HOW TO FIND AN EULER CIRCUIT.

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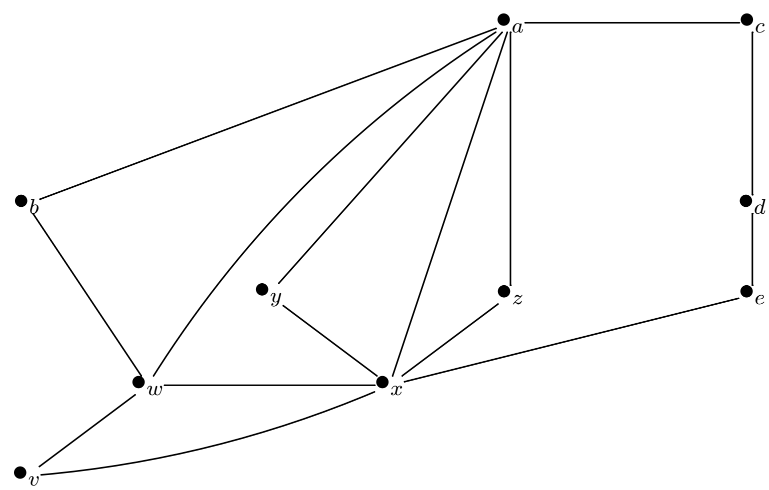
The book gives a proof that if a graph is connected, and if every vertex has even degree, then there is an Euler circuit in the graph. Buried in that proof is a description of an algorithm for finding such a circuit.

(a) First, pick a vertex to the the "start vertex."
(b) Find at random a cycle that begins and ends at the start vertex. Mark all edges on this cycle. This is now your "curent circuit."
(c) If there is not a vertex on the current circuit that is incident to an unmarked edge, you are done. If there is such a vertex, find a random cycle using unmarked edges that begin and ends at this vertex. Mark the edges in this cycle as you find it. Splice this cycle into the current circuit to make a new, larger current circuit that begins and ends at the start vertex. Repeat this step.

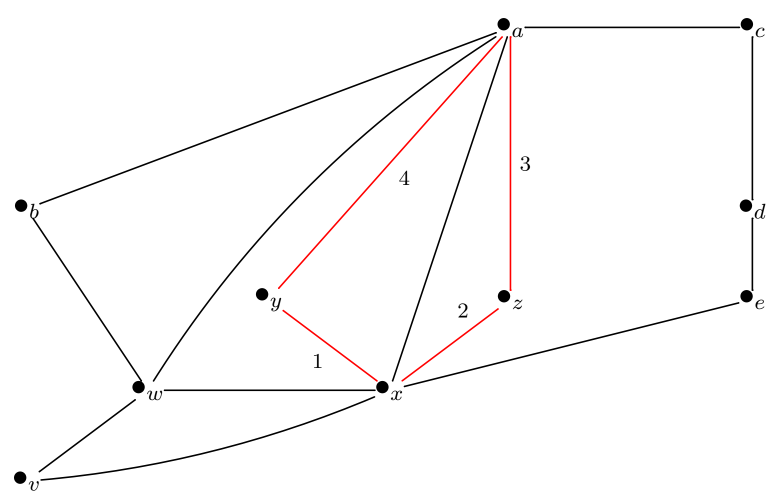
If you think about it, this algorithm can be applied to any graph. If it "gets stuck" at a vertex then you have discovered a vertex of odd degree. Otherwise, the algorithm will stop when if finds an Euler circuit of a connected component of the graph. If this is the whole graph, great, we found an Euler circuit for the original graph. Otherwise, we have shown that the graph is not connected. In this modified form, the algorithm tells you if a graph is Eulerian or not, and if so it produces an Euler circuit.

Here is a somewhat random graph:

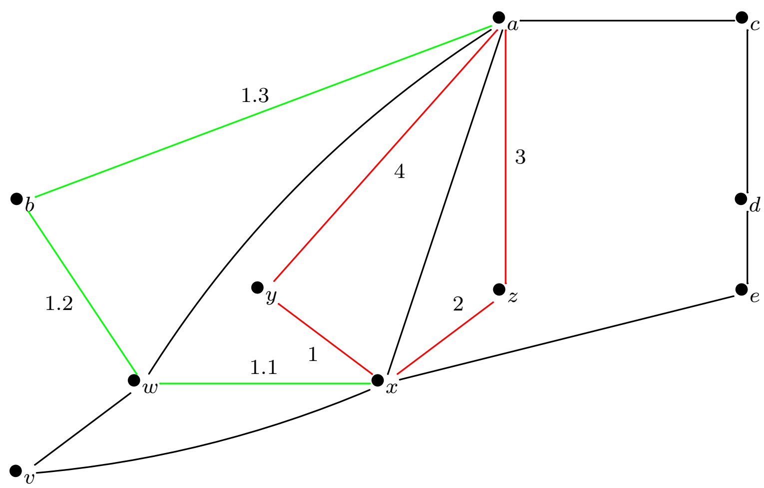


It has vertices all with even degree, so let's find an Euler circuit. (Plus, it is connected.)

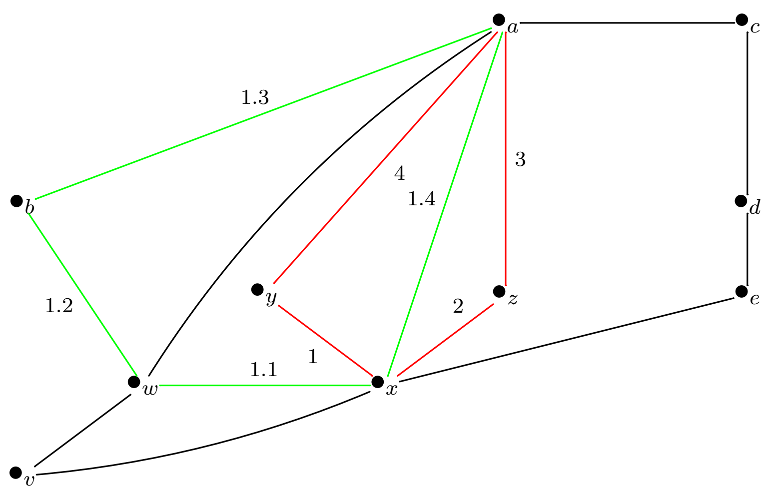
First just find a nice cycle, starting at vertex y :



There are no more ways out of y, so let's start retracing the cylce, looking for missed edges. We find one at x so we take it and wander off on roads not-yet-taken:



We can't stop here; we must continue back to our "second start" y :



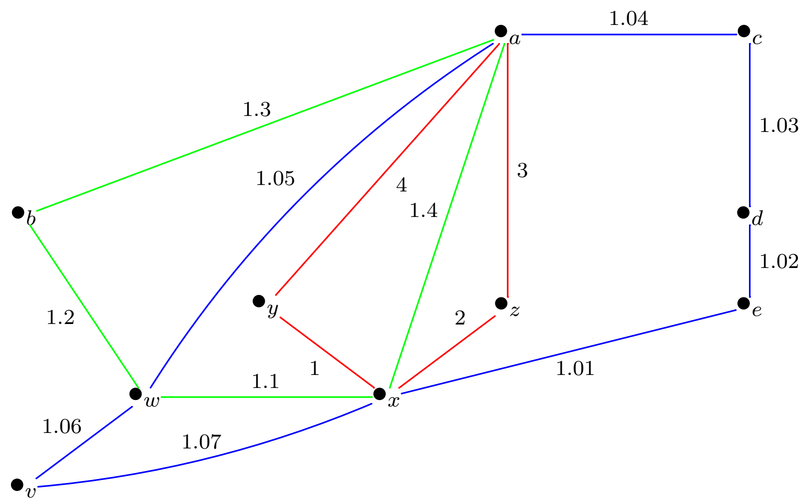
Our circuit is now described by the list of edges we have labeled

$$(1, 1.1, 1.2, 1.3, 1.4, 2, 3, 4)$$

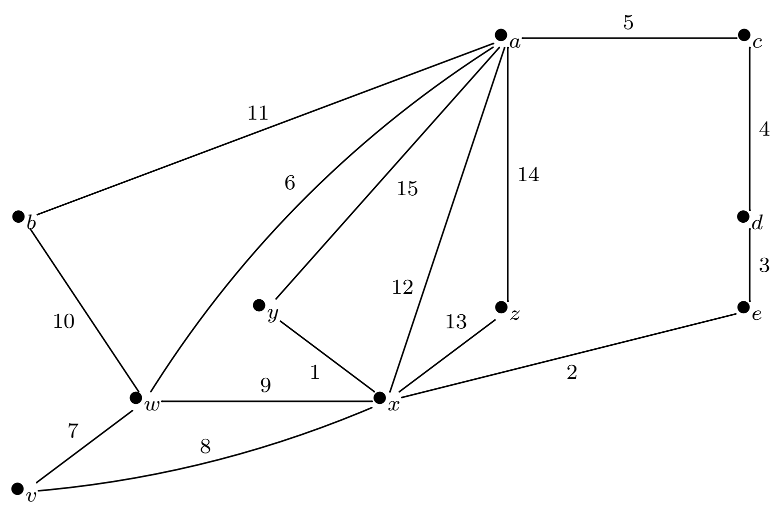
or by the list of vertices:

$$(y, x, w, b, a, x, z, a, y).$$

The first vertex on this list that has untaken edges incident to it is again x. So we go off on another cycle:



There are no more unmarked edges, so we are done. This looks better if relabel the edges 1,2,3 etc:



That is the easiest way to show your answer, but if you need a compact answer, you can answer in terms of the vertices (recall that the edges don't start with any names):

$$(y, x, e, d, c, a, w, v, x, w, b, a, x, z, a, y).$$

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