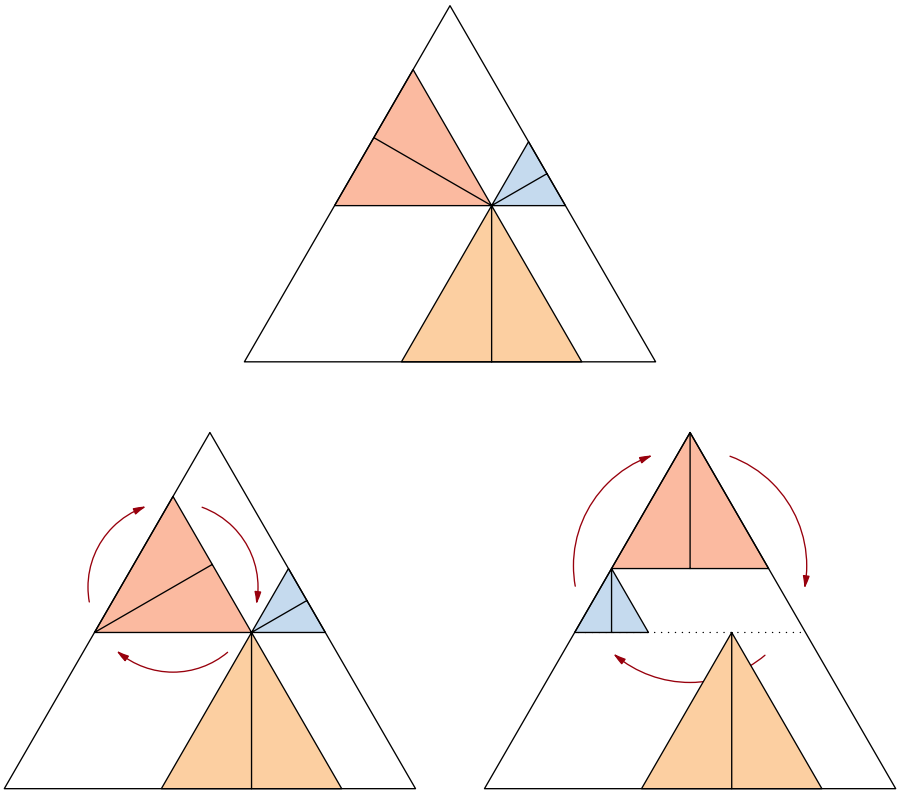


This shows a particular example, with C′GQ collinear, rather than the general case

— Samuel Wolf

Viviani's theorem II

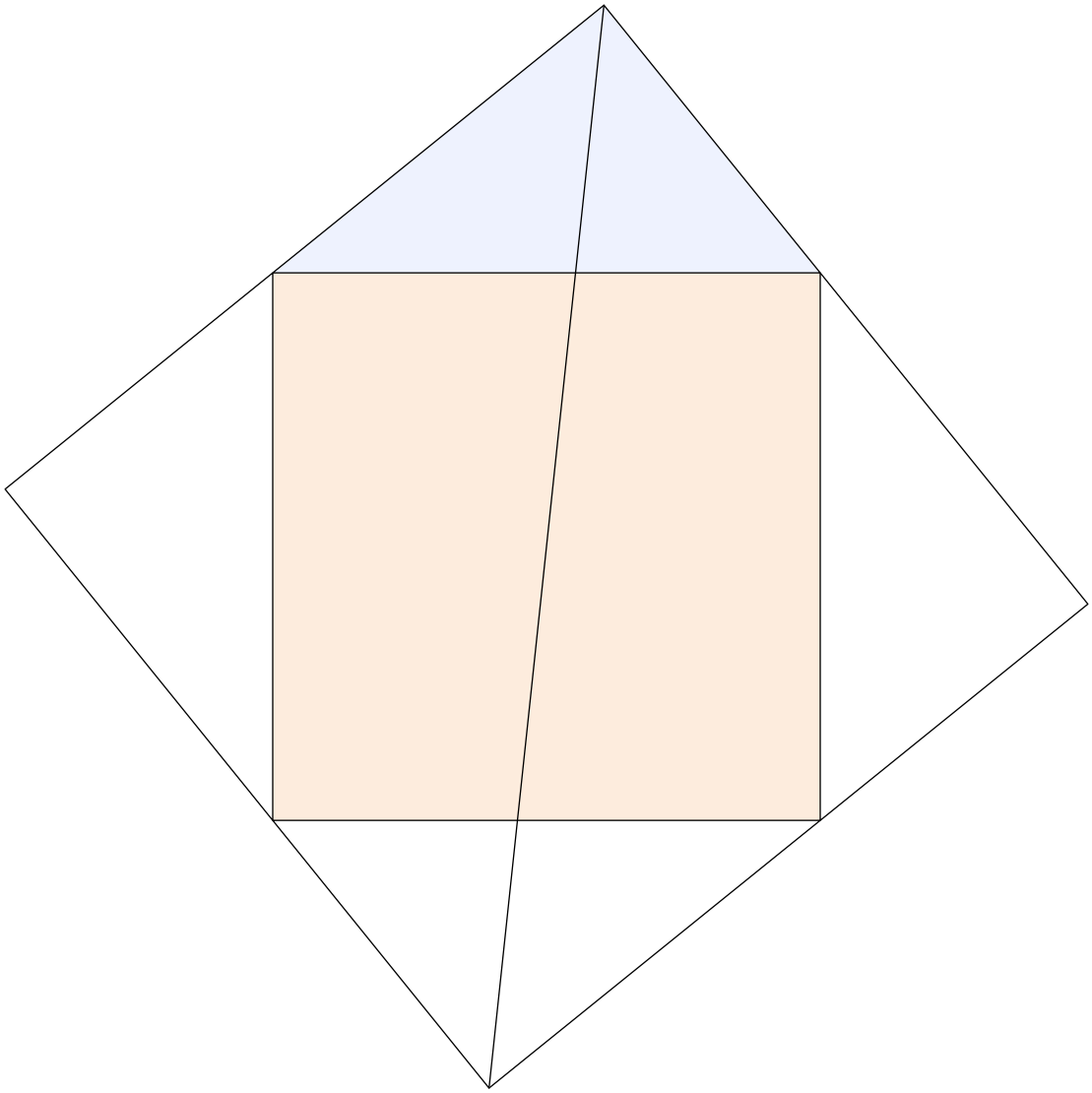
The perpendiculars to the sides from a point on
the boundary or within an equilateral triangle
add up to the height of the triangle.



— Ken-Ichiroh Kawasaki

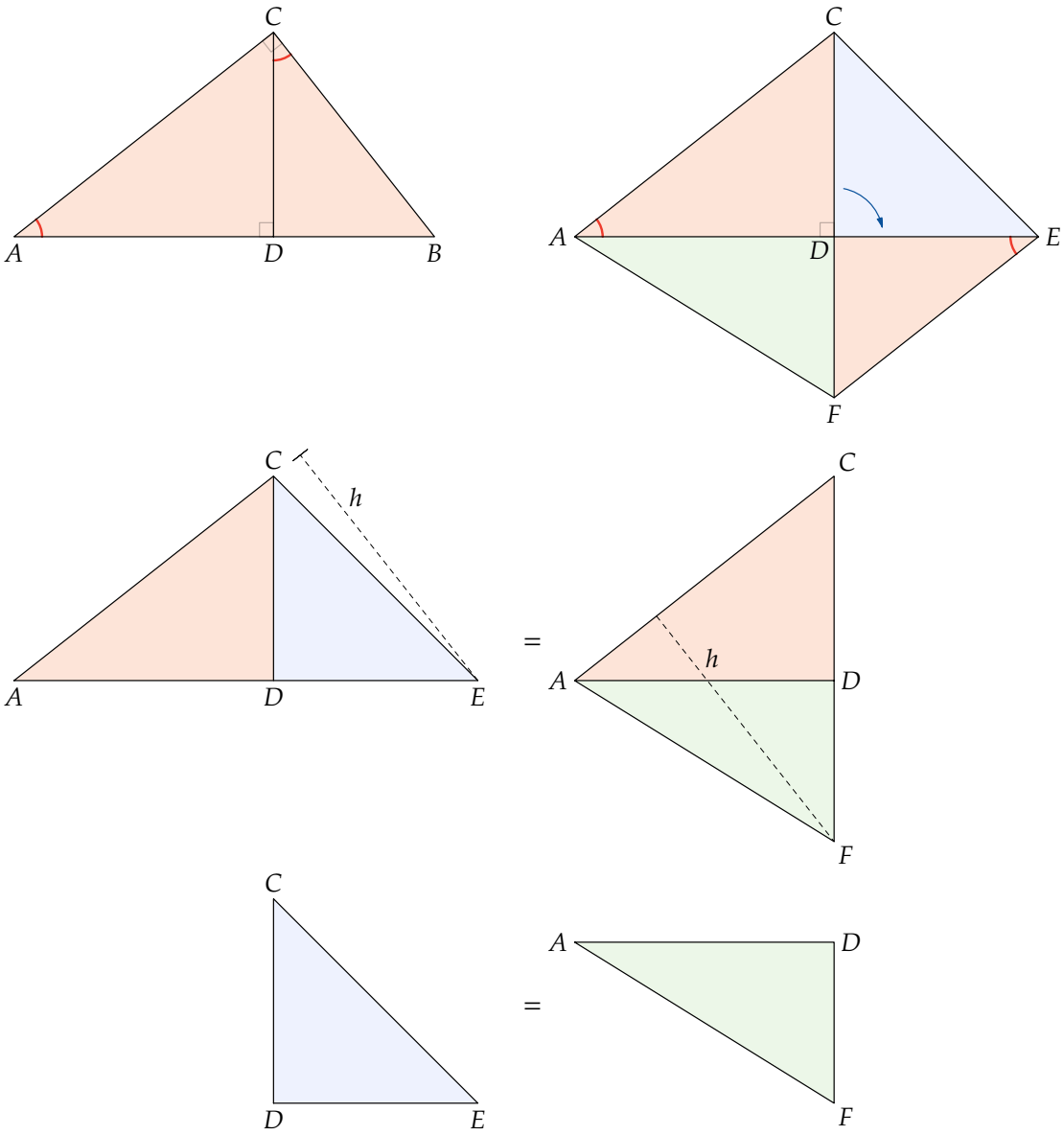
A theorem about right angles

The internal bisector of the right angle of a right triangle bisects the square on the hypotenuse



— Roland H. Eddy

Area and the projection theorem of a right triangle

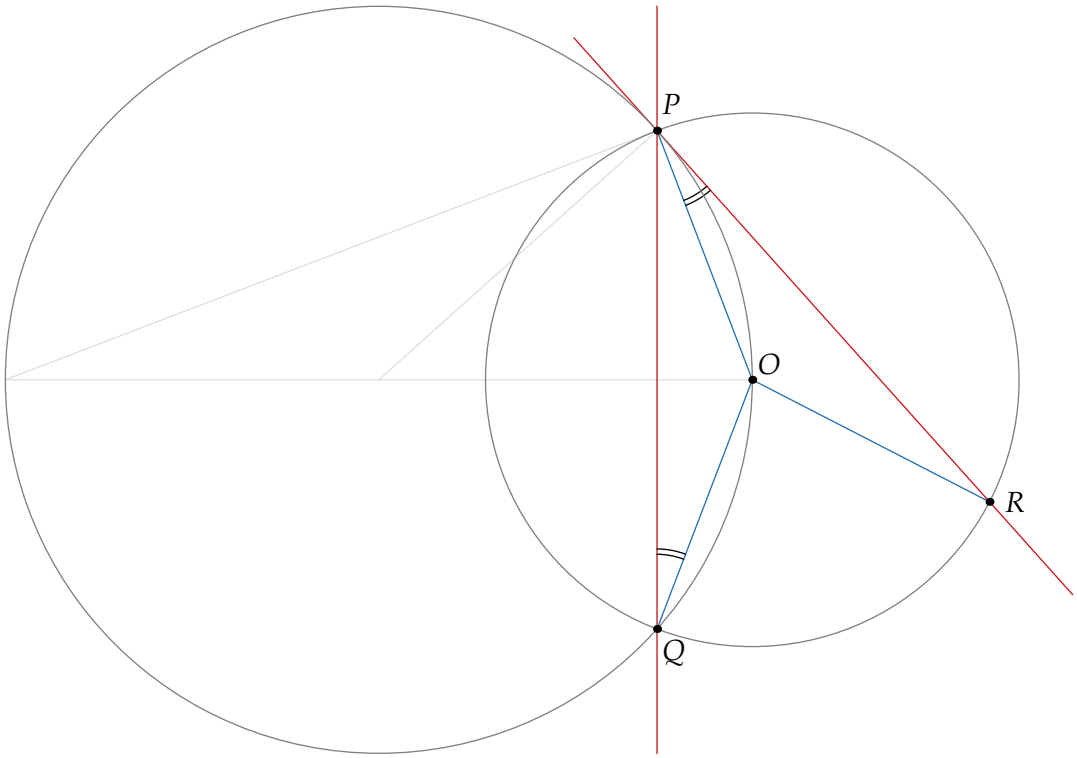


$$CD^2 = AD \cdot DB$$

— Sidney H. Kung

Chords and tangents of equal length

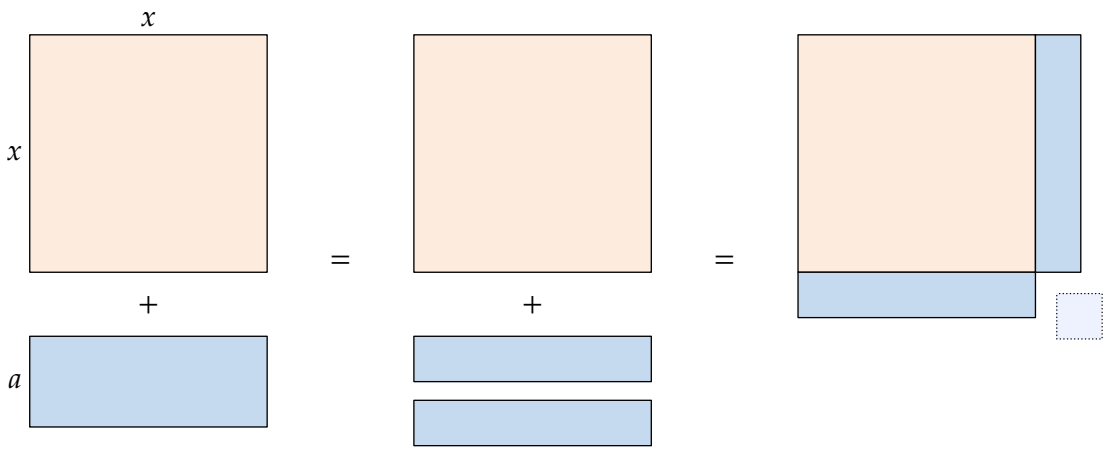
If circle C_1 passes through the center O of circle C_2, the length of the common chord $\overline{PQ}$ is equal to the tangent segment $\overline{PR}$.



— Roland H. Eddy

Completing the square

$$x^2 + ax = (x + a/2)^2 - (a/2)^2$$



— Charles D. Gallant