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Financial Ratios as Predictors of Failure

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At the turn of the century, ratio analysis was in its embryonic state. It began with the development of a single ratio, the current ratio,[1] for a single purpose—the evaluation of credit-worthiness. Today ratio analysis involves the use of several ratios by a variety of users—including credit lenders, credit-rating agencies, investors, and management.[2] In spite of the ubiquity of ratios, little effort has been directed toward the formal empirical verification of their usefulness.

The usefulness of ratios can only be tested with regard to some particular purpose. The purpose chosen here was the prediction of failure, since ratios are currently in widespread use as predictors of failure. This is not the only possible use of ratios but is a starting point from which to build an empirical verification of ratio analysis.

"Failure" is defined as the inability of a firm to pay its financial obligations as they mature. Operationally, a firm is said to have failed when any of the following events have occurred: bankruptcy, bond default, an overdrawn bank account, or nonpayment of a preferred stock dividend.[3] A "financial ratio" is a quotient of two numbers, where both num-

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[1] The author is deeply indebted to Harry Roberts, Professor of Statistics, University of Chicago, for his many helpful suggestions dealing with the statistical problems that were encountered in this study.

[2] The current ratio was cited in the literature as early as 1908: William M. Rosendale, "Credit Department Methods," *Bankers' Magazine* (1908), pp. 183–84. For an excellent review of the history and uses of ratios: James O. Horrigan, "An Evaluation of the Usage of Ratios in External Financial Statement Analysis" (unpublished dissertation proposal, Graduate School of Business, The University of Chicago, 1963).

[3] Of the 79 failed firms studied, 59 were bankrupt; 16 involved nonpayment of preferred stock dividends; 3 were bond defaults; and 1 was an overdrawn bank account. For a more complete discussion of the various definitions of failure: Charles W. Gerstenberg, *Financial Organization and Management of Business* (3rd ed., rev.; New York: Prentice-Hall, Inc., 1951), p. 569.

bers consist of financial statement items. A third term, *predictive ability,* also requires explanation but cannot be defined briefly. The various dimensions of predictive ability will be explored later.

The emphasis upon financial ratios does not imply that ratios are the only predictors of failure. The primary concern is not with predictors of failure per se but rather with financial ratios as predictors of important events—one of which is failure of the firm. Further, the primary concern is not with the ratios as a form of presenting financial-statement data but rather with the underlying predictive ability of the financial statements themselves. The ultimate motivation is to provide an empirical verification of the usefulness (i.e., the predictive ability) of accounting data (i.e., financial statements).

The study is intended to be a test of the status quo. It is based upon financial statements prepared under currently accepted reporting standards, and it employs the currently accepted form for analyzing the statements—financial ratios. The study is offered not as one of the last endeavors in this area but as one of the first. It is designed to be a benchmark for future investigations into alternative predictors of failure, into competing forms of presenting accounting data, and into other uses for accounting data.

The paper consists of five parts: (1) the sample design, (2) a comparison of means, (3) a dichotomous classification test, (4) an analysis of likelihood ratios, and (5) concluding remarks.

Selection of Failed Firms

The most difficult task of data collection was finding a sample of failed firms for which financial statements could be obtained. After this problem was discussed with representatives of several organizations who might have access to the information, it was decided that *Moody's Industrial Manual* was apparently the only source available.

Moody's Industrial Manual contains the financial-statement data for industrial, publicly owned corporations. The population excludes firms of noncorporate form, privately held corporations, and nonindustrial firms (e.g., public utilities, transportation companies, and financial institutions). The firms in *Moody's* tend to be larger in terms of total assets than are noncorporate firms and privately held corporations. Strictly speaking, inferences drawn from this study apply only to firms that are members of the population.

The choice of this population is admittedly a reluctant one. The probability of failure among this group of firms is not so high as it is among smaller firms. In this sense, it is not the most relevant population upon which to test the predictive ability of ratios. However, the situation may not be so bleak as it first appears. The population chosen is not a trivial one; it represents over 90 per cent of the invested capital of all industrial firms—invested capital representing the contributions of several million investors and creditors.

The findings may also possess some relevance for smaller firms as well. The crucial question is—does the *Moody's* population differ in some essential way from that of the smaller firms? The question cannot be directly answered within the context of the study. The answer can only be provided by a replication on other populations. However, the findings here can provide some evidence regarding the extent to which such a replication would be worthwhile.

The next task was to identify those firms in *Moody's* that had failed during the time period being studied (1954 to 1964, inclusive). In the front of *Moody's* there appears a list of firms—firms on whom *Moody's* has formerly reported but no longer does so. There are many reasons why a firm may be dropped—name change, merger, liquidation, lack of public interest, and, most importantly, failure. The list of several thousand names was condensed into a list of firms that had failed. Supplementing this basic list was a list of bankrupt firms provided by Dun and Bradstreet. The final list of failed firms contained 79 firms on which financial-statement data could be obtained for the first year before failure.[4]

Next, the failed firms were classified according to industry and asset size. Each firm was assigned a three-digit number that denoted its principal line of activity; the numbering system used was the Standard Industrial Classification (SIC) system of the United States Department of Commerce. The total asset size of each firm was obtained from the most recent balance sheet prior to the date of failure (the data were collected from *Moody's*). The industry and asset-size composition were heterogeneous. The 79 failed firms operated in 38 different industries. Eighteen of the industries contained only one failed firm; the most frequently represented industry was Manufacturers of Electronic Equipment (SIC number, 367), which consisted of six firms. The asset-size range was .6 million to 45 million dollars, and the mean asset size was approximately 6 million.[5]

Selection of Nonfailed Firms

The classification of the failed firms according to industry and asset size was an essential prerequisite to the selection of the nonfailed firms. The selection process was based upon a paired-sample design—that is, for each failed firm in the sample, a nonfailed firm of the same industry and asset size was selected. The names of the nonfailed firms were obtained from a list of 12,000 firms.[6] The list possessed two convenient

[4] A frequency distribution of the fiscal year of failure appears in Table A-1 in the Appendix.

[5] The industrial composition of the sample appears in Table A-2 in the Appendix.

[6] *12,000 Leading U.S. Corporations* (New York: News Front Magazine). The list

features: (1) the firms were grouped according to industry (using the SIC system) and (2) within each industry group, the firms were ranked according to size.

The following procedure was used to select the nonfailed firms: (1) take the industry number of a failed firm; (2) turn to that portion of the list where firms of that industry number are found; (3) within that industry group, tentatively select the firm whose asset size is closest to the asset size of the failed firm; (4) if that firm is in *Moody's* and is nonfailed, then accept it as one of the firms to be studied; (5) if the firm does not fulfill these two conditions, return to the list and tentatively select the firm which is next closest in asset size; (6) repeat this procedure until an acceptable candidate is found; and (7) repeat this procedure for each of the failed firms in the sample.

An adequate explanation of the motivation behind the paired-sample design involves a brief discussion of the history of this study. The findings presented here are an extension of an earlier effort based on the same body of data and the same sample design.[7]

The paired-sample design was selected in the earlier study to help provide a "control" over factors that otherwise might blur the relationship between ratios and failure. As early as 1923, the ratio literature suggested that industry factors must be incorporated in any complete ratio analysis.[8] The literature contends that "differences" exist among industries that prevent the direct comparison of firms from different industries. Another way of stating this argument is to say that the same numerical value of a ratio (e.g., a current ratio of 2.00) implies a different probability of failure in different industries. The evidence offered in behalf of industry differences is the fact that the ratio distributions differ among industries.[9] However, such evidence is not conclusive, since the failure rate may vary among industries to compensate for the differences in the ratios. No evidence has been offered to indicate whether or not the compensating differences exist.

Although less attention has been directed toward the influence of asset size, there are certain statistical reasons for believing that asset size alters the relationship between ratios and failure. It can be argued that the larger of two firms will have a lower probability of failure, even if

for years prior to 1959 was not available. The nonfailed mates of the firms that failed prior to 1959 tend to be older than their failed mates. There may be a possible bias here, but the evidence indicates that the ability to predict failure was unrelated to fiscal year before failure, and the ratio behavior of the nonfailed firms is quite stable and apparently independent of age.

[7] William H. Beaver, "Financial Ratios as Predictors of Failure" (unpublished Ph.D. dissertation, Graduate School of Business, University of Chicago, 1965).

[8] James H. Bliss, *Financial and Operating Ratios in Management* (New York: The Ronald Press, 1923), p. 59.

[9] A. C. Littleton, "The 2 to 1 Ratio Analyzed," *Certified Public Accountant*, August, 1926, pp. 244–46.

the ratios of the two firms are identical (i.e., even if the ratios have the same numerical values).

If firms are viewed as aggregates of assets and if asset returns are less than perfectly correlated with one another, statistical formulae suggest that the variability of total return to the firm will increase less than proportionately to the size of the firm.[10] The rate of return to the firm will become more stable as asset size increases. Empirical evidence indicates that the variability of rate of return does behave in this manner.[11] The implication is that larger firms are more solvent, even if the value of their ratios is the same as that of smaller firms.[12] Simply stated, the ratios of firms from different asset-size classes cannot be directly compared.

The paired-sample design in conjunction with the paired analysis is one way of compensating for the effects of industry and asset size. Whenever ratios are directly compared, the comparison involves two firms from the same industry and asset-size class. As previously indicated, each failed firm has a nonfailed "mate" in the sample. In the paired analysis, the ratios of the nonfailed firms are subtracted from the ratios of the failed firms; the analysis is then based upon the differences in the ratios rather than upon the ratios themselves. Since every difference in the ratios always involves a comparison of two firms (one failed and one nonfailed) from the same industry and asset-size class, the potentially disruptive effects of industry and asset size are mitigated. A paired design and a paired analysis were employed in the previous study.

Although the paired analysis is a legitimate approach to investigating predictive ability, it has one serious drawback. It cannot draw inferences regarding a single observation—only about pairs of observations. The earlier study is restricted to such statements as: Based upon an analysis of the ratios, firm A is more (or less) solvent than firm B. However, this is an ambiguous statement because it ignores the obvious question: How solvent is firm B? To be less solvent than firm B may not be so bad, if firm B is an extremely solvent firm. On the other hand, to be more solvent than firm B may be very bad, if firm B is extremely insolvent. In a decision-making situation, the user of ratios wishes to make inferences regarding a single firm—for example, what is the probability of failure for this firm? Such inferences are beyond the scope of the paired analysis.

In this study the paired analysis was dropped in favor of an unpaired

[10] W. J. Baumol, "The Transactions Demand for Cash: An Inventory Theoretic Approach," *Quarterly Journal of Economics*, November, 1962, p. 556.

[11] Sidney Alexander, "The Effect of Size of Manufacturing Corporation on the Distribution of the Rate of Return," *Review of Economics and Statistics*, August, 1949, pp. 229–35.

[12] "Solvency" is defined here in terms of probability of failure.

analysis, which does permit inferences regarding a single observation. The elimination of the paired analysis does not necessarily imply that the paired design should also be discarded. The problem of selecting nonfailed firms was re-examined to see if the paired analysis still served some useful purpose.

If the comparison of failed and nonfailed firms is to be meaningful, the sample of nonfailed firms should be drawn from the same population as that of the failed firms. For example, failure is virtually nonexistent among very large firms. A comparison of large firms with the failed firms would not be particularly meaningful, since the two groups of firms come from different populations. The failed firms were drawn from a population whose asset-size range does not include the very large firms.

A random selection of nonfailed firms without regard for asset size would be inappropriate. Another alternative would be a random selection of firms whose asset size is less than some specified value. However, it would be difficult to determine what that value should be. The paired-sample design offers a sampling technique for selecting nonfailed firms from a relevant population, because it draws nonfailed firms only from those asset-size (and industry) classes where failure has *actually* occurred. The paired-sample design is not the only acceptable approach but is a convenient one, in view of the earlier study.

The use of the paired design limits the scope of inferences in at least one respect. It is possible that industry or asset size may be an excellent predictor of failure. While the paired design mitigates the disruptive influence of the industry and asset-size factors, it also virtually eliminates any predictive power these factors may have had. Therefore, the findings will not provide any insight into the potential predictive power of industry and asset size.

The use of the unpaired analysis assumes that the residual effects of industry and asset size are not very great. If this assumption is incorrect, the findings could seriously misstate the predictive ability of ratios. Evidence will be offered later as to the validity of the assumption. If some residual effects are present, the size effect could be incorporated into some form of multivariate analysis, since size is easily quantifiable. The same is not true of industry, where an analysis of its effects would be more complex. The problem of the effects of industry and asset size is complicated in still another way, because the pairing for asset size was not perfect.

The mean asset size of the failed firms is $6.3 million, while the mean asset size of the nonfailed was $8.5 million.[13] This situation provides an interesting illustration of the subtle ways in which systematic differences can be introduced. In order to be accepted, a nonfailed firm must fulfill

[13] The median differences in asset size were $4.4 million (failed), and $5.8 million (nonfailed).

two conditions: (1) it must be closest in asset size to the failed firm—special effort was made to select the closest firm, whether that firm was smaller or larger than the failed firm, (2) the firm must be in *Moody's*. Since the probability that a firm will be in *Moody's* increases as asset size increases, an acceptable nonfailed firm was more likely to be larger, rather than smaller, than the failed firm.

The implications of the imperfect pairing are difficult to assess. A test will be conducted to discover the extent to which the imperfect pairing biases the findings regarding the predictive ability of the ratios.

Collection of Financial Statement Data

Financial-statement data of the failed firms were obtained from *Moody's* for five years prior to failure. The "first year before failure" is defined as that year included in the most recent financial statement prior to the date that the firm failed. Another condition also had to be fulfilled—the financial statements could not be more than six months old at the date of failure. The "second year before failure" is the fiscal year preceding the first year. The third, fourth, and fifth years are similarly defined. The financial statements of the nonfailed firms were obtained for the same fiscal years as those of their failed mates.

The financial-statement data were then grouped according to year before failure. For example, if two firms failed in 1964 and 1954, respectively, and their most recent financial statements were prepared on December 31, 1963 and 1953, respectively, the first year before failure would include the 1963 statements of the former and the 1953 statements of the latter. The fifth year before failure would include the 1959 and 1949 statements. Since the firms failed from 1954 to 1964, inclusive, the first year before failure includes financial statements from an eleven-year period, 1953 through 1963, while the fifth year includes 1949 through 1959.

The financial-statement data of the nonfailed firms were also stratified into years before failure, corresponding to the years that were assigned to their failed mates. For example, the 1963 (1953) statements of the nonfailed mate to the first (second) firm mentioned above would be assigned to the first year before failure. Similarly, the 1959 (1949) statements would be assigned to the fifth year before failure.

Because financial-statement data were not available for every year before failure the number of observations decreases as the time period before failure increases. The sample size is largest in the first year before failure—158 firms—and is smallest in the fifth year—117 firms.[14] Availability of data over the five-year period was not made a criterion for inclusion in the sample, since a possible bias might be introduced in this

[14] The number of observations in each year before failure appears in Table A-3 in the Appendix.

TABLE 1

List of Ratios Tested[a]

GROUP I (CASH-FLOW RATIOS)	GROUP V (LIQUID-ASSET TO CURRENT DEBT RATIOS)
1. Cash flow to sales	1. Cash to current liabilities
2. Cash flow to total assets	2. Quick assets to current liabilities
3. Cash flow to net worth	3. Current ratio (current assets to current liabilities)
4. Cash flow to total debt	GROUP VI (TURNOVER RATIOS)
GROUP II (NET-INCOME RATIOS)	1. Cash to sales
1. Net income to sales	2. Accounts receivable to sales
2. Net income to total assets	3. Inventory to sales
3. Net income to net worth	4. Quick assets to sales
4. Net income to total debt	5. Current assets to sales
GROUP III (DEBT TO TOTAL-ASSET RATIOS)	6. Working capital to sales
1. Current liabilities to total assets	7. Net worth to sales
2. Long-term liabilities to total assets	8. Total assets to sales
3. Current plus long-term liabilities to total assets	9. Cash interval (cash to fund expenditures for operations)
4. Current plus long-term plus preferred stock to total assets	10. Defensive interval (defensive assets to fund expenditures for operations)
GROUP IV (LIQUID-ASSET TO TOTAL-ASSET RATIOS)	11. No-credit interval (defensive assets minus current liabilities to fund expenditures for operations)
1. Cash to total assets	
2. Quick assets to total assets	
3. Current assets to total assets	
4. Working capital to total assets	

[a] The components of the ratios are defined in the following manner: cash flow—net income plus depreciation, depletion, and amortization; net worth—common stockholders' equity plus deferred income taxes; cash—cash plus marketable securities; quick assets—cash plus accounts receivable; working capital—current assets minus current liabilities; fund expenditures for operations—operating expenses minus depreciation, depletion, and amortization; and defensive assets—quick assets.

manner. The bias would arise in that the failed firms would have demonstrated an ability to survive for at least five years; the result might be the elimination of the most serious failures.

Computation of Ratios

For every set of financial statements available, 30 ratios were computed.[15] Three criteria were used to select the 30 ratios from the set of all possible combinations and permutations of financial statement items. The first criterion was popularity—frequent appearance in the literature.[16] The ratios advocated in the literature are perceived by many to reflect the crucial relationships. It will be interesting to see the extent to which popularity will be self-defeating—that is, the most popular ratios

[15] The 30 ratios appear in Table 1.

[16] The selection of ratios was based upon their frequency of appearance in nineteen financial-statement analysis texts.

will become those most manipulated by management (an activity known as window dressing) in a manner that destroys their utility.

The second criterion was that the ratios performed well in one of the previous studies. This criterion will enable the study to examine the consistency of its findings with those of the previous studies. The third criterion was that the ratio be defined in terms of a "cash-flow" concept. The cash-flow ratios offer much promise for providing ratio analysis with a unified framework, yet as of now they are untested. The presence of any one of the criteria was a sufficient condition for inclusion in the study. In every instance, the ratio selected had been previously suggested in the literature; the study restricted itself to testing existing ratios rather than to developing new ones.

Although I excluded any ratio that was simply a transformation of another ratio already selected, the ratios still had common numerators or common denominators. In a multiratio analysis it is desirable to have each ratio convey as much additional information as possible—that is, the common elements should be reduced to a minimum. Accordingly, the 30 ratios were divided into six "common element" groups. Only one ratio from each group will be selected as a focus for the analysis.[17]

The analysis of the data will be divided into three sections: (1) a comparison of mean values, (2) a dichotomous classification test, and (3) an analysis of likelihood ratios.

Comparison of Mean Values

The mean of the ratios was computed for the failed firms and for the nonfailed firms in each of the years before failure. The comparison of mean values shall be called a profile analysis. A profile is defined as an outline or a concise sketch. In psychology the term *personality profile* is used to summarize how an individual performed on various tests which attempt to measure several personality dimensions. The term has been adopted here because it provides an accurate description of the kind of analysis that is provided by a comparison of means.

Profile analysis is not a predictive test; it is merely a convenient way of outlining the general relationships between the failed and nonfailed firms. The profile analysis also facilitates a comparison with previous studies, since their analysis was based entirely upon the mean values.

Theory of Ratio Analysis

The "theory" of ratio analysis is an extremely simple one and can best be explained within the framework of a "cash-flow" (i.e., liquid-asset

[17] The six ratios were selected on the basis of the lowest percentage error for their group over the five-year period. Where two ratios predicted about equally well, other factors entered, such as frequency of appearance in the literature and superior performance in previous studies. The percentage error refers to the dichotomous classification test which is explained on pp. 83–85.

flow) model.[18] The purpose of introducing the cash-flow model is not to have the model develop an optimal set of ratios, but rather to use the model as a vehicle for explaining the ratios being tested.

The firm is viewed as a reservoir of liquid assets, which is supplied by inflows and drained by outflows. The reservoir serves as a cushion or buffer against variations in the flows. The solvency of the firm can be defined in terms of the probability that the reservoir will be exhausted, at which point the firm will be unable to pay its obligations as they mature (i.e., failure).

Four concepts are important in drawing the relationship between the liquid-asset-flow model and the ratios. The first is the size of the reservoir itself. The second is the net liquid-asset flow from operations, which measures the net amount of liquid assets supplied to (or drained from) the reservoir by current operations. The third is the debt held by the firm and is one measure of the potential drain upon the reservoir. The fourth is the fund expenditures for operations and is the amount of liquid assets drained from the reservoir by operating expenditures. Given these concepts, four *ceteris paribus* propositions can be stated:

(1) The larger the reservoir, the smaller the probability of failure.

(2) The larger the net liquid-asset flow from operations (i.e., cash flow), the smaller the probability of failure.

(3) The larger the amount of debt held, the greater the probability of failure.

(4) The larger the fund expenditures for operations, the greater the probability of failure.

The four propositions can be used to form predictions regarding the mean values of six financial ratios. The six ratios are cash flow to total debt, net income to total assets, total debt to total assets, working capital to total assets, current ratios, and the no-credit interval.[19] The predictions appear in Table 2.

Analysis of Evidence

The difference in the mean values is in the predicted direction for each ratio in all five years before failure.[20] Failed firms not only have lower cash flow than nonfailed firms but also have a smaller reservoir of liquid assets. Although the failed firms have less capacity to meet obligations, they tend to incur more debt than do the nonfailed firms.

The trend line of the nonfailed firms has a zero slope, and the devia-

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[18] A more detailed discussion appears in James E. Walter's, "The Determination of Technical Solvency," *Journal of Business*, January, 1957, pp. 30–43.

[19] The interval measures appear in George H. Sorter's and George Benston's "Appraising the Defensive Position of the Firm: The Interval Measure," *The Accounting Review*, October, 1960, pp. 633–40. The basis for the selection of the six ratios was given in n. 17 on p. 14.

[20] See Fig. 1.

TABLE 2

Prediction of the Mean Values of Failed and Nonfailed Firms

Ratio	Prediction[a]
Cash flow to total debt[b]	Nonfailed > failed
Net income to total assets	Nonfailed > failed
Total debt to total assets[b]	Failed > nonfailed
Working capital to total assets	Nonfailed > failed
Current ratio	Nonfailed > failed
No-credit interval	Nonfailed > failed

[a] Nonfailed > failed is a prediction that the mean value of the nonfailed firms will be greater than that of the failed firms.

[b] Debt is defined as current plus long-term liabilities plus preferred stock.

tions from the trend line are small. Yet the deterioration in the means of the failed firms is very pronounced over the five-year period. The difference in means is evident for at least five years before failure, with the difference increasing as the year of failure approaches.

The data demonstrate a substantial degree of consistency. The evidence overwhelmingly suggests that there is a difference in the ratios of failed and nonfailed firms.

Comparison of Mean Asset Size

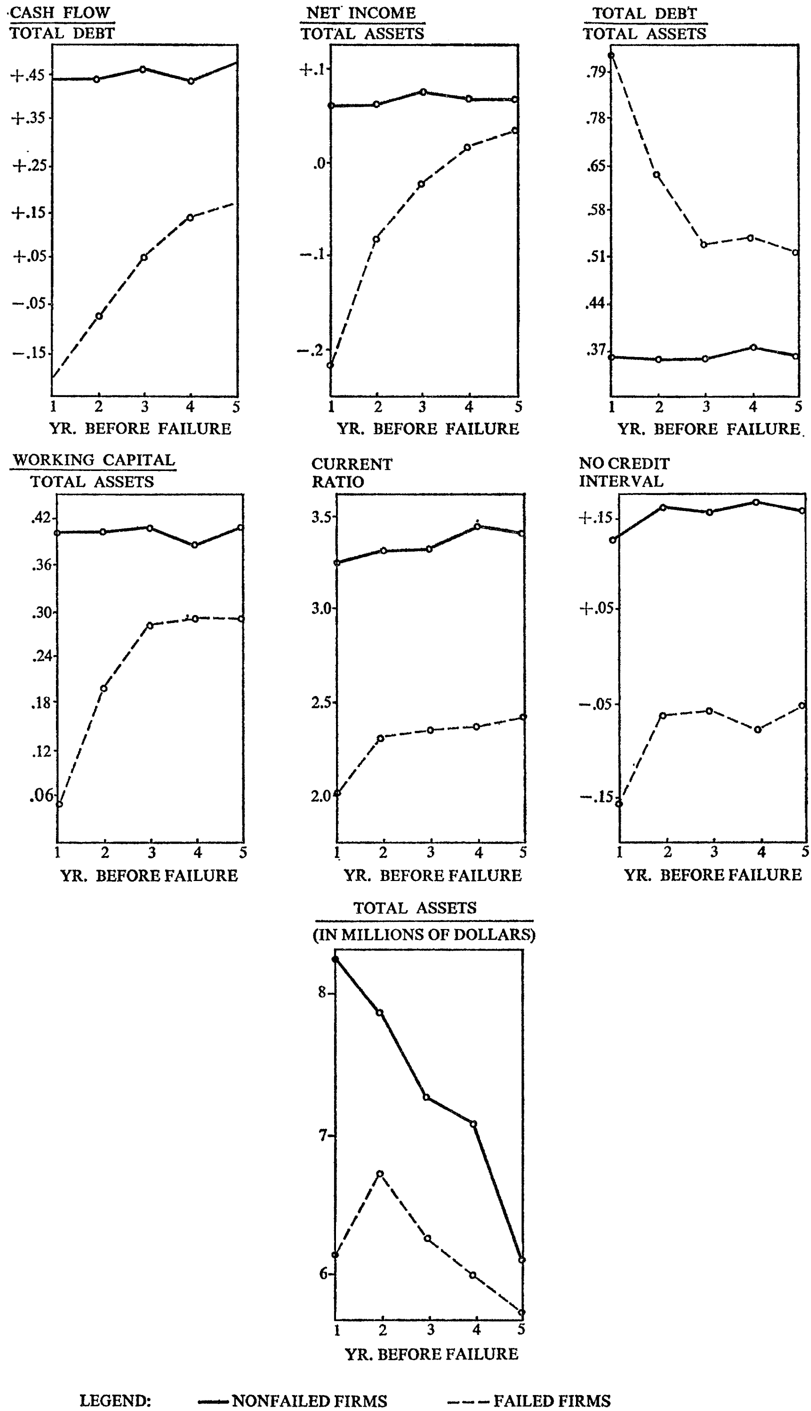
Both the failed and nonfailed firms grow in the second through fifth years, although the growth rate of the nonfailed firms is greater. In the first year before failure the nonfailed firms continue to grow while the total assets of the failed firms decline. The difference in asset size indicates that the pairing for size was not perfect. The implication of this finding will be examined later.

Comparison with Previous Studies

In 1932, Fitz Patrick published a study of nineteen pairs of failed and nonfailed firms.[21] His evidence indicated that there were persistent differences in the ratios for at least three years prior to failure. The Winakor and Smith study of 1935 investigated the mean ratios of failed firms for ten years prior to failure and found a marked deterioration in the mean values with the rate of deterioration increasing as failure approached.[22] In 1942, Merwin compared the mean ratios of continuing

[21] Paul J. Fitz Patrick, "A Comparison of Ratios of Successful Industrial Enterprises with Those of Failed Firms," *Certified Public Accountant*, October, November, and December, 1932, pp. 598–605, 656–62, and 727–31, respectively.

[22] Arthur Winakor and Raymond F. Smith, *Changes in Financial Structure of Unsuccessful Industrial Companies* (Bureau of Business Research, Bulletin No. 51 [Urbana: University of Illinois Press, 1935]).



LEGEND: —— NONFAILED FIRMS - - - FAILED FIRMS

FIG. 1. Profile analysis, comparison of mean values.

firms with those of discontinued firms for the period 1926 to 1936.[23] A difference in means was observed for as much as six years before discontinuance, while the difference increased as the year of discontinuance approached. The findings of all three studies support those of the profile analysis.

Limitations of Profile Analysis

Profile analysis can demonstrate that a difference between failed and nonfailed firms exists, but it cannot answer the crucial question: How large is the difference? The profile concentrates upon a single point on the ratio distribution—the mean. Without the additional knowledge of the dispersion about that point, no meaningful statement can be made regarding the predictive ability of a ratio.

Consider two ratio distributions, each of which is symmetrical, and assume that a difference in their means is observed. If the dispersion about the means is tight, there could be little or no overlap of one distribution on the other. Little or no overlap implies that the ratio would be an excellent predictor of failure. However, if the dispersion about the means is great, there could be quite a bit of overlap, which would indicate a lesser degree of predictive ability present.

If the distributions are skewed (i.e., nonsymmetrical), the extreme observations, which constitute a small proportion of the total number of firms, may be responsible for most of the difference in the means. Apart from two or three extreme firms, there could be a complete overlap of the distributions of failed and nonfailed firms. Previous studies indicate that ratio distributions do have pronounced skews.[24]

The preceding remarks imply that ratios may have little or no ability to predict failure, in spite of the differences in the means. The discussion suggests that some sort of predictive test is needed.[25]

Dichotomous Classification Test

The dichotomous classification test predicts the failure status of a firm, based solely upon a knowledge of the financial ratios. In contrast to the profile analysis, it is a predictive test, although it will not provide as much insight as likelihood ratios. Conducting this intermediate analysis has certain advantages.

The classification test is an intuitively appealing approach. It closely resembles the decision-making situation facing many users of ratios.

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[23] Charles L. Merwin, *Financing Small Corporations in Five Manufacturing Industries, 1926–36* (New York: National Bureau of Economic Research, 1942).

[24] *The Earning Power Ratios of Public Utility Companies* (Bureau of Business Research, Bulletin No. 15 [Urbana, Ill.: University of Illinois Press, 1927]).

[25] This discussion also implies that the use of standard ratios (i.e., average ratios for an industry) may have serious limitations.

For example, a bank's lending decision can be viewed as a dichotomous choice of accepting or rejecting a loan application. The object of the ratio analysis would be to classify firms as acceptable or not. Of course, the decision is not quite that simple. The bank must also decide how much to loan and at what rate to loan, if the firm is classified as acceptable. However, the example does illustrate the parallel between certain kinds of decisions and the classification test. The test also provides a convenient sifting device for selecting the six ratios that will serve as a focus for the analysis.

The classification test makes a dichotomous prediction—that is, a firm is either failed or nonfailed. In order to make the predictions, the data are arrayed (i.e., each ratio is arranged in ascending order). The array of a given ratio is visually inspected to find an optimal cutoff point—a point that will minimize the per cent of incorrect predictions. If a firm's ratio is below (or above, as in the case of the total debt to total-assets ratio) the cutoff point, the firm is classified as failed. If the firm's ratio is above (or below, for the total debt to total-assets ratio) the critical value, the firm is classified as nonfailed.

After each firm has been classified, the predictions are compared with the actual failure status, and the percentage of incorrect predictions is computed. This procedure was repeated for each of the 30 ratios; the results for six ratios appear in Table 3.[26] The percentage of misclassifications may be taken as a crude index of predictive ability—the smaller the error, the higher the degree of predictive ability present.

The process of finding the optimal cutoff point is largely one of trial and error. The test is open to criticism on the grounds that it selects *ex post* (i.e., after looking at the data) that point which will minimize the incorrect predictions. In a decision-making situation, the user of ratios does not have the benefit of such information and must predict on a new set of observations—that is, on a set of observations where the actual failure status is not known until sometime after the decision has been made.

The test was modified so that it more closely conformed to the decision-making situation. The sample was randomly divided into two subsamples. An optimal cutoff point was found for each subsample, and two tests were conducted. In the first test the firms in each subsample were classified using the cutoff point derived from their own subsample—which is the test outlined in the previous paragraphs. The percentage of incorrect predictions for this test are the numbers in parentheses in Table 3. In the second test the firms were classified using the cutoff point derived

[26] The six ratios were selected on the basis of the lowest percentage error for their group over the five-year period. Where two ratios predicted about equally well, other factors entered, such as frequency of appearance in the literature and superior performance in previous studies. The results for all 30 ratios appear in Table A-4 in the Appendix. The cutoff points used appear in Table A-5 in the Appendix.

TABLE 3

Percentage of Firms Misclassified[a]: Dichotomous Classification Test

Ratio	Year before Failure				
	1	2	3	4	5
Cash flow / Total debt	.13 (.10)	.21 (.18)	.23 (.21)	.24 (.24)	.22 (.22)
Net income / Total assets	.13 (.12)	.20 (.15)	.23 (.22)	.29 (.28)	.28 (.25)
Total debt / Total assets	.19 (.19)	.25 (.24)	.34 (.28)	.27 (.24)	.28 (.27)
Working capital / Total assets	.24 (.20)	.34 (.30)	.33 (.33)	.45 (.35)	.41 (.35)
Current ratio	.20 (.20)	.32 (.27)	.36 (.31)	.38 (.32)	.45 (.31)
No-credit interval	.23 (.23)	.38 (.31)	.43 (.30)	.38 (.35)	.37 (.30)
Total Assets	.38 (.38)	.42 (.42)	.45 (.42)	.49 (.41)	.47 (.38)

[a] The top row represents the results of the second test. The bottom row refers to the first test.

from the other subsample. The second test is similar to the classification problem facing the users of ratios. The results of the second test are those numbers which are not enclosed in parentheses in Table 3.

Analysis of Evidence (Percentage Error)

The ability to predict failure is strongest in the cash-flow to total-debt ratio. Unless otherwise indicated, the comments will refer to the second test, which is a more meaningful test. In the first year before failure the error is only 13 per cent, while in the fifth the error percentage is 22. Both error percentages are much smaller than would be expected from a random-prediction model, where the expected percentage error would be approximately 50 per cent. There is an extremely small probability that random prediction could have done as well.[27]

Clearly all ratios do not predict equally well. The net-income to total-assets ratio predicts second best, which is to be expected, since its correlation with the best ratio is higher than the correlation of any other ratio

[27] In both the first and fifth years, the probability is less than one in ten thousand.

with the best ratio. The total-debt to total-assets ratio predicted next best, with the three liquid-asset ratios performing least well. The most crucial factor seems to be the net liquid-asset flow supplied to the reservoir, while the size of the reservoir is the least important factor.

The explanation of relative predictive ability is an interesting topic, but is not discussed here because of the additional amount of analysis that would be required. Part of the answer can be found in an investigation into the behavior of the components of the ratios.[28]

Industry Effects

One crude test of the residual effects of industry is the amount of increase in percentage error from the first test to the second test, since the industry composition differs between the two subsamples. The increase in error is small, especially for the better ratios, and this finding is consistent with the assumption that the residual effects are not great.

Another indirect test is the increase in percentage error from the paired analysis to the unpaired analysis.[29] In most ratios the increase is small but persistent. Although the reduction in predictive power is by no means overwhelming, there seems to be a small residual effect of industry. To the extent that the effects are still present, the predictive ability of ratios is understated. Predictive ability would be expected to improve if the effect of industry were fully taken into account.

Both tests are not direct measures of the residual effects, because the increase in percentage errors could be due to other factors such as residual asset-size effects. The tests offer evidence of an indirect sort—that is, nothing has been observed to suggest that the residual effects are great.

Implications of Imperfect Pairing for Asset Size

The profile analysis indicated that the mean asset size of the nonfailed firms was greater than that of the failed firms. If the ratios are correlated with total assets and if total assets has some explanatory power of its own, the findings would overstate the predictive ability of ratios.

The correlation coefficients between asset size and the ratios are shown in Table 5. The square of the coefficient (r^2) can be interpreted as the proportion of the variance of the ratio that is explained by the variation in total assets. In three of the ratios, the r^2 is below 1 per cent for both subsamples, and the highest value of r^2 is 4.8 per cent. The r^2 can be properly interpreted as the proportion of variance explained, only if the relationship between the variables is bivariate normal. Bivariate normality is difficult to determine, since the joint distribution may not be bivariate normal even if the marginal distributions are. However, the

[28] Beaver, *op cit.*, p. 46.

[29] See Table 4.

TABLE 4

Comparison of Percentage Errors for Paired and Unpaired Classification Tests

Ratio	Year before Failure				
	1	2	3	4	5
Cash flow to total assets[a]					
Unpaired	.10	.20	.24	.28	.28
Paired	(.10)	(.13)	(.20)	(.23)	(.28)
Difference	.00	.07	.04	.05	.00
Net income to total assets					
Unpaired	.13	.21	.23	.29	.28
Paired	(.11)	(.15)	(.16)	(.38)	(.30)
Difference	.02	.06	.07	(.09)	(.02)
Total debt to total assets					
Unpaired	.19	.25	.34	.27	.28
Paired	(.15)	(.27)	(.30)	(.27)	(.39)
Difference	.04	(.02)	.04	.00	(.11)
Working capital to total assets					
Unpaired	.24	.34	.33	.45	.41
Paired	(.16)	(.22)	(.34)	(.26)	(.33)
Difference	.08	.12	(.01)	.19	.08
Current ratio					
Unpaired	.20	.32	.36	.38	.45
Paired	(.15)	(.25)	(.32)	(.32)	(.39)
Difference	.05	.07	.04	.06	.06
No-credit interval					
Unpaired	.23	.38	.43	.38	.37
Paired	(.21)	(.38)	(.43)	(.43)	(.43)
Difference	.02	.00	.00	(.05)	(.06)

[a] The cash-flow to total-debt ratio was not computed for the paired analysis. The cash-flow to total-asset ratio is offered as a substitute.

evidence presented in the next section will indicate that even the marginal distributions are not normal, which suggests that inferences based upon the r^2 must be treated with reservation. The findings can be interpreted in a negative sense—at least no evidence of strong correlation has been observed. The results are consistent with the hypothesis that the ratios are uncorrelated with asset size.

The implication of little or no correlation is that the predictive ability of the ratios is not overstated by the imperfect asset-size pairing. However, if strong correlation had been found, it would be impossible to isolate the marginal predictive ability of individual variables. The minimum possible overstatement would be zero; the maximum possible would

TABLE 5

Correlation of Total Assets with Six Ratios, First Year Before Failure

Ratio	Correlation[a] Coefficient (r)		Proportion of Variance Explained (r^2)	
	Subsample		Subsample	
	A	B	A	B
Cash flow / Total debt	.12	.20	.0144	.0400
Net income / Total assets	.22	.18	.0484	.0324
Total debt / Total assets	−.09	−.06	.0081	.0036
Working capital / Total assets	−.15	−.01	.0225	.0001
Current ratio	−.04	.02	.0016	.0004
No-credit interval	−.02	.15	.0004	.0225

[a] Coefficients rounded to nearest .01. Subsamples A and B refer to the two subsamples discussed on p. 84.

be the predictive ability of total assets. Table 3 suggests that the residual predictive power of total assets is small and is virtually nonexistent in the third through the fifth years.

Ultimately, I am unconcerned about the marginal predictive ability of ratios versus asset size, since my primary concern is with the utility of accounting data regardless of the form in which it is introduced into the analysis. To the extent that total assets has predictive power of its own, the study understates the predictive power of accounting data, because the effect of the paired design was to reduce any predictive power that asset size may have had.

Contingency Tables

The percentage of incorrect predictions suffers from two limitations. (1) In a world where the costs of misclassifying a failed firm are likely to be much greater than the costs associated with misclassifying a nonfailed firm, it is important to know the probability of misclassifying a failed firm (Type I error) versus the probability of misclassifying a nonfailed firm (Type II). (2) The difference in percentage error between the ratios and the random-prediction model can vary substantially by altering the probability of failure. If the probability of failure for the sample differs from that of the total population, a comparison of percentage of total errors is not very meaningful.

The Type I and Type II errors for a given sample will be the same as that for the population, even if the probability of failure for the population is different from that of the sample.[30] The previous statement requires one assumption—namely, that the ratio distributions of the sample accurately describe the ratio distributions of the population. Note also that the Type I and II errors discussed are those obtained when the cutoff points were selected on the basis of minimizing the total number of incorrect predictions without regard for the Type I and Type II errors implied. Since this might be a disastrous decision rule where the misclassification costs are asymmetrical, the cutoff points can be easily altered to any desired ratio of Type I to Type II errors.

One way of presenting Type I and Type II errors is through contingency tables, as shown in Table 6.[31] The row denotes the prediction made; the column denotes the actual status of the firm; and each cell contains the number of firms fulfilling each condition. For example, in the first year before failure, the total number of failed firms was 79, of which 62 were correctly classified as failed and 17 were not. The total number of nonfailed firms was also 79, of which 75 were correctly classified and 4 were not. The Type I error was 17/79 or 22 per cent, and the Type II error was 4/79 or 5 per cent. The total error was 21/158 or 13 per cent—the number previously referred to as the percentage error.

The Type I and Type II errors are probabilities of error conditional upon the actual status of the firm. Another set of conditional probabilities of error is the probability of error conditional upon the prediction made. The probability of error given the prediction, *failure,* is 4/66 or 6 per cent. The probability of error given the prediction, *nonfailure,* is 17/72 or 18 per cent. The two sets of conditional probabilities are merely different ways of expressing the same underlying relationships. Either can be appropriate depending upon the inferences desired.

It is interesting to compare the Type I and II errors of a ratio with those obtained from a random-prediction model. For example, consider a model constrained so that the ratio of failed to nonfailed predictions is the same as that of the financial ratio. In the first year before failure the Type I and II errors would be 58 per cent and 42 per cent, respectively, while in the fifth year the Type I and II errors would be 71 and 29 per cent, respectively. The random-prediction model could be altered in many other ways, but the tenor of the inferences would remain the same.

Analysis of Type I and Type II Errors

In the first year before failure the cash-flow to total-debt ratio has a striking ability to classify both failed and nonfailed firms to a much

[30] Unless otherwise stated, the paper shall refer to the probability of a Type I error simply as a Type I error, and similarly for a Type II error.

[31] The results for the cash-flow to total-debt ratio are shown in Table 6. The results for the other five ratios and for total assets appear in Table A-6 in the Appendix.

TABLE 6

Cash Flow to Total Debt: Contingency Table

Predicted Outcome	Year One			Predicted Outcome	Year Two			Predicted Outcome	Year Three		
	Actual Outcome				Actual Outcome				Actual Outcome		
	FLD	NFLD	Total		FLD	NFLD	Total		FLD	NFLD	Total
FLD	62	4	66	FLD	50	6	56	FLD	47	6	53
NFLD	17	75	92	NFLD	26	71	97	NFLD	28	69	97
Total	79	79	158	Total	76	77	153	Total	75	75	150
$\frac{21}{158} = .13$				$\frac{32}{153} = .21$				$\frac{34}{150} = .23$			

Predicted Outcome	Year Four			Predicted Outcome	Year Five		
	Actual Outcome				Actual Outcome		
	FLD	NFLD	Total		FLD	NFLD	Total
FLD	33	2	35	FLD	31	3	34
NFLD	29	64	93	NFLD	23	60	83
Total	62	66	128	Total	54	63	117
$\frac{31}{128} = .24$				$\frac{26}{117} = .22$			

greater extent than would be possible though random selection. The Type I and Type II errors are 22 per cent (vs. 58 per cent) and 5 per cent (vs. 42 per cent). In the fifth year before failure the errors are also smaller than would be obtained through random selection. The Type I and Type II errors are 42 per cent (vs. 71 per cent) and 4 per cent (vs. 29 per cent). There is an extremely small probability that random prediction could have done as well.[32]

The evidence further indicates that the ratio cannot classify failed and nonfailed firms with equal success. In each year before failure, the Type I error is greater than the Type II error. In fact, the Type II error is remarkably stable over the five-year period, while the Type I error increases as the time period before failure increases. The differences are 22 per cent versus 5 per cent (year one); 34 per cent versus 8 per cent (year two); 36 per cent versus 8 per cent (year three); 47 per cent versus 3 per cent (year four); and 42 per cent versus 4 per cent (year five). The analysis of the other ratios would have the same tenor, but the relationships

[32] In both years the probability is less than one in ten thousand.

would not be as strong as in the case of the cash-flow ratios. Both the Type I and Type II errors would tend to be higher for these ratios.

The findings suggest that ratio analysis can be useful for at least five years before failure, but ratios cannot be used indiscriminately. (1) Not all ratios predict equally well. The cash-flow to total-debt ratio predicts quite well five years before failure, but the same is not true of the liquid-asset ratios. (2) Ratios do not correctly predict failed and nonfailed firms with the same degree of success. Nonfailed firms can be correctly classified to a greater extent than the failed firms can. The implication is important. Even with the use of ratios, investors will not be able to completely eliminate the possibility of investing in a firm that will fail. This is a rather unfortunate fact of life, since the costs in that event are rather high.

Limitations of the Classification Test

One limitation of the test is that it treats the predictions made by the ratio as dichotomous. Even if the decision does involve a dichotomous choice, the conditional error given the prediction may differ depending upon the magnitude of the ratio. If the ratio is very far away from the cutoff point, more confidence will be placed in the prediction than if it were close. In other words, the magnitude of the ratio can also provide important information regarding the probability of error. This is not revealed by a dichotomous classification test, even when it is presented in the form of contingency tables.

A second limitation is that the specific values of the cutoff points obtained from the sample cannot be used in a decision-making situation. One reason is that the cutoff points have been selected without regard to the asymmetrical loss functions of Type I and Type II errors. Another is that the probability of failure for the population is not the same as that of the sample. Under these circumstances it is unlikely that the same cutoff point would be optimal. This does not imply that the conclusions regarding the predictive ability of ratios were incorrect, only that the specific values of the cutoff points cannot be directly carried over to a decision-making situation.

Analysis of Likelihood Ratios

Financial ratios can be viewed as a way of assessing the likelihood of failure. This approach possesses neither of the limitations of the classification test, and the ability to generalize about the predictive ability of ratios is facilitated. In order to assess the likelihood ratios from the financial ratios, histograms are prepared. Figure 2 contains histograms for the cash-flow to total-debt ratio.[33] The horizontal axis shows the

[33] Relative-frequency distributions have been prepared for the other five ratios and

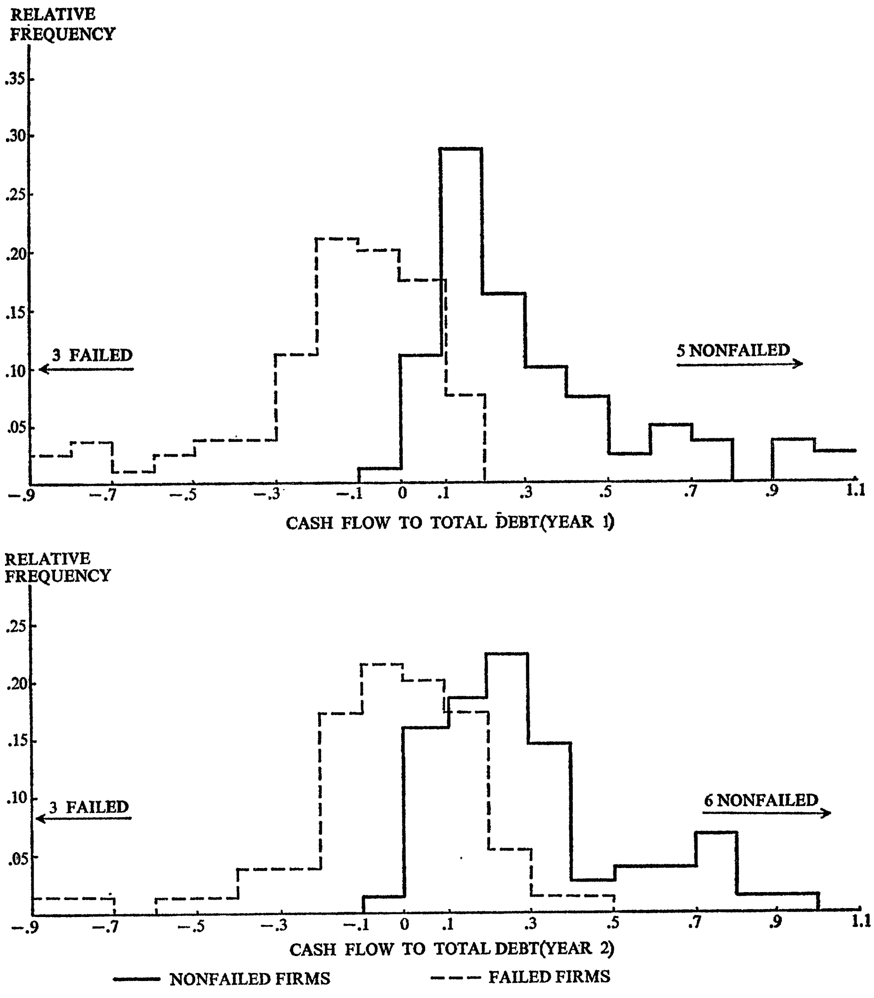

FIG. 2. Histogram.

values of the ratio, while the vertical axis indicates the relative frequency with which the ratios of failed (or nonfailed) firms fall into each class interval. For example, in the first year, 28 per cent of the ratios of the nonfailed firms fall in the interval .1 to .2, and 21 per cent of the ratios of failed firms fall in the interval −.1 to −.2.

Consistency of Data

Histograms can provide insights into financial ratios, apart from a likelihood ratio interpretation. For example, the inferences drawn from

total assets. The data appear in Table A-7 in the Appendix and contain the same information that a histogram contains.

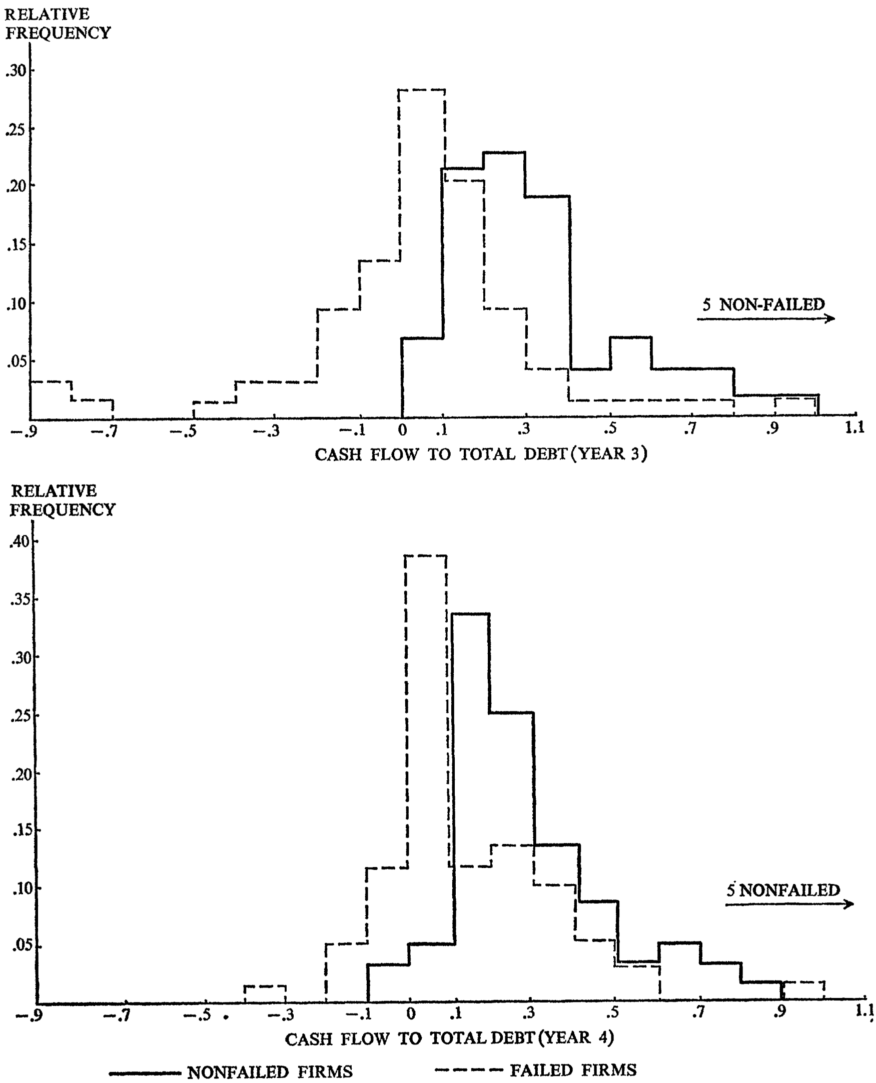

FIG. 2—Continued

the profile and the classification test are also suggested by an inspection of the histograms.

Figure 2 indicates that the distribution of nonfailed firms is remarkably stable in each of the years before failure. No shifting over time is apparent and, if the distributions from different years were superimposed, the overlap would be virtually complete. In all instances, there is a pronounced skew to the right, while the distribution drops off abruptly on the left. The stability suggests that the sample of nonfailed firms is comparable over time, even though their industry and asset-size composition changes. The distribution of failed firms shifts farther to the left as failure ap-

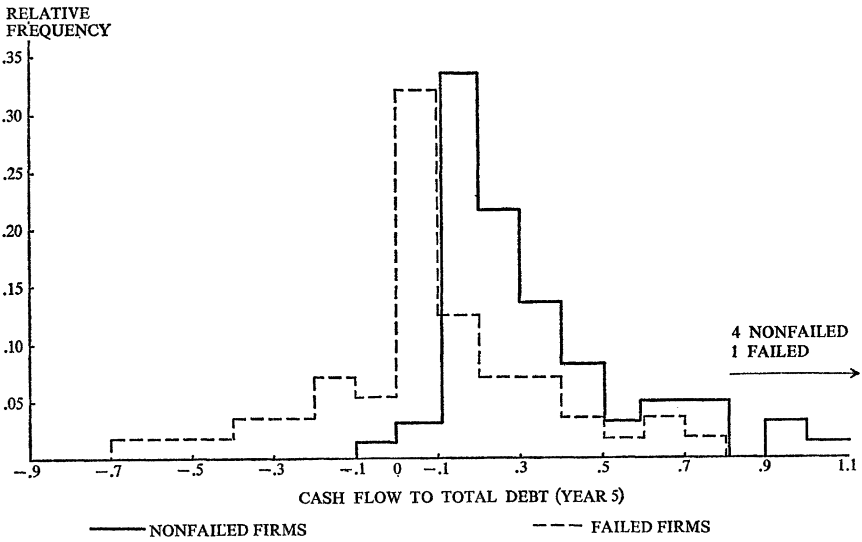

FIG. 2—Continued

proaches, while the gap between the failed and nonfailed firms becomes greater. Similar behavior was noted in the comparison of means in the profile analysis.

In the first year, the overlap of the two distributions is small, which leads to the low percentage error in the classification test. By the fifth year, the degree of overlap is greater and is reflected in the increase in the percentage of error. The tail area of the failed firms overlaps onto the nonfailed distribution to a greater extent than the tail area of the nonfailed firms overlaps onto the failed distribution. The result is the difference in Type I and Type II errors mentioned earlier. Similar relationships exist in the other five ratios but not to the same extent as in the cash-flow to total-debt ratio. The overlap is greater, an indication that their predictive power is not as strong.

An inspection of the histogram reinforces the conclusions reached in the earlier sections. However, the histograms reveal additional aspects of the ratios. Rather pronounced skews were evident in most of the distributions. The skews suggest that the data are not normally distributed.

Normality of Data

The normality of the data is best analyzed through the use of cumulative density functions (cdf). The vertical axis indicates the fractile associated with the corresponding value of the ratio, which is shown on the horizontal axis. The fractile is the proportion of the distribution whose values are equal to or below a given value of the ratio. The fractile for the i^{th} observation is the "$i/n + 1$" fractile, where "n" is the number of failed (or nonfailed) firms. The i^{th} observation is determined

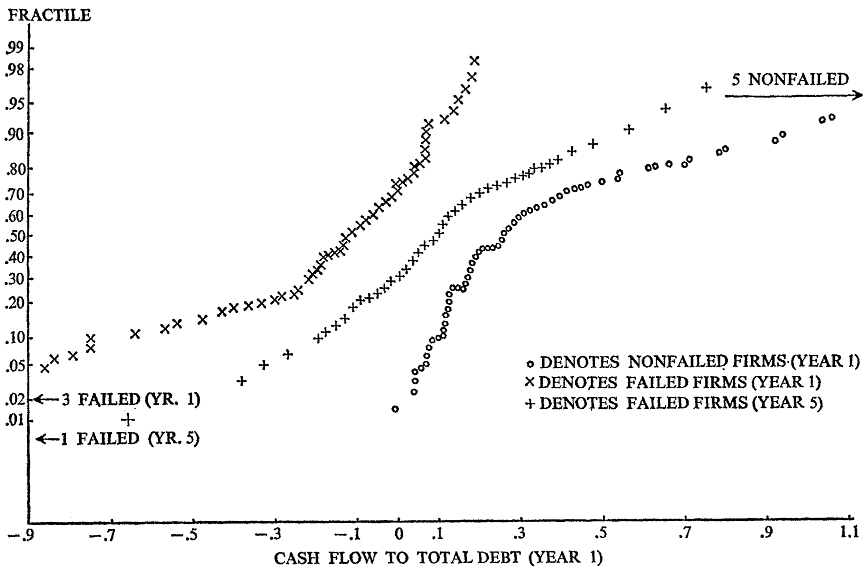

FIG. 3. CDF.

by arraying the data in ascending order. The lowest value is the first observation, while the highest value is the n^{th}. The .70 fractile of the failed firms (Fig. 3) in the first year is zero, while the .70 fractile for the nonfailed firms is .40. The vertical axis is calibrated according to the arithmetic probability scale. If the data are normal, the distribution will appear as a straight line. Departure from linearity can be interpreted as departure from normality.

The cash-flow to total-debt ratio has pronounced skews in each of the years before failure. The other five ratios also do not conform to the normality assumption.[34] Simple transformations (e.g., log and square root) have been performed on the ratios in the hope that the transformed variables would be approximately normal. The initial results indicate that the transformed variables are almost as badly skewed as the original variables. The departure from normality has serious implications for attempts to derive a multiratio model, since most of the existing multivariate techniques rely upon the normality assumption.

Likelihood Ratios

None of the discussion thus far has been directed toward the use of ratios as assessments of likelihood ratios. The study of financial ratios as predictors of failure is placed in its broadest context through the discussion of likelihood ratios. Their use is essentially a Bayesian approach and can be explained most conveniently by referring to the

[34] The data in Table A-7 can be used to form cdf's for the other five ratios. Table A-7 appears in the Appendix.

following tableau:

Event	Prior Probability	Likelihood	Joint Probability	Posterior Probability
Fail	$P(F)$	$P(R/F)$	$P(R,F)$	$P(F/R)$
Not fail	$P(\bar{F})$	$P(R/\bar{F})$	$P(R,\bar{F})$	$P(\bar{F}/R)$
	1.00		P(R)	1.0

The problem of predicting failure can be viewed as a problem in assessing the probability of failure conditional upon the value of the ratio—$P(F/R)$. The conditional probability of failure is shown in the last column, which is headed posterior probability. In arriving at estimates of the conditional probability of failure, the possible events are viewed as being dichotomous—either the firm will fail or it will not. Before looking at the financial ratios of the firm, certain prior probabilities are formed. The prior probabilities—$P(F)$ and $P(\bar{F})$—may be based upon several factors, such as the unconditional probability of failure for all firms, industry, asset size, or quality of management. Since failure and nonfailure are the only two events that can occur, the sum of prior probabilities is 1.00.

After the financial ratio is observed, assessments of the likelihood of failure and nonfailure are formed. The likelihood of failure is the probability that the observed numerical value of the ratio would appear if the firm were failed—$P(R/F)$; the likelihood of nonfailure is the probability that the specific value of the ratio would be observed if the firm were nonfailed—$P(R/\bar{F})$. The joint probabilities are the product of the prior probabilities times the likelihood estimates.[35] The sum of the joint probabilities is the marginal probability—$P(R)$—the probability that a ratio of the observed numerical value could occur.

The posterior probability is the quotient of the joint probability divided by the marginal probability.[36] The sum of the posterior probabili-

[35] The calculation is

$$P(F)P(R/F) = P(R,F)$$
$$P(\bar{F})P(R/\bar{F}) = P(R,\bar{F})$$

and is based upon Bayes' Theorem which states: Given two events A and B,

$$\frac{P(A,B)}{P(B)} = P(A/B).$$

[36] The calculation is

$$\frac{P(R,F)}{P(R)} = P(F/R)$$

$$\frac{P(R,\bar{F})}{P(R)} = P(F/\bar{R})$$

and is also based upon Bayes' Theorem.

ties must be 1.00. The posterior probability is the probability of failure (or nonfailure) after the ratio analysis.

The tableau can also be expressed in terms of odds ratios rather than probabilities. For example, if the probability an event will occur is .75 (which implies the probability of nonoccurrence is .25), it can also be said that the odds are 3 to 1 (i.e., .75/.25) that the event will occur. In fact, in many cases it is common practice to state the relationships in terms of odds rather than probabilities—the prediction of the outcome of horse races and boxing matches is a good example. The prior probabilities are replaced by the prior-odds ratio, the likelihood estimates by the likelihood-odds ratio, and the posterior probabilities by the posterior-odds ratio.[37] The following relationship exists among the three odds ratios:[38]

$$(\text{Prior-odds ratio}) \times (\text{likelihood-odds ratio}) = (\text{posterior-odds ratio}).$$

The discussion could be conducted in terms of the posterior-odds ratio. However, the ratio would largely be affected by the probability of failure for this particular sample (i.e., .50), which is vastly different from the probability for all firms in the economy (i.e., less than .01). The likelihood ratios are unaffected by the probability of failure and therefore carry with them a degree of generality. For example, consider a sample whose financial-ratio distributions accurately reflect the financial-ratio distributions of the population. The numerical values of the likelihood ratios, which are derived from the financial ratios of the sample, are the same ones that would apply to the population, even though the frequency of failure in the sample is vastly different from that of the entire population.

If the likelihood-odds ratio in favor of failure is greater than 1 (one), the user of the ratio, after having looked at the firm's ratio, will feel that the firm is more likely to fail—the higher the likelihood ratio, the stronger the feeling. If the likelihood ratio is less than 1, the user of the ratio will feel that the firm is less likely to fail—the lower the ratio, the stronger the feeling. If the likelihood ratio is exactly 1, the prior feelings of the user are unchanged after looking at the ratio—the poste-

[37] The odds are defined as follows:

$$\text{Prior-odds ratio} = \frac{P(F)}{P(\bar{F})}$$

$$\text{Likelihood-odds ratio} = \frac{P(R/F)}{P(R/\bar{F})}$$

$$\text{Posterior-odds ratio} = \frac{P(F/R)}{P(\bar{F}/R)}\,.$$

[38] $$\frac{P(F)}{P(\bar{F})} \times \frac{P(R/F)}{P(R/\bar{F})} = \frac{P(R,F)}{P(R,\bar{F})} = \frac{P(R,F)}{P(R)} \times \frac{P(R)}{P(R,\bar{F})} = \frac{P(F/R)}{P(\bar{F}/R)}\,.$$

rior-odds ratio will be numerically equal to the prior-odds ratio. The role of financial ratios is to provide assessments of the likelihood ratio. The information content of the ratios can be evaluated in terms of the degree to which they change the prior feeling.

The discussion is based upon estimates derived from the histograms. The likelihood estimates—$P(R/F)$ and $P(R/\bar{F})$—are obtained by measuring the heights of the failed and nonfailed distributions at a given value of the ratio. The likelihood ratio is the ratio of those two heights. For example, in the first year before failure, for the cash-flow to total-debt ratio, the likelihood estimates for the ratio interval .0 to .1 are .18 (failed firms) and .11 (nonfailed firms). The likelihood ratio would be 1.64 (.18/.11). In the interval $-.1$ to .0, the likelihood ratio would be 16.00, while in the interval $-.2$ to $-.1$, the likelihood ratio is undefined since there are not any nonfailed firms in that interval. Presumably, the likelihood ratio would be extremely high, judging by the behavior of the distributions in the surrounding class intervals.

If the ratio of the heights of the distributions is blindly used to compute the likelihood ratio, some rather erratic numbers will be obtained. A more reasonable approach would be to smooth the data by means of a moving average technique or an exponential function. No attempt was made to smooth the data nor were the numerical values of the likelihood ratio computed. The discussion shall be in terms of the general behavior of likelihood ratios rather than their specific numerical values. The data are available for the reader to carry out the numerical computations if he so desires.

Analysis of Evidence (Likelihood Ratios)

In the first year before failure, the likelihood ratio in favor of failure is either very large or very small over most of the range of the ratio. Looking at the financial ratio will lead the user to change his prior impressions a great deal. Even five years before failure, the likelihood ratio still takes on high values. The implication is that the ratio can convey useful information in determining solvency for at least five years prior to failure. In the tail areas of the distributions, the likelihood ratios are difficult to quantify because of the scarceness of observations in these areas. The development of smoothing techniques and an increasing of the sample size would produce more accurate measures of the likelihood ratios.

In the first year, the relationship between the likelihood ratio and the financial ratio is monotonic—the higher the financial ratio, the lower the likelihood ratio. This simple relationship makes the interpretation of the financial ratio relatively straightforward. However, in the fifth year the likelihood ratio decreases as the financial ratio increases over most, *but not all*, values of the ratio. In the highest values of the ratio, the likelihood ratio increases slightly. The implication is that it is slightly more risky for a firm to have a very high cash-flow to total-debt ratio than to have a lower one in a range where the bulk of the nonfailed firms appear. Here the interpretation of the ratio is not quite so simple as it was in the first year. The U-shaped behavior of the likelihood ratio was also observed in the other five ratios, even in the first year before failure. The phenomenon deserves more attention, and its explanation is one of the areas planned for future research.

The Classification Test Revisited

Since the probability of failure for the sample was .50 and the costs of misclassification were implicitly treated as being symmetrical, the optimal cutoff point was located where the likelihood ratio equaled 1.00. If either of these conditions is altered, the cutoff point will have to be shifted to another value of the likelihood ratio, which can be determined once the costs of misclassification and the probability of failure have been specified.[39] When the likelihood-ratio approach is used, the shifting of the cutoff point involves no substantive change in the analysis. The same likelihood ratios still apply, subject of course to sampling error.

Once a classification has been made, the likelihood ratios can be used to assess the conditional probability of error, which takes into account how far the financial ratio is from the cutoff point. This error assessment is superior to the conditional error derived from the contingency table, because the latter conditional error ignores how far a given firm's ratio may be from the cutoff point.

CONCLUDING REMARKS

Implication for Accounting

In a very real sense, the title of this paper should not be "Financial Ratios as Predictors of Failure" but rather "Accounting Data as Predictors of Failure." My primary concern is not with ratios per se but with the accounting data that comprise the ratios. One premise is crucial to an acceptance of this viewpoint. The premise is that accounting data can be evaluated in terms of their utility and that utility can be defined in terms of predictive ability. Possibly the most important contribution to be made by this paper is to suggest a methodology for the evaluation of accounting data for any purpose, not merely for solvency determination. I feel this approach will be helpful in resolving many of accounting's controversies.

The financial-lease controversy could be subjected to tests similar to the ones used here. The efficacy of capitalizing financial leases could

[39] A more complete discussion of this topic appears in Professor Neter's comments and in my reply to his comments.

be evaluated by computing two sets of financial ratios. One set would include the capitalized value of leases as debt, while the other set would not. The set of ratios that best predicted failure would suggest whether or not financial leases should be capitalized. A similar approach could be used for other controversies and for other predictive purposes.[40]

Suggestions for Future Research

The analysis conducted here has been a univariate analysis—that is, it has examined the predictive ability of ratios, one at a time. It is possible that a multiratio analysis, using several different ratios and/or rates of change in ratios over time, would predict even better than the single ratios. Some preliminary efforts have been undertaken to develop multiratio models, but the results have not been very encouraging in the sense that the best single ratio appears to predict about as well as the multiratio models.

As stated in the introductory remarks, the immediate purpose of this study was not to find the best predictor of failure but rather to investigate the predictive ability of financial ratios. That evaluation is enhanced by a comparison with other predictors of failure. Initially, efforts will be directed toward investigating market rates of return (i.e., dividends plus price changes) as predictors of failure. Market rates of return will provide a very powerful test since they reflect all of the sources of information available to investors—sources that not only include accounting data but other kinds of information as well.

A replication on a population of smaller firms would be another avenue for additional research. The Merwin study was based upon a sample of firms whose average asset size was $50,000, and yet the study produced differences in means comparable to those observed here. This suggests that the replication on smaller firms would be worthwhile, if the data were accessible.

The profile analysis indicated that the mean current ratio of the failed firms was above the magic "2:1" standard in all five years. In fact, in the final year the mean value is 2.02—which is about as close to 2.00 as possible. The evidence hints that failed firms may attempt to window dress. However, when the failed firms are compared with the nonfailed, their weaker position is evident. Perhaps the nonfailed firms window dress more successfully than do the failed firms. If so, attempts to window dress may tend to improve the predictive power of ratios rather than impair it, as is often suggested. No pretense is made that this is anything other than a causal glance at the data. However, it does suggest an intriguing hypothesis to be investigated at some future date.

[40] An example would be Phillip Brown's "The Predictive Abilities of Alternative Income Concepts" (unpublished manuscript presented before the Workshop in Accounting Research, University of Chicago, Graduate School of Business, April, 1966).

Summary and Conclusions

Before restating the conclusions, I would like to provide one final word of warning regarding the uncertainty surrounding this empirical study. There are many factors that have prevented a measurement of the "true" predictive ability of ratios. In addition to the factors previously mentioned, two more come to mind. Both stem from the fact that financial ratios are in widespread use as measures of solvency.

If ratios are used to detect the financial "illness" of a firm, there may be many firms whose illnesses were detected before failure occurred. In these cases, the proper treatment was applied, and the firms did not fail. The sample of failed firms will include those firms whose illnesses were not detectable through ratios. This is a biased sample for investigating the usefulness of ratios. An important piece of information is missing—how many firms were saved from failure because their problems were detected in time through the use of ratios? Such information would be difficult, if not impossible, to obtain. However, this argument suggests that the study may be understating the usefulness of ratios.

On the other hand, ratios are used by financial institutions to determine the credit-worthiness of its borrowers. In many cases, lines of credit may be severed by the lending institutions because the borrowing firm failed to improve its financial ratio to a "respectable" level. This argument suggests that the predictive ability of ratios may be overstated, relative to what it would be if ratios were not so popular. There exists a countless number of arguments on both sides, regarding the possible biases in the data. This brief discussion should suggest that the conclusions that follow are arrived at with much less than perfect certainty.

The ratio distributions of nonfailed firms are quite stable throughout the five years before failure. The ratio distributions of the failed firms exhibit a marked deterioration as failure approaches. The result is a widening gap between the failed and nonfailed firms. The gap produces persistent differences in the mean ratios of failed and nonfailed firms, and the difference increases as failure approaches.

For the cash-flow to total-debt ratio the overlap of the two distributions is small in the first year before failure. By the fifth year, the overlap has increased but the two distributions are still distinguishable. The degree of overlap is reflected in the ability to correctly predict the failure status of firms. The cash-flow to total-debt ratio has the ability to correctly classify both failed and nonfailed firms to a much greater extent than would be possible through random prediction. This ability exists for at least five years before failure.

Although ratio analysis may provide useful information, ratios must be used with discretion: (1) Not all ratios predict equally well. The cash-flow to total-debt ratio has excellent discriminatory power throughout the five-year period. However, the predictive power of the liquid asset ratios is much weaker. (2) The ratios do not predict failed and nonfailed firms with the same degree of success. Nonfailed firms can be correctly classified to a greater extent than can failed firms. The implication is that the investor will not be able to completely eliminate the possibility of investing in a firm that will fail. This is a rather unfortunate fact of life, since the costs in that event are rather high.

When financial ratios are used to assess likelihood ratios, the cash-flow to total-debt ratio produces large likelihood ratios, even five years before failure. Looking at the financial ratio will lead the user to shift his prior feelings a great deal. In general, the relationship between the likelihood ratio and the cash-flow to total-debt ratio is a monotonic one. However, in the fifth year before failure, there is a slight increase in the likelihood ratio in the high values of the cash-flow ratio. This finding implies that it is more risky to have a very high ratio than to have a lower one in a range where most of the nonfailed firms appear.

The data exhibit a remarkable degree of consistency among themselves and with previous studies. Subject to the reservations that must accompany inferences drawn from this study, the evidence indicates that ratio analysis can be useful in the prediction of failure for at least five years before failure.

APPENDIX

TABLE A-1

Year of Failure

Year of Failure	No. of Failed Firms
1954	2
1955	6
1956	14
1957	7
1958	6
1959	4
1960	7
1961	10
1962	9
1963	10
1964	4
Total	79

TABLE A-2
Industrial Classification

Industry[a]	No. of Failed Firms
Manufacturing	
Food products (201)	1
Grain mill products (204)	1
Bakery products (205)	3
Confectionery (207)	1
Beverages (208)	3
Textile (221)	4
Women's apparel (233)	1
Industrial chemicals and plastics (281)	3
Petroleum refining (291)	3
Rubber products (301)	1
Leather products (311)	2
Clay products (325)	1
Concrete, gypsum (327)	1
Iron and Steel foundries (331)	2
Nonferrous metals (333)	1
Cutlery, etc. (342)	3
Heating and plumbing (343)	2
Structural metal products (344)	1
Etching, engraving (347)	2
Engines, industrial machinery (351)	3
Service, industrial machines (358)	1
Household appliances, etc. (363)	5
Commercial equipment (366)	1
Electronics (367)	6
Motor vehicles (371)	5
Aircraft (372)	3
Scientific instruments (381)	1
Watches (387)	2
Toys, sporting goods (394)	1
Nonmanufacturing	
Public warehouse (422)	1
Hardware, wholesale (507)	1
General merchandise (532)	3
Grocery stores (541)	2
Apparel stores (561)	3
Furniture stores (571)	1
Drug stores (591)	2
Personal services (721)	1
Motion pictures (781)	1

[a] The number in parentheses indicates the three-digit SIC number.

TABLE A-3

Number of Observations on Which Financial-Statement Data Could Be Obtained

Item	Year before Failure				
	1	2	3	4	5
Failed firms	79	76	75	62	54
Nonfailed firms	79	77	75	66	63
Total number	158	153	150	128	117

TABLE A-4

Percentage Error for 30 Ratios on Dichotomous Classification Test[a]

Ratio	Year before Failure 1	2	3	4	5
Group I					
Cash flow / Sales	.14 (.11)	.20 (.19)	.29 (.24)	.37 (.33)	.44 (.31)
Cash flow / Total assets	.10 (.10)	.20 (.17)	.24 (.20)	.28 (.26)	.28 (.25)
Cash flow / Net worth	.13 (.11)	.21 (.16)	.28 (.24)	.38 (.31)	.37 (.32)
Cash flow / Total debt	.13 (.10)	.21 (.18)	.23 (.21)	.24 (.24)	.22 (.22)
Group II					
Net income / Sales	.13 (.09)	.22 (.16)	.28 (.24)	.35 (.28)	.31 (.27)
Net income / Total assets	.13 (.12)	.21 (.15)	.23 (.22)	.29 (.28)	.28 (.25)
Net income / Net worth	.13 (.10)	.26 (.18)	.26 (.26)	.34 (.31)	.40 (.28)
Net income / Total debt	.15 (.08)	.20 (.16)	.22 (.20)	.26 (.26)	.32 (.26)
Group III					
Current liabilities / Total assets	.30 (.27)	.41 (.28)	.36 (.34)	.40 (.33)	.46 (.30)
Long-term liabilities / Total assets	.36 (.32)	.45 (.40)	.42 (.40)	.47 (.38)	.51 (.41)
Current + long term liabilities / Total assets	.23 (.19)	.30 (.26)	.36 (.29)	.39 (.31)	.38 (.33)
Current + long term + preferred / Total assets	.19 (.19)	.25 (.24)	.34 (.28)	.27 (.24)	.28 (.27)
Group IV					
Cash / Total assets	.28 (.25)	.29 (.28)	.30 (.30)	.36 (.34)	.38 (.31)
Quick assets / Total assets	.38 (.34)	.42 (.36)	.36 (.36)	.48 (.37)	.40 (.34)

[a] Top row of numbers are the results of the second test; the bottom row of numbers, enclosed in parentheses, refer to the results of the first test.

TABLE A-4—Continued

Ratio	Year before Failure				
	1	2	3	4	5
Group IV—*Continued*					
Current assets / Total assets	.38 (.37)	.48 (.44)	.48 (.43)	.47 (.43)	.49 (.38)
Working capital / Total assets	.24 (.20)	.34 (.30)	.33 (.33)	.45 (.35)	.41 (.35)
Group V					
Cash / Current liabilities	.22 (.22)	.28 (.24)	.36 (.28)	.38 (.34)	.38 (.29)
Quick assets / Current liabilities	.24 (.24)	.32 (.30)	.40 (.28)	.34 (.34)	.37 (.29)
Current assets / Current liabilities	.20 (.20)	.32 (.27)	.36 (.31)	.38 (.32)	.45 (.31)
Group VI					
Cash / Sales	.34 (.30)	.24 (.24)	.36 (.34)	.43 (.39)	.45 (.41)
Receivables / Sales	.46 (.40)	.45 (.39)	.46 (.38)	.43 (.37)	.42 (.38)
Inventory / Sales	.47 (.40)	.50 (.50)	.54 (.47)	.48 (.42)	.53 (.42)
Quick assets / Sales	.46 (.40)	.47 (.42)	.45 (.40)	.52 (.48)	.44 (.42)
Cur. assets / Sales	.44 (.42)	.51 (.40)	.48 (.42)	.49 (.47)	.51 (.47)
Working capital / Sales	.26 (.23)	.33 (.27)	.42 (.35)	.46 (.40)	.40 (.37)
Net worth / Sales	.32 (.28)	.36 (.33)	.44 (.38)	.42 (.38)	.40 (.38)
Total assets / Sales	.37 (.34)	.42 (.40)	.38 (.34)	.42 (.38)	.44 (.39)
Cash Interval	.33 (.26)	.27 (.27)	.35 (.32)	.42 (.42)	.43 (.38)
Defensive Interval	.39 (.34)	.41 (.40)	.46 (.41)	.48 (.47)	.51 (.43)
No-credit Interval	.23 (.23)	.38 (.31)	.43 (.30)	.38 (.35)	.37 (.30)

[a] Top row of numbers are the results of the second test; the bottom row of numbers, enclosed in parentheses, refer to the results of the first test.

TABLE A-5

Cutoff Points Used in Classification Test for Six Ratios[a]

Ratio	Year before Failure 1		2		3		4		5	
	A	B	A	B	A	B	A	B	A	B
Cash flow / Total debt	.03	.07	.05	.07	.10	.09	.09	.09	.11	.11
Net income / Total assets	.00	.02	.01	.02	.03	.03	.02	.02	.04	.03
Total debt / Total assets	.57	.57	.51	.49	.53	.50	.58	.57	.57	.57
Working capital / Total assets	.19	.27	.33	.28	.26	.26	.40	.24	.43	.29
Current assets / Current liab.	1.6	1.6	2.3	1.7	2.3	1.8	2.6	1.8	2.8	2.1
No credit interval	−.04	−.04	.03	−.02	.01	−.01	.00	−.01	.04	−.02
Total assets[b]	4.6	4.6	5.2	5.2	4.4	4.4	5.5	4.0	4.8	2.5

[a] A and B columns refer to cutoff points used on the two subsamples, A and B.
[b] Expressed in millions of dollars.

TABLE A-6

Contingency Tables: Five Ratios and Total Assets

Predicted Outcome	Actual Outcome								
	Net Income to Total Assets			Total Debt to Total Assets			Working Capital to Total Assets		
	F[a]	$\bar{F}$	T	F	$\bar{F}$	T	F	$\bar{F}$	T
Year One									
F	66	9	75	56	7	63	54	13	67
$\bar{F}$	13	70	83	23	72	95	25	66	91
T	79	79	158	79	79	158	79	79	158
Year Two									
F	57	13	70	53	16	69	48	24	72
$\bar{F}$	19	64	83	23	61	84	28	53	81
T	76	77	153	76	77	153	76	77	153
Year Three									
F	49	9	58	36	12	48	38	13	51
$\bar{F}$	26	66	92	39	63	102	37	62	99
T	75	75	150	75	75	150	75	75	150
Year Four									
F	34	9	43	43	14	57	30	25	55
$\bar{F}$	28	57	85	19	52	71	32	41	73
T	62	66	128	62	66	128	62	66	128
Year Five									
F	36	15	51	25	4	29	36	30	66
$\bar{F}$	18	48	66	29	59	88	18	33	51
T	54	63	117	54	63	117	54	63	117

Predicted Outcome	Actual Outcome								
	Current Ratio			No-Credit Interval			Total Assets		
	F	$\bar{F}$	T	F	$\bar{F}$	T	F	$\bar{F}$	T
Year One									
F	55	7	62	52	9	61	46	27	73
$\bar{F}$	24	72	96	27	70	97	33	52	85
T	79	79	158	79	79	158	79	79	158
Year Two									
F	44	18	62	49	31	80	46	34	80
$\bar{F}$	32	59	91	27	46	73	30	43	73
T	76	77	153	76	77	153	76	77	153
Year Three									
F	42	21	63	46	36	82	40	32	72
$\bar{F}$	33	54	87	29	39	68	35	43	78
T	75	75	150	75	75	150	75	75	150
Year Four									
F	34	21	55	35	21	56	31	32	63
$\bar{F}$	28	45	73	27	45	72	31	34	65
T	62	66	128	62	66	128	62	66	128
Year Five									
F	33	31	64	34	23	57	25	26	51
$\bar{F}$	21	32	53	20	40	60	29	37	66
T	54	63	117	54	63	117	54	63	117

[a] F denotes failed, $\bar{F}$ denotes nonfailed, and T denotes total. Cells contain number of firms fulfilling conditions regarding predicted and actual outcomes.

TABLE A-7

Relative Frequency Distribution of Five Ratios and Total Assets

Class Interval of Net Income to Total Assets	Year before Failure 1 F[a]	1 $\bar{F}$	5 F	5 $\bar{F}$
Below −.65	.051	.000	.000	.000
−.65 to −.60	.025	.000	.000	.000
−.60 to −.55	.000	.000	.000	.000
−.55 to −.50	.000	.000	.000	.000
−.50 to −.45	.038	.000	.018	.000
−.45 to −.40	.013	.000	.000	.000
−.40 to −.35	.013	.000	.000	.000
−.35 to −.30	.051	.000	.018	.000
−.30 to −.25	.038	.000	.018	.000
−.25 to −.20	.089	.000	.018	.000
−.20 to −.15	.127	.000	.037	.000
−.15 to −.10	.076	.000	.037	.000
−.10 to −.05	.102	.000	.037	.016
−.05 to −.00	.202	.038	.093	.032
.00 to .05	.152	.367	.425	.318
.05 to .10	.025	.418	.223	.430
.10 to .15	.000	.140	.037	.127
.15 to .20	.000	.025	.000	.048
.20 to .25	.000	.013	.018	.032
.25 to .30	.000	.000	.018	.000
Above .30	.000	.000	.000	.000
Total Relative Frequency	1.000	1.000	1.000	1.000

Class Interval ot Total Debt to Total Assets[b]	Year before Failure 1 F	1 $\bar{F}$	5 F	5 $\bar{F}$
0 to .1	.013	.063	.018	.032
.1 to .2	.025	.152	.093	.206
.2 to .3	.038	.152	.112	.143
.3 to .4	.089	.240	.112	.143
.4 to .5	.076	.165	.093	.221
.5 to .6	.102	.152	.183	.201
.6 to .7	.152	.051	.183	.015
.7 to .8	.038	.025	.074	.032
.8 to .9	.076	.000	.074	.000
.9 to 1.0	.102	.000	.037	.000
1.0 to 1.1	.063	.000	.000	.000
1.1 to 1.2	.038	.000	.000	.000
1.2 to 1.3	.076	.000	.000	.000
1.3 to 1.4	.025	.000	.000	.000
1.4 to 1.5	.013	.000	.018	.000
1.5 to 1.6	.025	.000	.000	.000
1.6 to 1.7	.000	.000	.000	.000
1.7 to 1.8	.000	.000	.000	.000
1.8 to 1.9	.000	.000	.000	.000
1.9 to 2.0	.000	.000	.000	.000
above 2.0	.038	.000	.000	.000
Total Relative Frequency	1.000	1.000	1.000	1.000

Class Interval of Working Capital to Total Assets	Year before Failure 1 F	1 $\bar{F}$	5 F	5 $\bar{F}$
Below −1.0	.025	.000	.000	.000
−1.0 to −.9	.013	.000	.000	.000
−.9 to −.8	.000	.000	.000	.000
−.8 to −.7	.000	.000	.000	.000
−.7 to −.6	.013	.000	.000	.000
−.6 to −.5	.025	.000	.000	.000
−.5 to −.4	.025	.000	.000	.000
−.4 to −.3	.051	.000	.000	.000
−.3 to −.2	.038	.000	.000	.000
−.2 to −.1	.038	.000	.037	.000
−.1 to 0	.114	.000	.037	.000
0 to .1	.139	.025	.148	.048
.1 to .2	.177	.089	.130	.095
.2 to .3	.114	.152	.148	.111
.3 to .4	.063	.254	.183	.238
.4 to .5	.063	.228	.148	.206
.5 to .6	.038	.127	.111	.190
.6 to .7	.051	.114	.018	.048
.7 to .8	.013	.013	.037	.064
.8 to .9	.000	.000	.000	.000
.9 to 1.0	.000	.000	.000	.000
Total Relative Frequency	1.000	1.000	1.000	1.000

Class Interval of Current Ratio	Year before Failure 1 F	1 $\bar{F}$	5 F	5 $\bar{F}$
0 to .5	.139	.000	.018	.000
.5 to 1.0	.202	.000	.055	.000
1.0 to 1.5	.254	.013	.223	.079
1.5 to 2.0	.165	.152	.202	.143
2.0 to 2.5	.038	.254	.186	.159
2.5 to 3.0	.038	.139	.129	.190
3.0 to 3.5	.038	.152	.055	.095
3.5 to 4.0	.013	.063	.037	.048
4.0 to 4.5	.038	.038	.000	.048
4.5 to 5.0	.013	.038	.055	.048
5.0 to 5.5	.000	.025	.000	.037
5.5 to 6.0	.000	.013	.000	.048
6.0 to 6.5	.000	.051	.000	.037
6.5 to 7.0	.000	.013	.000	.048
7.0 to 7.5	.000	.025	.000	.000
7.5 to 8.0	.000	.000	.000	.000
8.0 to 8.5	.038	.013	.018	.000
8.5 to 9.0	.000	.000	.000	.000
9.0 to 9.5	.013	.000	.000	.000
9.5 to 10.0	.013	.000	.000	.000
Above 10.0	.000	.013	.018	.018
Total Relative Frequency	1.000	1.000	1.000	1.000

TABLE A - 7—Continued

Class Interval of No-Credit Interval	Year before Failure				Class Interval of Total Assets[b]	Year before Failure			
	1		5			1		5	
	F	$\bar{F}$	F	$\bar{F}$		F	$\bar{F}$	F	$\bar{F}$
Below −.9	.013	.000	.037	.000	0 to 1	.051	.000	.074	.016
−.9 to −.8	.013	.000	.000	.000	1 to 2	.127	.051	.185	.111
−.8 to −.7	.013	.000	.000	.000	2 to 3	.139	.114	.111	.175
−.7 to −.6	.013	.000	.000	.000	3 to 4	.114	.127	.148	.175
−.6 to −.5	.025	.000	.000	.000	4 to 5	.238	.114	.130	.048
−.5 to −.4	.051	.000	.000	.000	5 to 6	.051	.114	.074	.064
−.4 to −.3	.102	.000	.018	.000	6 to 7	.000	.051	.055	.032
−.3 to −.2	.076	.013	.055	.000	7 to 8	.089	.063	.074	.048
−.2 to −.1	.165	.000	.129	.048	8 to 9	.038	.038	.018	.095
−.1 to 0	.263	.316	.315	.286	9 to 10	.025	.051	.037	.079
0 to .1	.190	.392	.260	.334	10 to 11	.051	.089	.018	.032
.1 to .2	.063	.165	.074	.206	11 to 12	.000	.025	.000	.016
.2 to .3	.000	.063	.055	.032	12 to 13	.000	.000	.000	.000
.3 to .4	.013	.000	.018	.032	13 to 14	.013	.025	.000	.016
.4 to .5	.000	.000	.000	.032	14 to 15	.013	.013	.000	.032
.5 to .6	.000	.025	.018	.000	15 to 16	.013	.013	.018	.016
.6 to .7	.000	.000	.000	.000	16 to 17	.000	.038	.000	.016
.7 to .8	.000	.013	.000	.000	17 to 18	.013	.013	.018	.000
.8 to .9	.000	.000	.000	.000	18 to 19	.000	.013	.000	.016
.9 to 1.0	.000	.000	.000	.000	19 to 20	.000	.000	.018	.000
1.0 to 1.1	.000	.000	.018	.000	Above 20	.038	.051	.018	.016
Above 1.1	.000	.013	.000	.032					
Total Relative Frequency	1.000	1.000	1.000	1.000	Total Relative Frequency	1.000	1.000	1.000	1.000

[a] F denotes distribution of failed firms, while $\bar{F}$ denotes distribution of nonfailed firms.

[b] In millions of dollars.