

The Sub-column determination problem is essentially one of bit width β search. Our proposed algorithm is a cost-model-based algorithm to find optimal bit width of sub-columns. The unique challenges specific to this problem is to find optimal storage cost of sub-columns with less compression time. We replace the exhaustive search for the optimal sub-column bit width with a Particle-Swarm-Optimization (PSO) based procedure in Algorithm 4. In Lines 4-9, algorithm initial i -th particle's position p_i , its velocity v_i , the best i -th position $p_{best,i}$, the minimum cost $Cost_{min,i}$, the global best position p_{best} , and the global minimum cost C_{min} . Then, after t_{max} iterations, the algorithm updates the minimum cost for each particle and the global minimum cost. Particles update their velocities and positions using the standard PSO rule with inertia w and cognitive/social coefficients c_1, c_2 in Lines 12-13. The global minimum cost C_{min} is updated whenever any particle finds a lower cost in Lines 14-18. Finally, in the vicinity of the optimal solution β' found by the global search of PSO, another local refinement is performed in Lines 19-23.

Since Algorithm 4 updates s particles with t_{max} iterations, the total runtime is approximately $s * t_{max} * n$ and the time complexity is $O(n)$. The parameters s and t_{max} could be tuned to trade off computational cost against solution quality, which influences compression ratio. This method requires far fewer time cost evaluations than a full exhaustive search for large M in Algorithm 1, when $s * t_{max}$ is far smaller than M .

Algorithm 4: Sub-column Determination with PSO

Input: Series $X = (x_1, x_2, \dots, x_n)$, PSO hyperparams: swarmSize s , maxIter t_{max} , inertia w , cognitive coefficients c_1 , social coefficients c_2 , localRadius R

Output: Bit width β'

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1  $M = \lceil \log(x_{max} - x_{min} + 1) \rceil$  ;
2 for  $i \leftarrow 1$  to  $s$  do
3    $p_i = U(1, M)$  ;
4    $v_i = U(-(M-1)/2, (M-1)/2)$ ;
5    $p_{best,i} = p_i$ ;
6    $Cost_{min,i} = Cost(X, round(p_i))$ ;
7  $p_{best} = \arg \min_i Cost_{min,i}$  ;
8  $C_{min} = \min Cost_{min,i}$ ;
9  $v_{max} = M$ ;
10 for  $t \leftarrow 1$  to  $t_{max}$  do
11   for  $i \leftarrow 1$  to  $s$  do
12      $r_1 = U(0, 1), r_2 = U(0, 1)$ ,
13      $v_i = w \cdot v_i + c_1 r_1 (p_{best,i} - p_i) + c_2 r_2 (p_{best} - p_i)$ ,
14     clamp  $v_i$  to  $[-v_{max}, v_{max}]$ ;
15      $p_i = p_i + v_i$ , clamp  $p_i$  to  $[1, M]$ ;
16      $\beta = round(p_i)$ ;
17     if  $C(X, \beta) < p_{best,i}$  then
18        $Cost_{min,i} = C(X, \beta), p_{best,i} = p_i$ ;
19     if  $C(X, \beta) < C_{min}$  then
20        $C_{min} = C(X, \beta), p_{best} = p_i$ ;
21  $\beta' = round(p_{best})$ ;
22 for  $b \leftarrow \beta' - R$  to  $\beta' + R$  do
23   if  $1 \leq b \leq M$  then
24     if  $Cost(X, b) < C_{min}$  then
25        $C_{min} = Cost(X, b), \beta' = b$  ;
26 return  $\beta'$ ;
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