Universidad Nacional de Ingeniería
Facultad de Ciencias
Escuela Profesional de Matemática

Ciclo 2022–II

Lista 1 - CM4F1

1. Para $A = \begin{pmatrix} 3 & -2 \\ 0 & 3 \\ 4 & 4 \end{pmatrix}$ and $b = \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix}$, solve mín $\|b - Ax\|$.

2. Use transformaciones de Householder para triangulizar la siguiente matriz:

$$\begin{pmatrix} 2 & 2 & 4 & 18 \\ 1 & 3 & -2 & 1 \\ 3 & 1 & 3 & 14 \end{pmatrix}.$$

3. Use factorización QR de Gram-Schimdt para encontrar una función $f(x) = a_1x + a_0$ tal que $\sum_{i=1}^{4}(f(x_i) - y_i)^2$ sea mínimo, donde $x_1 = -2, x_2 = -1, x_3 = 0, x_4 = 1$ e $y_1 = 1, y_2 = 2, y_3 = 0, y_4 = 1$. Formule el problema como de mínimos cuadrados.

4. For the following points on a plane $(-1, 1), (0, 0), (1, 1), (1, -1)$, we look for a polynomial $p(x) = a + bx$ such that $\sum_{i=1}^{4}(p(x_i) - y_i)^2$ is minimum. How do you formulate this as problem as a linear least squares problem and then solve it using Householder's method?

5. Find and orthonormal basis for the column space of the matrix $\begin{pmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{pmatrix}$.

6. Show that the matrix $\begin{pmatrix} 2 & 1 & -1 \\ 1 & 0 & 2 \\ 2 & -1 & 3 \end{pmatrix}$ is nonsingular. Find the QR factorization of the matrix.

7. Find the closest point to $b = (1, 0, 2)^T$ in the subspace $W = span\{(1, -1, 1)^T, (1, 2, -1)^T\}$.

8. Find the QR factorization of $A = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.

9. Find the QR factorization of $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

10. Find the QR factorization of $A = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{pmatrix}$.

11. Find the least squares solution $\hat{x}$ for the system:

$$\begin{pmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix} x = \begin{pmatrix} 0 \\ 0 \\ 10 \\ 0 \end{pmatrix}.$$

12. Find the solution $\hat{x}$ for the system:
$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix} x = \begin{pmatrix} 9 \\ 6 \\ 3 \end{pmatrix}.$$

13. Find the solution $\hat{x}$ for the system:
$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

14. Use Gram-Schmidt to compute the QR factorization of the matrix $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \\ 1 & -1 \end{pmatrix}$.

15. Use Gram-Schmidt to compute the QR factorization of the matrix $A = \begin{pmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{pmatrix}$.

16. Use Gram-Schmidt to compute the QR factorization of the matrix $A = \begin{pmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -10 & \end{pmatrix}$.

17. Use Gram-Schmidt to compute the QR factorization of the matrix $A = \begin{pmatrix} 2 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$. Use it to find a least squares solution of $Ax = b$ where $b = \begin{pmatrix} 3 \\ 2 \\ -5 \end{pmatrix}$.

El profesor[1]
Lima, 17 de Noviembre del 2022.

[1] Hecho en LaTeX