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Omega performance measure and portfolio insurance

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ABSTRACT

We analyze the performance of the two main portfolio insurance methods, the OBPI and CPPI strategies, using downside risk measures. For this purpose, we introduce Kappa performance measures and especially the Omega measure. These measures take account of the entire return distribution. We show that the CPPI method performs better than the OBPI. As a-by-product, we determine the set of threshold values for these risk/reward performance measures.

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1. Introduction

Portfolio insurance (PI) has been used extensively by the financial management industry, in equities, bonds, and hedge funds. It is particularly useful during a financial drop, allowing a given percentage of the initial portfolio value to be recovered at maturity. The two main standard portfolio insurance methods are *Option Based Portfolio Insurance* (OBPI) and *Constant Proportion Portfolio Insurance* (CPPI). The OBPI method was introduced by Leland and Rubinstein (1976). The portfolio is invested in a risky reference asset S covered by a listed put written on it. The strike K is equal to a predetermined proportion of the initial investment. This amount corresponds to the capital insured at maturity. Indeed, whatever the value of S at the terminal date T, the portfolio value will always be above the strike of the put.[1] The CPPI was introduced by Perold (1986) for fixed-income instruments and Black and Jones (1987) for equity instruments (see also Perold and Sharpe, 1988). In this method, the investor allocates assets dynamically over time. She chooses a *floor* equal to the lowest acceptable value of her portfolio. Then, she invests an amount, called *the exposure*, in the risky asset, which is proportional to the excess of the portfolio value over the floor, usually called *the cushion*. The remaining funds are invested in cash, usually T-bills. The proportional factor is defined as *the multiple*. Both floor and multiple depend on the investor's risk tolerance and are exogenous to the model. This portfolio strategy implies that, if the cushion value converges to zero, then exposure approaches zero too. In continuous time, this prevents portfolio value from falling below the floor, except if there is a very sharp drop in the market before the investor can modify her portfolio weights.

Some of the properties of portfolio insurance have previously been studied by Black and Rouhani (1989) and Black and Perold (1992) when the risky asset follows a geometric Brownian motion (GBM) and by Bertrand and Prigent (2003), when the volatility is stochastic. When we compare the two portfolio payoffs, the OBPI method performs better if the financial market increases moderately, as illustrated by Bookstaber and Langsam (2000). The CPPI method is the best strategy when the market drops or increases by a significant amount. Bird et al. (1990) examine OBPI properties under various market conditions by using simulations. Cesari and Cremonini (2003) compare also different portfolio insurance properties by means of Monte Carlo simulations. Bertrand and Prigent (2002, 2003, 2005) compare CPPI with OBPI by systematically introducing the probability distributions of the two portfolio values and by comparing them by means of various criteria: the four first moments of their returns, the cumulative distribution of their ratio and some of its quantiles. The main conclusion is that, when both the CPPI and OBPI portfolios have to be dynamically hedged, it is not easy to discriminate between these two strategies, except by their sensitivity Vega to the volatility of the risky asset. Therefore, it appears interesting to search for an existing criterion which might be able to reveal their differences.

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[1] Equivalently, we can buy a call option on S with strike K and hold cash equal to the discounted value of K.

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One of the main criteria that can be used is the standard expected utility maximization, introduced by von Neumann and Morgenstern (1944).[2] It has been widely applied to analyze optimal portfolio insurance and PI equilibrium (see Brennan and Schwartz, 1989; Basak, 1995; Grossman and Zhou, 1996). Results of Leland (1980) and Brennan and Solanki (1981) allow to prove for example that, for an investor having an HARA utility, the optimal portfolio corresponds to a CPPI strategy when the risky asset is a GBM. This feature is mentioned in Basak (1995), which illustrates how optimal PI depends on risk aversion, as already mentioned by Benniga and Blume (1985). Brennan and Schwartz (1988) further develop alternative time invariant portfolio insurance strategies. Grossman and Vila (1992) show that the CPPI strategy is optimal in the framework of long-term risk-sensitive portfolio optimization. Nevertheless, if the guarantee constraint is exogeneous, the optimal strategy looks like an OBPI one. For instance, let us assume that the investor has a CRRA utility, that the risky asset is a GBM, and let us impose that the portfolio value at maturity is above a predetermined amount K. In this framework, the optimal portfolio is a combination of long positions on the risk-free asset and on a call option defined on a underlying which is a power of the risky asset. Additionally, if the relative risk aversion is equal to the inverse of an instantaneous type Sharpe ratio, then the optimal solution is exactly the OBPI strategy (see Bertrand et al., 2001; El Karoui et al., 2005). To summarize, the expected utility criterion does not allow to clearly discriminate CPPI and OBPI stategies since the ranking depends crucially on utiliy specification.

This is related to the non first-order stochastic dominance (SD) of one of these strategies by the other, as proved in Bertrand and Prigent (2005) for the GBM case. The notion of stochastic dominance, introduced by Hadar and Russell (1969), belongs to the family of stochastic orderings (see also Bawa, 1975). One of its main properties is that it does not require a precise knowledge of preferences. According to its order, SD is linked to general properties of utility functions. For instance, the first-order corresponds to increasing utility and the second-order to concave utility, meaning risk aversion (for details about these properties, see Levy, 1992). However, stochastic dominance does not provide a complete ordering: for example, for some pairs of portfolio values, none stochastically dominates the other. This second criterion type has been further developed to compare PI strategies. Zagst and Kraus (forthcoming) analyze and compare both OBPI and CPPI methods. Using various SD criteria up to third-order and assuming that the risky underlying asset follows a GBM, they provide very specific parameter conditions implying the second-and third-order SD of the CPPI strategy. While they show that second-order stochastic dominance is based on the value $m = 1$, they determine an interval for the value of the multiplier m that implies third-order stochastic dominance. This latter result depends of course on the parameters of the underlying financial market, more especially on the volatility. Annaert et al. (2009) evaluate also PI performances using SD criteria but without assuming that the underlying asset follows a GBM. They use block-bootstrap simulations from the empirical distributions to take account of heavy tails and volatility clustering. They conclude in particular that both OBPI and CPPI can be prefered to simple buy-and-hold strategy but that PI is less attractive for low volatility levels. They suggest also to use a daily portfolio rebalancing. Stochastic dominance can be also consistent with reward-risk portfolio selection as illustrated by De Giorgi (2005): without market frictions, the market portfolio can be efficient in the sense of second-order stochastic dominance.

[2] More recently, Dierkes et al. (2010) show that portfolio insurance is rather attractive for an investor having preferences described by the Cumulative Prospect Theory (CPT). They find that probability weighting is a key factor for insurance strategies attractiveness.

Since 2000, downside risk measures have been intensively used in portfolio management. They are linked to economical capital allocation as recommanded by Basel II for banking laws and regulations (see Goovaerts et al., 2002). Jarrow and Zhao (2006) argue that it is due to the increasing development of derivatives, for instance in equity portfolio management. They show that, when asset returns are not Gaussian but with large left tails, there exist significant differences in mean–variance and mean-lower partial moment optimal portfolios. Rachev et al. (2007) show also differences between momentum strategies based on the standard Sharpe ratio based on the variance as a risk measure, and other reward-risk ratios associated to the expected shortfall as a risk measure. Actually, a third way to compare portfolios is to introduce adequate performance measures since risk and performance measurement are nowadays fundamental in quantitative finance. For example, for standard asset allocation, we can use Sharpe's ratio, Treynor's ratio or Jensen's Alpha. However, the payoffs of portfolio insurance strategies are typically non-linear with respect to the risky reference asset, which induces asymmetric return distributions. Therefore, we need a performance measure which overcomes the inadequacy of traditional performance measures when they are used to analyze return distributions which are not normally distributed. Generally, such performance measures correspond to "reward/risk" ratios. For the Sharpe ratio, the risk measure is the standard deviation. For the Treynor ratio, the risk measure is the CAPM beta. However, these risk measures do not take account of the whole return distribution. To meet this need, downside risk measures were introduced and analyzed (see e.g. Pedersen and Satchell, 1998; Artzner et al., 1999; Szegö, 2002; Acerbi, 2004). Keating and Shadwick (2002) use risk measures to define a new performance measure, called the Omega measure, based on a gain-loss approach. Using the downside lower partial moment, it takes account of investor loss aversion, as supported by the works of Tversky and Kahneman (1992) and of Hwang and Satchell (2010) who illustrate the important role of loss aversion in particular for asset allocation problems. The Omega measure splits the return into two sub-parts according to a threshold which corresponds to a minimum acceptable return. This measure is defined as the ratio of the expectation of gains ("the return is above the threshold") and the expectation of losses ("the return is below the threshold"). More precisely, as noted by Kazemi et al. (2004), the Omega measure is (mathematically) equal to the ratio of the expectations of a call option payoff to a put option payoff written on the risky reference asset with a strike price corresponding to the threshold. Note that these expectations are similar to option prices but both expectations are evaluated under the historical probability measure. As mentioned for example in Bacmann and Scholz (2003), the main advantage of the Omega measure is that it involves all the moments of the return distribution, including skewness and kurtosis. Moreover, ranking is always possible, whatever the "rational" threshold, in contrast to the Sharpe ratio where this level is fixed and equal to the riskless return. The Omega measure has been applied across a broad range of models in financial analysis, in particular to examine hedge fund style or equity funds. More generally, we introduce Kappa (n) measures, which are based on n-order lower partial moments as risk measures. For first-order and second-order, it corresponds respectively to the Omega measure and to the well-known Sortino ratio. Farinelli and Tibiletti (2008) and Zakamouline and Koekebakker (2009) introduce also portfolio performance evaluation with generalized Sharpe ratios, in particular the Omega and Sortino ratios. As proved by Pedersen and Satchell (1998, 2002), the Sortino ratio is related to utility function with lower risk aversion. More generally, Zakamouline (2010) proves that Kappa measures correspond to performance measures based on piecewise linear plus power utility functions. Note also that Darsinos and Satchell (2004) prove that n-order SD implies Kappa ($n-1$) dominance. For example, they show that second-order SD implies Omega dominance while third-order SD implies Sortino dominance.

The current paper compares standard portfolio insurance strategies by means of the Kappa performance measure, especially the Omega measure. Since portfolio insurance payoffs are not linear in the risky asset, which induces asymmetric returns, downside risk measures are required for such products. We begin by examining the problem of determining the threshold. Usually, this level must be chosen lower than the expected portfolio returns. In the portfolio insurance framework, additional constraints must be taken into account, for instance the threshold must be higher than the guaranteed amount. Then, we calculate the Omega measures for both the CPPI and the OBPI strategies. We study the Omega call and put components of both the standard and the capped OBPI portfolio value, which correspond to specific compound options. To illustrate the comparison of the two methods, we begin by considering two basic examples that model the risky asset price. The first one is the standard geometric Brownian motion, with various drift and volatility values. The second one assumes that the logarithm of the risky asset price process is the sum of a Brownian motion and a compound Poisson process with jump sizes double-exponentially distributed. Models based on Lévy processes allow us to take account of possible jumps in the asset dynamics, which is of particular interest when dealing with portfolio insurance. As mentioned by Kou (2002), the double-exponential distribution for jump sizes is quite easy to implement and analytical solutions for option pricing can be deduced. It also provides an explanation for the asymmetric leptokurtic feature and the volatility smile.[3] For the geometric Brownian case, we analyze the sensitivities of the Omega measures of both OBPI and CPPI methods to the drift and volatility parameters, for various insured amounts and thresholds. We also focus on the special case of the equality of expected returns of both OBPI and CPPI portfolio values. In this framework, we show that the CPPI strategy always dominates the OBPI strategy. This result is robust to change of financial parameters and also when we introduce other Kappa measures, such as the Sortino ratio. For the jump case, we compare the two strategies by using a similar approach as in Ramezani and Zeng (2007) for the parameter estimations of the daily S & P 500 returns from the beginning of 1970 through late June 2010.[4] In all the cases, we find that CPPI generally performs better than OBPI. To confirm this result, we also provide backtesting on daily S & P 500 returns on the same time period, as in Annaert et al. (2009).

The paper is organized as follows. Section 2 recalls the main properties of the Omega measure and of the more general Kappa measures. In particular, the Omega measure is computed for the buy-and-hold strategy and for both the OBPI and the CPPI methods. Section 3 provides numerical comparisons and interpretations of the Omega and Kappa performances of such strategies. [5]

2. The Omega and Kappa performance measures applied to portfolio insurance

2.1. The financial market

In what follows, we suppose that the investor invests the amount V_0 at the initial time 0 and trades on two basic assets : a money market account, denoted by B, and a financial index, denoted by S. The time period is equal to $[0, T]$. To illustrate some particular properties, we assume that the value of the riskless asset B evolves according to:

$$dB_t = B_t r dt, \tag{1}$$

where r is the deterministic interest rate, and that the risky asset price S_t follows a diffusion process with jumps, characterized by the following stochastic differential equation (SDE) [6]:

$$dS_t = S_{t-}[\mu(t, S_t)dt + \sigma(t, S_t)dW_t + \delta(t, S_t)dN_t], \tag{2}$$

where $(W_t)_t$ is a standard Brownian motion which is independent from the Poisson process with the measure of jumps N.[7]

Two basic cases are considered:

- The first corresponds to the standard case: the risky asset logreturn is a Brownian motion with linear drift. This case corresponds to constant coefficients $\mu(t,S_t) \equiv \mu, \sigma(t,S_t) \equiv \sigma$, and $\delta(t,S_t) \equiv 0$. The numerical base example that we use corresponds to the following parameter values:

$$T = 1,\ \mu = 8\%,\ S_0 = 100,\ r = 3\%,\ \sigma = 17\%.$$

- The second corresponds to the case where the risky asset logreturn is a Lévy process with double-exponential jumps. [8] This model introduces two independent components: a diffusion part which is a Brownian motion with drift; a jump part where the jump times are exponentially distributed (Poisson process) and the jump sizes are double-exponentially distributed. As shown by Kou and Wang (2003), Sepp (2004), such a model is quite tractable since it allows explicit calculation for option prices by means of Laplace transform. [9]

The risky return satisfies the following SDE:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t + d\left(\sum_{i=1}^{N_t}\left(\frac{\Delta S_{T_n}}{S_{T_n}}\right)\right), \tag{3}$$

where W_t is a standard Brownian motion, N_t is a Poisson process with intensity $\lambda > 0$. The relative jumps $\frac{\Delta S_{T_n}}{S_{T_n}}$ take their values in the set $(-1,\infty)$. They are independent and identically distributed (i.i.d.). The random variables $Z_n = \ln\left(1 + \frac{\Delta S_{T_n}}{S_{T_n}}\right)$ are double-exponentially distributed with probability density function (pdf) given by:

$$f_Z(z) = p_z \cdot \eta_1 e^{-\eta_1 z}\mathbb{1}_{\{z\geqslant 0\}} + q_z \cdot \eta_2 e^{\eta_2 z}\mathbb{1}_{\{z<0\}},\ \eta_1 > 1,\ \eta_2 > 0, \tag{4}$$

where $p_z, q_z \geqslant 0$ respectively denote the probabilities of moving upside or downside ($p_z + q_z = 1$).

We also have:

$$\ln\left(1 + \frac{\Delta S_{T_n}}{S_{T_n}}\right) = Z_{T_n} = \begin{cases} \xi^+, & \text{with probability } p_z \\ -\xi^-, & \text{with probability } q_z \end{cases},$$

[3] It leads to analytical solutions to many option-pricing problems: European call and put options; interest rate derivatives (see Glasserman and Kou, 2003); path-dependent options, such as barrier, lookback and perpetual American options (see Kou and Wang, 2003, 2004; Sepp, 2004).

[4] This is the price return index that is used since this is the only one available on the entire period of time considered here. Not taking into account the dividend can be understood as compensation for management fees. Note that the same data are used in the comparison of both methods.

[5] Proofs of all propositions are available at request.

[6] The functions $\mu(\cdot)$, $\sigma(\cdot)$ and $\delta(\cdot)$ are assumed to satisfy usual conditions to ensure the existence, the uniqueness and the positivity of the solution of this stochastic differential equation (see Jacod and Shiryaev (2003) for such conditions).

[7] Recall that the sequence of times $(T_n)_n$ at which jumps occur has the following properties: The jump interarrival times $(T_{n+1} - T_n)$ are independent with the same exponential distribution associated with parameter λ. The relative jumps $\frac{\Delta S_{T_n}}{S_{T_n}}$ are equal to $\delta(T_n, S_{T_n})$, which are assumed to be strictly higher than (-1) (to guarantee the positivity of asset price S). The integral $\int_{-\infty}^{+\infty}\int_0^t S_{u-}\cdot\delta(u, S_u)dN$ is equal to the sum $\sum_{T_n\leqslant t}\Delta S_{T_n}$ of all jumps before time t (see Jacod and Shiryaev, 2003).

[8] If the relative jumps $\frac{\Delta S_{T_n}}{S_{T_n}}$ are Gaussian distributed, then this model corresponds to the Merton's model (1976), but in this case, no analytical solution exists for path-dependent options.

[9] From an econometric point of view, Ramezani and Zeng (2007) independently introduce the same jump-diffusion model in order to improve the empirical fit of Merton's Gaussian jump-diffusion model to stock price data.

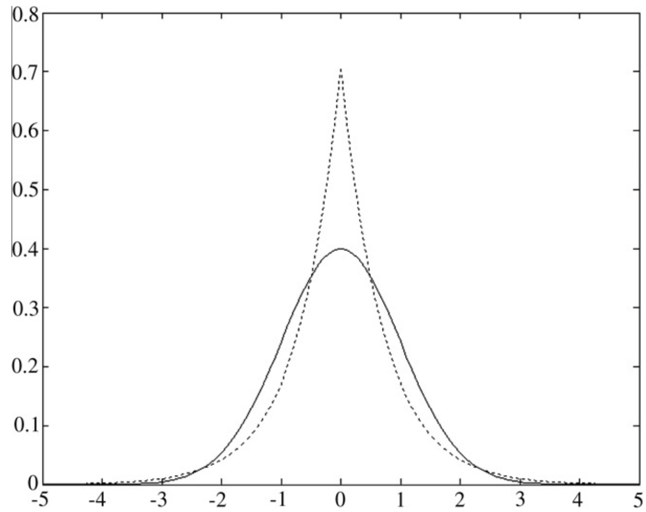

Fig. 1. Comparison between a Gaussian pdf and a double-exponential pdf.

where ξ^+ and ξ^- are exponentially distributed with means $\frac{1}{\eta_1}$ and $\frac{1}{\eta_2}$. In this model, the processes W_t,N_t and the sequence $(Z_n)_n$ are independent.

The solution of the SDE (3) is given by:

$$S_t = S_0 \cdot exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W_t + \sum_{i=1}^{N_t} \ln\left(1 + \frac{\Delta S_{T_n}}{S_{T_n}}\right)\right]. \tag{5}$$

Note that $\mathbb{E}[Z] = \frac{p_z}{\eta_1} - \frac{q_z}{\eta_2}, \mathbb{V}[Z] = 2\left(\frac{p_z}{\eta_1^2} + \frac{q_z}{\eta_2^2}\right) - \left(\frac{p_z}{\eta_1} - \frac{q_z}{\eta_2}\right)^2$ and:

$$\mathbb{E}\left[\frac{\Delta S_{T_n}}{S_{T_n}}\right] = \mathbb{E}[e^Z] - 1 = q_z \frac{\eta_2}{\eta_2 + 1} + p_z \frac{\eta_1}{\eta_1 - 1} - 1,\ \eta_1 > 1,\ \eta_2 > 0.$$

The condition $\eta_1 > 1$ is necessary to ensure that $\mathbb{E}\left[\frac{\Delta S_{T_n}}{S_{T_n}}\right] < \infty$and $\mathbb{E}[S_t] < \infty$.

Fig. 1 illustrates the comparison between the pdf of the Gaussian distribution and the pdf of the double-exponential distribution when both the expectations are equal to *0* and both the variances are equal to 1. We also take: $p = 0.5$ and $\eta_1 = \eta_2 = \sqrt{2}$.

The numerical base example that we consider in the following corresponds to the estimations of the S & P 500 Composite index daily returns (without dividends) which span the period from the beginning of 1970 through late June 2010.[10] The parameter values (daily returns) are given by[11]:

$T = 1(\text{year}); \mu = 0.052\%; \quad S_0 = 100; \quad r = 3\%(\text{annual});$
$\sigma = 0.40\%; \quad \eta_1 = 182.08; \quad \eta_2 = 172.86; \quad \lambda = 1.4615;$
$p_z = 0.4960; \quad q_z = 0.5040.$

2.2. Definition and general properties of the Omega and Kappa measures

The Omega performance measure has been first introduced by Keating and Shadwick (2002) and Cascon et al. (2003). It is designed to overcome the shortcomings of performance measures based only on the mean and the variance of the distribution of the returns. Omega measure takes account of the entire return distribution while requiring no parametric assumption on the distribution. The returns both below and above a given loss threshold are considered. More precisely, Omega is defined as the probability weighted ratio of gains to losses relative to a return threshold. The exact mathematical definition is given by:

$$\Omega_X(L) = \frac{\int_L^b (1 - F(x))dx}{\int_a^L F(x)dx}, \tag{6}$$

where $F(\cdot)$ is the cumulative distribution function (cdf) of the asset return X defined on the interval (a,b), with respect to the probability distribution $\mathbb{P}$ and L is the return threshold selected by the investor. For any investor, returns below her loss threshold are considered as losses and returns above as gains. At a given return threshold, the investor should always prefer the portfolio with the highest value of Omega.

The Omega function exhibits the following properties:

- First, as shown for example in Kazemi et al. (2004), Omega can be written as:

$$\Omega_X(L) = \frac{\mathbb{E}_{\mathbb{P}}\left[(X - L)^+\right]}{\mathbb{E}_{\mathbb{P}}\left[(L - X)^+\right]}. \tag{7}$$

It is the ratio of the expectations of gains above the threshold L to the expectations of the losses below the threshold L.[12]

Kazemi et al. (2004) define the Sharpe Omega measure as:

$$Sharpe_\Omega(L) = \frac{\mathbb{E}_{\mathbb{P}}[X] - L}{\mathbb{E}_{\mathbb{P}}\left[(L - X)^+\right]} = \Omega_X(L) - 1. \tag{8}$$

Note that if $\mathbb{E}_{\mathbb{P}}[X] < L$, the Sharpe Omega will be negative, otherwise it will be positive.

- For $L = \mathbb{E}_{\mathbb{P}}[X],\ \Omega_X(L) = 1$,
- $\Omega_X(\cdot)$ is a monotone decreasing function.
- $\Omega_X(\cdot) = \Omega_Y(\cdot)$ if and only if $F_X = F_Y$.

Typically, consider a strategy which consists in investing 100% of the initial amount in the risky asset. In that case, the portfolio payoff is equal to the stock payoff S at time T which is modelled by a geometric Brownian motion. Therefore, here we have $X = S_0$ exp $[(\mu - \sigma^2/2)T + \sigma W_T]$, where W_T has the Gaussian distribution $\mathcal{N}(0, T)$. Then, $\mathbb{E}_{\mathbb{P}}[X] = S_0 \exp[\mu T]$ does not depend on the volatility. Thus, if $S_0 \exp[\mu T] < L$ then the Sharpe Omega is an increasing function of the volatility σ (due to the Vega of the put option). If $S_0 \exp[\mu T] > L$, the Sharpe Omega is a decreasing function of the volatility σ.

The level must be specified exogenously. It varies according to investment objective and individual risk aversion. As proved by Unser (2000), we are often only interested in an evaluation of outcomes which are "risky", i.e. their values are smaller than a given target, thus reflecting the attitude towards downside risk. Examples would be an inflation rate for pension incomes, or the rate of a benchmark financial index. Such downside risk measures have been examined for instance in Ebert (2005), and are linked to the measures proposed by Fishburn (1977, 1984).

Actually, the Sharpe Omega measure is one of the Kappa measures considered in Kaplan and Knowles (2004). These latter ones are defined by: for $l = 1,2,\ldots$

[10] As noted by Ramezani and Zeng (2007), the up and down jumps arrive roughly once every 2 days. Their mean magnitudes are approximately equal to 0.60% and 0.70%, respectively. The value of λshows that high-frequency jumps are needed to fit the index return data better, which is consistent with the findings of Huang and Wu (2004) to fit the S & P-500 index.

[11] We use a similar statistical approach as in Ramezani and Zeng (2007), except that we use Maximum Likelihhod Estimation (MLE) of the characteristic function instead of MLE of the pdf, as proposed by Press (1972) and further developed by Chan et al. (2009).

[12] Kazemi et al. (2004) note that, by multiplying both numerator and denominator by the discount factor, Omega can be considered as the ratio of the prices of a call option to a put option written on X with strike price L but both evaluated under the historical probability $\mathbb{P}$ instead of the risk neutral one. For example, if the risky asset S follows a GBM, then, mathematically speaking, the Omega value of a whole investment in S is the ratio of the Black–Scholes call value upon the put value with strike L and value of the drift of S instead of the riskless return.

$$Kappa_l(L) = \frac{\mathbb{E}_{\mathbb{P}}[X] - L}{\left(\mathbb{E}_{\mathbb{P}}\left[\left[(L-X)^+\right]^l\right]\right)^{\frac{1}{l}}}. \tag{9}$$

For $l = 1$, we get the Sharpe Omega measure and, for $l = 2$, we recover the Sortino ratio. Zakamouline (2010) proves that Kappa measures correspond to performance measures based on piecewise linear plus power utility functions. To prove such result, consider the following utility function:

$$U_L(v) = (v-L)^+ - \left(\left[(L-v)^+\right] + \frac{\Phi}{n}\left[(L-v)^+\right]^n\right),$$

with $\Phi > 1$ and n a nonzero integer. Then, as shown in Zakamouline (2010), the investor's capital allocation problem yields to the following relation:

$$E[U_L^*(V)] = \frac{n-1}{n}\left(\frac{E[V]-L}{(E[(L-V)^{+n}])^{\frac{1}{n}}}\right)^{\frac{n}{n-1}},$$

where $E[U_L^*(V)]$ denotes the utility of the optimal allocation. Therefore, $E[U_L^*(V)]$ is an increasing transformation of the Kappa (n) ratio, which proves that this latter one is based on the utility U_L. Note that U_l is convex on $[-\infty, L]$ if and only if $n = 1$. This corresponds to the Omega measure, which is a limiting case when $n \to 1$. Therefore, the Omega measure is linked to the maximization of an expected utility with loss aversion, as introduced by Tversky and Kahneman (1992).

2.3. The choice of the threshold

We examine the choice of the threshold for the standard buy-and-hold strategy, which indicates the "rationale" set of values for this reference level. Assume that the investor wants a guaranteed level equal to pV_0 (with $p \leqslant e^{rT}$) and uses a buy-and-hold strategy. Thus, her portfolio value at maturity is given by:

$$V_T = pV_0 + \theta S_T, \text{with } \theta = \frac{V_0(1-pe^{-rT})}{S_0}.$$

Then, the Omega of her portfolio is given by:

$$\Omega_{V_T}(L) = \frac{\mathbb{E}_{\mathbb{P}}\left[(S_T - aS_0)^+\right]}{\mathbb{E}_{\mathbb{P}}\left[(aS_0 - S_T)^+\right]}, \tag{10}$$

where $a = (L/V_0 - p)/(1 - pe^{-rT})$ and the threshold L is chosen obviously higher than the guaranteed amount pV_0. Thus, it can be considered as the ratio of the prices of a call option to a put option written on S_T with strike price aS_0, evaluated under the historical probability $\mathbb{P}$.

Note that the ratio a is higher than 1 (out-of-the-money call) if and only if the ratio L/V_0 is higher than $1 + p(1 - e^{-rT})$. Additionally, the threshold L is smaller than the expectation $\mathbb{E}_{\mathbb{P}}[V_T]$ if and only if $L/V_0 \leqslant [1 + p(1 - e^{-rT})]e^{\mu T}$.

Thus, to compare the ratio L/V_0 with the riskless return e^{rT}, we have to consider three cases:

(1) The ratio L/V_0 satisfies: $p \leqslant L/V_0 \leqslant 1 + p(1 - e^{-rT})$;
(2) The ratio L/V_0 satisfies: $1 + p(1 - e^{-rT}) \leqslant L/V_0 \leqslant e^{rT}$; and,
(3) The ratio L/V_0 satisfies: $e^{rT} \leqslant L/V_0 \leqslant 1 + p(1 - e^{-rT})e^{\mu T}$.

Proposition 1. *The Omega performance measure for the previous buy-and-hold strategy is a monotonic function of the guaranteed percentage p. If the ratio L/V_0 satisfies:*

(1) *$L/V_0 < e^{rT}$, then Omega is an increasing function of the percentage p.*
(2) *$L/V_0 > e^{rT}$, then Omega is a decreasing function of the percentage p.*
(3) *$L/V_0 = e^{rT}$, then Omega is a constant function of the percentage p.*

Proposition 1 can be interpreted as follows: In part 1 of Proposition 1, the low level of the threshold means that the investor is mainly concerned about risk control. As a result, the Omega becomes increasing with respect to the insured percentage p. As the threshold is higher (part 2 of Proposition 1), the investor worries more about the performance of her fund. As a result, the Omega becomes decreasing with respect to the insured percentage p. If the threshold is exactly equal to the initial capitalized portfolio value, $L = V_0e^{rT}$, the Omega measure is independent of the insured percentage p. Suppose for example that the risky asset S satisfies:

$$S_T = S_0 \exp\left[(\mu - \sigma^2/2)T + \sigma W_T\right], \tag{11}$$

where W_t is a standard Brownian motion. Therefore, the expectation of the portfolio value is given by $\mathbb{E}_{\mathbb{P}}[V_T] = pV_0 + \alpha \cdot S_0 \exp[\mu T]$, which shows that it does not depend on the volatility σ. Since the threshold L is assumed to be smaller than the expectation $\mathbb{E}_{\mathbb{P}}[V_T]$, both Sharpe Omega and Omega ratios are decreasing functions of the volatility σ.

Fig. 2 shows how the Omega measure depends on the threshold, for numerical values such as in Section 2.1. It also illustrates the results of Proposition 1: for levels L smaller than the riskless return, the Omega performance measure is an increasing function of the guaranteed percentage p.

2.4. The Omega measure of the OBPI strategy

As mentioned by Leland (1980), "general insurance policies are those that provide strictly convex payoff functions". Indeed, the simple buy-and-hold strategy does not allow enough profit to be made from market rises. This is the main reason why portfolio insurance methods OBPI and CPPI were introduced.

2.4.1. The OBPI strategy

The OBPI method consists basically in purchasing an amount $q.K$ invested in the money market account, and q shares of European call options written on asset S with maturity T and exercise price K. These options may be capped at level H, if the portfolio manager wants to increase her profit from average performances of asset S while potentially discarding very high values of S. Generally, the OBPI method is capped as follows. Consider a parameter H higher than strike K. The profile of the capped OBPI with strike K and parameter H is given by:

$$\begin{aligned} V_{cap,T}^{OBPI} &= qMin\left[K + (S_T - K)^+, H\right] \\ &= q\left(K + (S_T - K)^+ - (S_T - H)^+\right). \end{aligned} \tag{12}$$

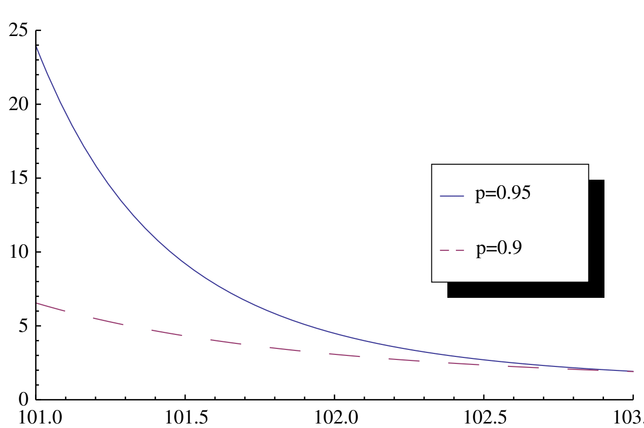

Fig. 2. Ω as a function of threshold for $p = 1$ and $\sigma = 20\%$.

This relation shows that the insured amount at maturity is the exercise price multiplied by the number of shares ($q{\cdot}K$). The value $V^{OBPI}_{cap,t}$ of this portfolio at any time t in the period $[0,T]$ is:

$$V^{OBPI}_{cap,t} = q\left(Ke^{-r(T-t)} + C(t,K) - C(t,H)\right), \tag{13}$$

where $C(t,x)$ denotes the no-arbitrage value of a European call option with strike x, calculated under a given risk-neutral probability Q (if coefficient functions μ,a and b are constant, $C(t,x)$ is the usual Black–Scholes value of the European call). Note that, for all dates t before T, the portfolio value $V^{OBPI}_{cap,t}$ is always above the deterministic level $qKe^{-r(T-t)}$. The investor is still willing to recover a percentage p of her initial investment V_0. Then, her portfolio manager has to choose the three appropriate parameters, q, K and H.

- First, since the insured amount is equal to $q{\cdot}K$, it is required that K satisfies the relation: [13]

  $$pV_0 = pq(K\cdot e^{-rT} + C(0,K) - C(0,H)) = qK, \tag{14}$$

  which implies that:

  $$\frac{C(0,K)-C(0,H)}{K} = \frac{1-pe^{-rT}}{p}. \tag{15}$$

  Therefore, strike K is an increasing function $K(p)$ of percentage p.
- Second, the number of shares q is given by:

  $$q = \frac{V_0}{Ke^{-rT} + C(0,K) - C(0,H)}. \tag{16}$$

  Thus, for any investment value V_0, number of shares q is a decreasing function of percentage p.

2.4.2. Computations of the OBPI omega

Since we are analyzing portfolio insurance, the threshold of the Omega measure must be greater than the insured amount at maturity: $L > q{\cdot}K = p{\cdot}V_0$. Additionally, to avoid a degenerate case, we assume that $L < q{\cdot}H$ (otherwise, $\Omega^{OBPI}(L)$ would not be defined).

Proposition 2. *For the capped OBPI method, the Omega measure $\Omega^{OBPI}_{cap}(L)$ is defined by:*

$$\Omega^{OBPI}_{cap}(L) = \frac{\mathbb{E}_{\mathbb{P}}\left[[Min(S_T,H) - L/q]^+\right]}{\mathbb{E}_{\mathbb{P}}\left[(L/q - S_T)^+\right] - \mathbb{E}_{\mathbb{P}}\left[(K - S_T)^+\right]}. \tag{17}$$

For the standard OBPI strategy (not capped), the Omega measure $\Omega^{OBPI}(L)$ is given by:

$$\Omega^{OBPI}(L) = \frac{\mathbb{E}_{\mathbb{P}}\left[[S_T - L/q]^+\right]}{\mathbb{E}_{\mathbb{P}}\left[(L/q - S_T)^+\right] - \mathbb{E}_{\mathbb{P}}\left[(K - S_T)^+\right]}. \tag{18}$$

As proved by Relation (18), the risk measure associated with the Omega performance measure at a given level $\widetilde{L}$ is the difference between expectations of the European puts $(\widetilde{L} - S_T)^+$ and $(K - S_T)^+$. Thus, for the OBPI strategy, the risk component of the Omega measure can be viewed as that of asset S with the risk of falling below level K removed. Since the insured amount at maturity is $q.K$, the risk reduction is clearly due to portfolio insurance.

Fig. 3 illustrates this risk reduction which induces a capped put. This figure also shows that the profiles of the portfolio value and the Omega-call component for both capped and non-capped OBPI show similar patterns. Looking at relations (17) and (18), the Omega measures of the standard and capped OBPI strategies have similar risk components (the European puts), whereas the call component of the standard OBPI method seems higher than the corresponding call component for the capped OBPI method. However, for the same initial investment V_0, strikes K and shares q are different for capped and non-capped OBPI strategies.

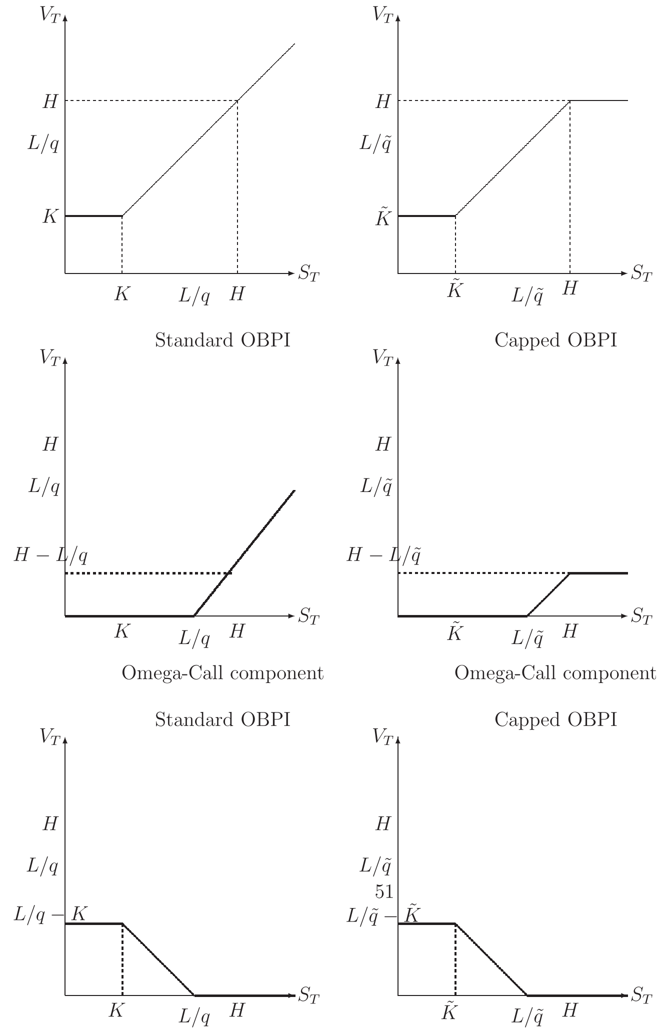

Fig. 3. OBPI and CPPI payoffs and their corresponding call and put Omega components.

Taking this property into account, the comparison between different OBPI strategies can be illustrated for example for the geometric Brownian case and when the risky asset return is a double-exponential Lévy process with parameter values given in Section 2.1.[14]

As shown in Tables 1 and 2, no general monotonicity property exists, except with respect to the threshold L (Ω is decreasing w.r.t. L).

Fig. 3 hereunder shows the portfolio profiles, the call component (the "reward") and the put component (the "risk") of the Omega measures of OBPI portfolios.

2.5. The Omega measure of the CPPI strategy

2.5.1. The CPPI strategy

The CPPI method consists in managing a dynamic portfolio so that its value is above a floor F at any time t. The value of the floor

[13] This relation could also be adjusted to take account of the smile effect.

[14] The jump premium is set equal to *0* as in Merton (1976). Other assumptions about the option pricing such as "the risky asset is also double-exponentially distributed under the risk-neutral probability" do not significantly change the comparison results.

Table 1
Omega of OBPI (Brownian case).

		Capped OBPI			OBPI
		$H = 115$	$H = 120$	$H = 130$	$H = +\infty$
$L = 101$	$p = 0.9$	2.49	2.45	2.43	2.47
	$p = 1$	6.87	6.54	6.23	6.24
$L = 102$	$p = 0.9$	2.03	2.02	2.03	2.09
	$p = 1$	2.89	2.82	2.77	2.87
$L = 103$	$p = 0.9$	1.66	1.67	1.70	1.78
	$p = 1$	1.58	1.60	1.64	1.76

Table 2
Omega of OBPI (Levy case with double-exponential jumps).

		Capped OBPI			OBPI
		$H = 115$	$H = 120$	$H = 130$	$H = +\infty$
$L = 101$	$p = 0.9$	4.58	3.91	3.12	2.35
	$p = 1$	7.16	5.78	4.14	2.46
$L = 102$	$p = 0.9$	3.57	3.08	2.49	1.88
	$p = 1$	2.93	2.42	1.78	1.07
$L = 103$	$p = 0.9$	2.69	2.44	1.99	1.52
	$p = 1$	1.56	1.33	1.01	0.62

gives the dynamically insured amount. It is assumed to evolve according to:

$$dF_t = F_t r dt, \tag{19}$$

where r denotes the instantaneous riskless rate. The initial floor F_0 is chosen such as to recover a guaranteed amount pV_0 at maturity T ($p \leqslant e^{rT}$). Thus, $F_0 = pV_0e^{-rT}$. The initial floor F_0 must be chosen smaller than the initial portfolio value V_0^{CPPI}. The difference $V_0^{CPPI} - F_0$ is called the cushion, denoted by C_0. Its value C_t at any time t in $[0,T]$ is given by:

$$C_t = V_t^{CPPI} - F_t. \tag{20}$$

Denote by e_t the exposure, which is the total amount invested in the risky asset. The standard CPPI method consists in letting $e_t = mC_t$ where m is a constant called the multiple. The interesting case is when $m > 1$, that is, when the payoff function is convex. Assuming that the risky asset S has the dynamics given by Eq. (2), then the cushion value C_t at time t is given by [15]:

$$C_t = C_0 exp\left((1-m)rt + m\left[\int_o^t (\mu - 1/2m\sigma^2)(s,S_s)ds + \int_o^t \sigma(s,S_s)dW_s\right]\right) \times \prod_{0\leqslant T_n \leqslant t}(1 + m\delta(T_n, S_{T_n})), \tag{21}$$

and the portfolio value is given by:

$$V_t^{CPPI}(m,S_t) = F_0.e^{rt} + C_t. \tag{22}$$

Relation (21) shows that the guarantee is satisfied as soon as the relative jumps satisfy $\delta(T_n, S_{T_n}) \geqslant -1/m$. Moreover, if the relative jumps satisfy the condition that $\delta(T_n, S_{T_n})$ are higher than a non-positive parameter d, then the condition $0 \leqslant m \leqslant -1/d$ implies the positivity of the cushion. For example, if d is equal to -10%, then $m \leqslant 10$.

First case: the stock logreturn follows a Brownian motion.

This case corresponds to constant coefficients $\mu(t,S_t) \equiv \mu$, $\sigma(t,S_t) \equiv \sigma$, and $\delta(t,S_t) \equiv 0$. Therefore, the value of the portfolio V_t^{CPPI} at any time t in the period $[0,T]$ is equal to:

$$V_t^{CPPI}(m,S_t) = F_0 \cdot e^{rt} + \alpha_t \cdot S_t^m, \tag{23}$$

where $\alpha_t = \left(\frac{C_0}{S_0^m}\right)\exp[\beta t]$ and $\beta = \left(r - m(r - \frac{1}{2}\sigma^2) - m^2\frac{\sigma^2}{2}\right)$.

Thus, the CPPI method is parametrized by F_0 and m, and the cushion value is given by:

$$C_t = C_0 e^{m\sigma W_t + \left[r + m(\mu - r) - \frac{m^2\sigma^2}{2}\right]t} \quad \text{with } C_0 = V_0 - F_0.$$

Then, the cushion and portfolio values are independent of the paths of asset price S. Additionally, they have Lognormal distributions (up to a linear transformation for the portfolio value) and the guarantee is always satisfied.

Second case: the stock logreturn follows a Lévy process with jumps that are double-exponentially distributed.

The first two moments are given by: $\xi = \mathbb{E}[Z]$; $\delta^2 = \mathbb{V}[Z]$.

$$\mathbb{E}[V_t] = (V_0 - F_0)e^{T[r + m(\mu + \xi\lambda - r)]} + F_0e^{rT}$$
$$\mathbb{V}(V_t) = (V_0 - F_0)^2 e^{2T[r + m(\mu + \xi\lambda - r)]}\left(e^{m^2T(\sigma^2 + \delta^2\lambda)} - 1\right).$$

For the double-exponential distribution, the cushion may be negative since the negative jumps are not lower-bounded. We can control the probability of such an event by using a quantile condition:

$$\mathbb{P}[\forall t \in [0,T],\ C_t > 0] \geqslant 1 - \varepsilon, \text{for a small } \varepsilon(\varepsilon = 1\%), \tag{24}$$

and/or by controlling the level of the following expected shortfall: [16]

$$\forall t \in [0,T], \mathbb{E}[\Gamma_t - C_t \mid C_t < \Gamma_t, C_{t-} > 0, \mathcal{F}_{t-}] \leqslant \epsilon C_{t-}, \tag{25}$$

where Γ_t denotes a reference threshold. Usually, we take $\Gamma_t = \gamma C_{t-}$ where $\gamma$is a fixed parameter satisfying: $\gamma < 0$.

Proposition 3. *For the double-exponential distribution, the condition (24) leads to the following upper bound on the multiple:*

$$m \leqslant \frac{1}{1 - \left(\frac{Log\left[\frac{1}{1-\varepsilon}\right]}{q_Z\lambda T}\right)^{\frac{1}{\eta_2}}}. \tag{26}$$

For the expected shortfall condition, we deduce:

$$m \leqslant \epsilon(\eta_2 + 1) + (1 - \gamma). \tag{27}$$

These conditions are not so stringent. For $\varepsilon = 1\%$, the upper bound on the multiple determined from the quantile condition is approximately equal to 46 and the upper bound associated with the expected shortfall condition, with $\gamma = 0$ (i.e. the cushion C_t becomes negative) and $\epsilon = 10\%$, is approximately equal to 19. For this latter upper bound corresponding to the failure of protection, we recover standard values of the multiple for level ϵsmaller than 5%.

2.5.2. Computations of the CPPI omega

Assume that the risky asset price S_t follows the diffusion process with jumps defined in Eq. (2). Then, the CPPI Omega measure can be determined.

Proposition 4. *For the CPPI strategy, the Omega measure is defined by*

$$\Omega^{CPPI}(L) = \frac{\sum_{n=0}^{\infty} e^{-\lambda t}\frac{(\lambda t)^n}{n!}\mathbb{E}\left[(X_{n,T} - L)^+\right]}{\sum_{n=0}^{\infty} e^{-\lambda t}\frac{(\lambda t)^n}{n!}\mathbb{E}\left[(L - X_{n,T})^+\right]}, \tag{28}$$

[15] See Prigent (2007).

[16] $\mathcal{F}_{t-}$ denotes the left hand limit at time t of the filtration generated by the observations of the processes W, N and the sequence of relative jumps $\left(\frac{\Delta S_{T_n}}{S_{T_n}}\right)_{T_n \leqslant t}$.

Table 3
Omega of CPPI (geometric Brownian case).

		CPPI			
		$m = 3$	$m = 5$	$m = 7$	$m = 9$
$L = 101$	$p = 0.9$	3.93	3.01	2.75	2.68
	$p = 1$	367	41.45	18.48	12.48
$L = 102$	$p = 0.9$	2.70	2.39	2.32	2.33
	$p = 1$	11.84	5.87	4.50	4.00
$L = 103$	$p = 0.9$	1.92	1.94	1.98	2.05
	$p = 1$	2.01	2.00	2.03	2.09

Table 4
Omega of CPPI (Lévy case with double-exponential jumps).

		CPPI			
		$m = 3$	$m = 5$	$m = 7$	$m = 9$
$L = 101$	$p = 0.9$	8.47	5.37	4.47	4.07
	$p = 1$	6937	379	87.82	40.94
$L = 102$	$p = 0.9$	4.66	3.76	3.45	3.32
	$p = 1$	51.52	15.37	9.47	7.36
$L = 103$	$p = 0.9$	2.70	2.71	2.73	2.76
	$p = 1$	2.94	2.84	2.82	2.83

where the probability distribution of the random variable $X_{n,T}$ corresponds to the conditional distribution of the portfolio value given that the number $\mathcal{N}_T$ of jumps before maturity T is equal to n.

For the geometric Brownian motion, we deduce by using relation (23):

Proposition 5. *The Omega performance measure of the CPPI strategy is defined by:*

$$\Omega^{CPPI}(L) = \frac{\mathbb{E}_{\mathbb{P}}\left[\left(p \cdot V_0 + \alpha_T \cdot S_T^m - L\right)^+\right]}{\mathbb{E}_{\mathbb{P}}\left[\left(L - p \cdot V_0 - \alpha_T \cdot S_T^m\right)^+\right]}. \tag{29}$$

The expectation of the CPPI portfolio value is given by:

$$\mathbb{E}_{\mathbb{P}}[V_T^{CPPI}] = p \cdot V_0 + C_0 e^{[r+m(\mu-r+b\lambda)]T},$$

where b denotes the expectation of the relative jumps of risky asset S. Note that this expression does not depend on the volatility σ induced by the continuous component (the diffusion). Thus, as for stock S, the Sharpe Omega ratio $Sharpe_{\Omega}^{CPPI}(L)$ of the CPPI depends on volatility σ only through its denominator, which is equal to a put option. Thus, it is a decreasing function of volatility σ, as is the Omega measure for the CPPI.

For the numerical base example ($V_0 = 100$, $S_0 = 100$, $r = 3\%$, $T = 1$, $\sigma = 17\%$, $\mu = 8\%$), we illustrate the influence of both the threshold L and the multiple m, for two guaranteed percentages: $p = 0.9$ and $p = 1$ (see Table 3).

We now consider the Lévy case with jumps that are double-exponentially distributed (see Table 4).

According to the Omega criterion, CPPI performs better for relatively low threshold levels. It also performs better for relatively high levels of the insured percentage of the initial portfolio value, p and/or for relatively low levels of multiple m.

3. Comparison of portfolio insurance performances with omega measure

3.1. Sensitivity analysis

The standard OBPI method is based on the choice of one unique parameter, the strike K of the put. In order to compare the two methods, first the initial amounts V_0^{OBPI} and V_0^{CPPI} are assumed to be equal, secondly the two strategies are assumed to provide the same guarantee $qK = pV_0$ at maturity T. Hence, $F_T = qK$ and then $F_0 = qKe^{-rT}$. Additionally, the initial value C_0 of the cushion is equal to the call price $C(0,S_0,K)$. Note that these two conditions do not impose any constraint on the multiple m. In what follows, we analyze the sensitivities of both OBPI and CPPI Omega functions to volatility σ (geometric Brownian motion case) and to the threshold L. This leads us to consider CPPI strategies for various values of the multiple m.[17] The portfolio payoffs for both strategies are illustrated in Fig. 4, for the numerical values in Section 2.1.

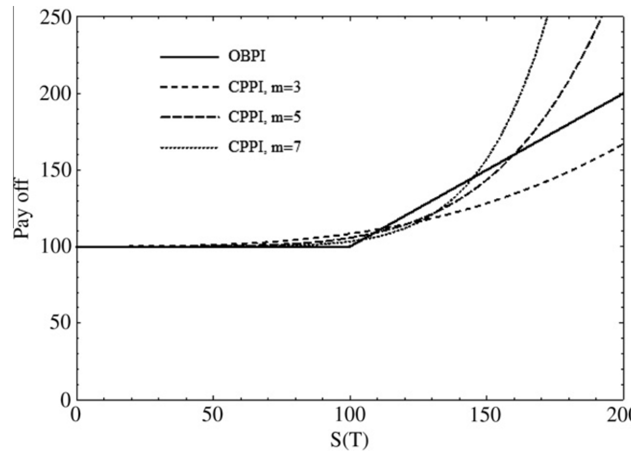

Fig. 4. CPPI and OBPI Payoff as functions of S_T.

Note that the value of the level L corresponding to the first intersection of both graphs is about 103, which is approximately the value of the riskless return. In what follows, we compare the CPPI and OPBI methods for "rational" thresholds which are in fact around this value. As a first step, we compare numerically the Omega of the OBPI and of the CPPI without any constraint on their expectations. As a second step, we impose the equality of their expectations.

The financial parameter values are those given in Section 2.1. In what follows, the threshold level is chosen lower than the lowest expectation values of both CPPI and OBPI portfolios.

3.1.1. Omega as a function of volatility σ

We first analyze the effect of volatility on OBPI Omega and on CPPI Omega for different values of the multiple m. For the usual volatility values, both OBPI and CPPI Omega functions are decreasing w.r.t. volatility σ. Additionally, the CPPI strategy most often dominates the OBPI strategy, as illustrated by Figs. 5 and 6, for the case $L = 102$. For this numerical example, CPPI strategy with a smaller multiple than another CPPI strategy dominates this latter. However if we consider theoretical values for volatility σ higher than 40%, the CPPI curves intersect each other beyond the 40% level. This means that Omega dominance of one CPPI strategy w.r.t. another does not hold for very high volatility levels. Note also that for other values of the drift μ, for example between 5% and 10%, the CPPI dominance over the OBPI is still verified.

3.1.2. Omega as a function of threshold L

Assume standard volatility levels such as $\sigma = 20\%$. For relative low values of the threshold, for example $L = 102$, the CPPI strategies dominate the OBPI strategy at all volatility values, as can be seen on Figs. 7 and 8. The effect of L becomes sensitive to high values

[17] Note that the multiple must not be too high as shown for example in Prigent (2001) or in Bertrand and Prigent (2002).

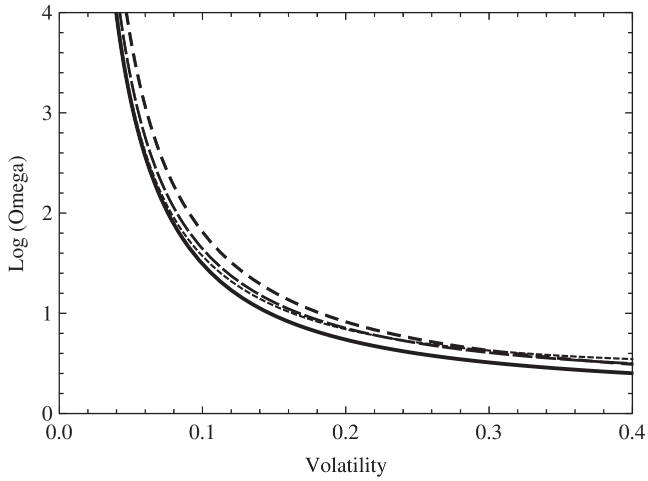

Fig. 5. Ω as a function of sigma for p = 0.9 and L = 102.

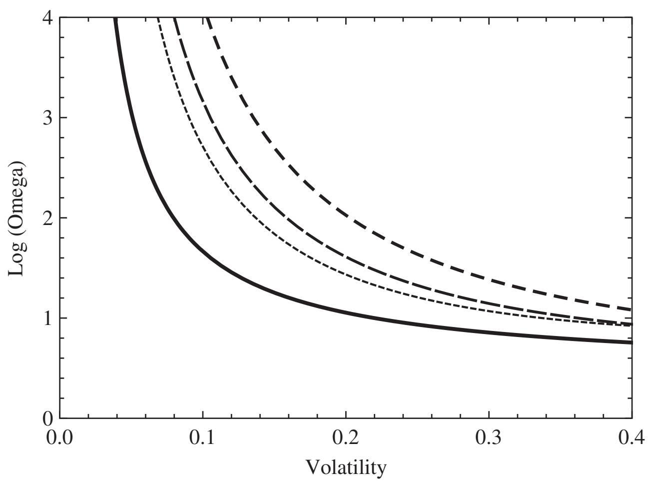

Fig. 6. Ω as function of sigma for p = 1 and L = 102.

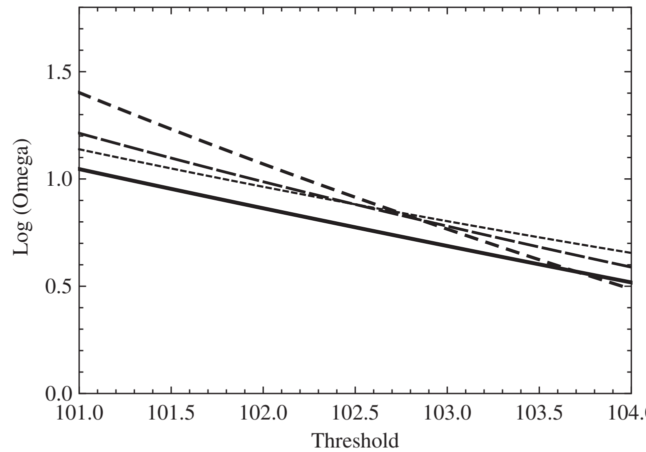

Fig. 7. Ω as a function of threshold for p = 0.9 and σ = 20%.

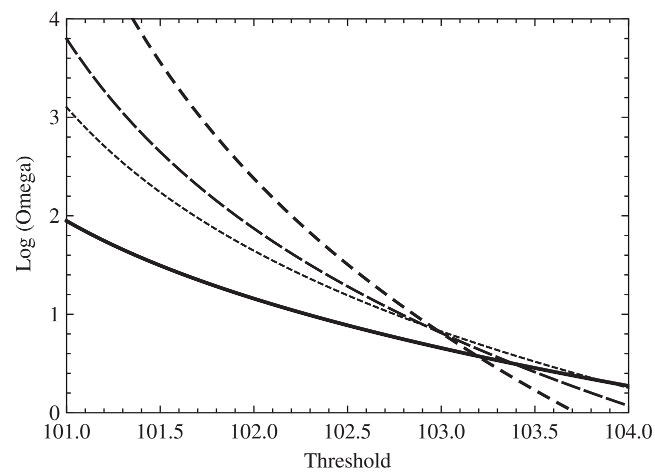

Fig. 8. Ω as a function of threshold for p = 1 and σ = 20%.

of L as already shown in Fig. 2. It is only for L higher than 103.5 that the ranking between OBPI and CPPI is inverted, particularly for low values of m. As the percentage p decreases, OBPI tends to dominate certain CPPI strategies: for example for low volatility levels and for L = 104. As soon as the threshold is low, CPPI strategies dominate OBPI strategy.

3.1.3. A special case

We now consider the case where both OBPI and CPPI portfolios values have the same expectation. Recall that the value of the multiple such that the expectations of the two portfolio values are equal is given by [18]:

$$m^*(K) = 1 + \left(\frac{1}{\mu - r}\right) \ln\left(\frac{C(0, S_0, K, \mu)}{C(0, S_0, K, r)}\right). \tag{30}$$

Figs. 9 and 10 show that, according to the Omega performance criterion, CPPI most often dominates OBPI. Indeed, when expectations are equal, OBPI and CPPI Omega functions differ only in their put components, which measure the risk of falling below the threshold. For the geometric Brownian case with μ = 10%, due to convexity as illustrated in Fig. 4, the CPPI portfolio value is higher than the OBPI one for risky asset values lower than a level of about 103.

These results are still verified when introducing other Kappa downside risk measures and lower values for the drift μ, as illustrated by Tables 5 and 6. They provide the expected portfolio values E, which represent the upper bounds of the threshold L, and the risk measures $m_l = (\mathbb{E}_{\mathbb{P}}[[(L - X)^+]^l])^{\frac{1}{l}}$.

3.2. Simulation on the US market

In this section, we provide bootstrap simulation of the Omega and Kappa performance measures of these two portfolio insurance strategies as in Annaert et al. (2009). The simulation is conducted on the S & P 500 Composite index. We consider the daily data of this index from the beginning of 1970 through late June 2010, as in previous sections. The CPPI and OBPI portfolios are simulated on a daily basis: portfolio rebalancing takes place at the daily closing price. The portfolio management period for both methods is set at 1 year (252 days). The short rate used to gross up the risk-free part of the CPPI portfolio is the Fed Fund rate. Option prices are computed according to the Black–Scholes (BS) formula. The volatility that entered the BS formula is computed on the 252 daily index returns prior to the starting date of management. The CPPI initial floor is computed with the 1 year interest rate prevailing at the

[18] See Bertrand and Prigent (2005).

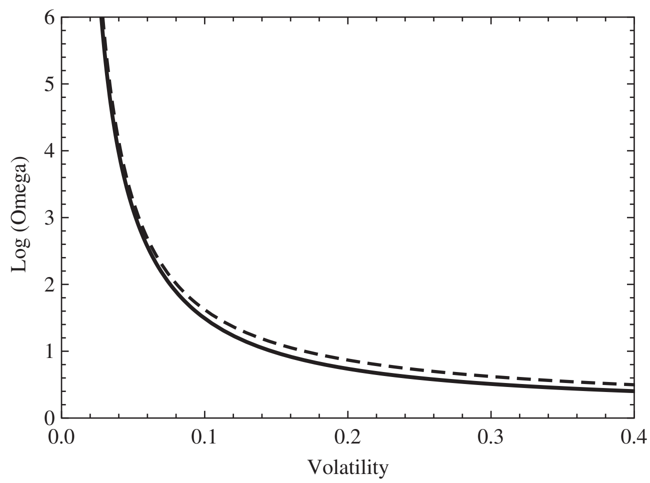

Fig. 9. Ω as a function of sigma for $p = 0.9$ and $L = 102$.

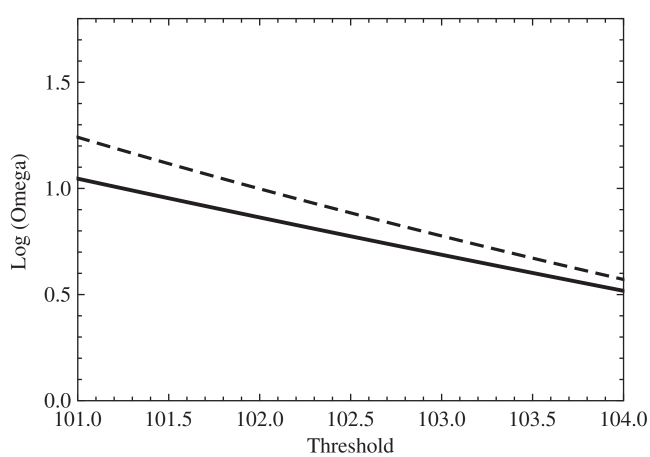

Fig. 10. Ω as a function of threshold for $p = 0.9$ and $\sigma = 20\%$.

Table 5
OMEGA for a threshold of $L = 102$.

	l	1	2	3	4
$p = 0.9$	$L = 102$		Kappa exponent l		
	$\mu = 4\%$		$E = 103.77$		
OBPI	m_l	5.43	7.70	8.78	9.41
CPPI	m_l	4.47	6.04	6.86	7.41
	$\mu = 10\%$		$E = 108.64$		
OPPI	m_l	4.08	6.60	7.88	8.67
CPPI	m_l	3.14	4.84	5.79	6.42
$p = 1$	$L = 102$		Kappa exponent l		
	$\mu = 4\%$		$E = 103.32$		
OBPI	m_l	1.53	1.74	1.81	1.86
CPPI	m_l	1.02	1.28	1.41	1.49
	$\mu = 10\%$		$E = 105.38$		
OPPI	m_l	1.33	1.62	1.74	1.80
CPPI	m_l	0.74	1.06	1.22	1.32

Table 6
OMEGA for a threshold $L = 103$.

	l	1	2	3	4
$p = 0.9$	$L = 103$		Kappa exponent l		
	$\mu = 4\%$		$E = 103.77$		
OBPI	m_l	6.02	8.41	9.55	10.23
CPPI	m_l	5.15	6.80	7.65	8.21
	$\mu = 10\%$		$E = 108.64$		
OPPI	m_l	4.54	7.22	8.58	9.43
CPPI	m_l	3.69	5.50	6.50	6.97
$p = 1$	$L = 103$		Kappa exponent l		
	$\mu = 4\%$		$E = 103.32$		
OBPI	m_l	2.31	2.62	2.74	2.80
CPPI	m_l	1.77	2.10	2.26	2.35
	$\mu = 10\%$		$E = 105.38$		
OPPI	m_l	2.01	2.44	2.61	2.70
CPPI	m_l	1.37	1.80	2.01	2.13

Table 7
SP 500 Returns.

Mean	0.000299
Standard deviation	0.00108
Skewness	−0.650
Kurtosis	23.65

starting date of portfoliio management. The same interest rate enters the BS formula. We only use index return data for which both short term and 1 year interest rates are also available.

3.2.1. S & P 500 statistics

The following statistics on the S & P 500 returns highlight the non-normality of these daily returns (see Table 7).

The mean is 0.0299% per trading day or about 7.85% per year. The standard deviation corresponds to annualized volatility of 17.15%. The skewness is negative and significant. The most interesting feature is the kurtosis which measures the magnitude of the extremes. If returns were normally distributed, then the kurtosis should be three. Here, the kurtosis is 23.262. This is strong evidence that extremes are more substantial than would be expected from a normal random variable. The Jarque–Bera test confirms this with a p-value of 0. The presence of autocorrelation and heteroskedasticity in S & P 500 returns is confirmed by performing a Ljung-Box test and an Engle's ARCH test.

Thus, it is interesting to test the robustness of previous theoretical results on real index returns data that are not normally distributed. Note also that when the return of the risky asset price is no longer a geometric Brownian motion, the payoff comparison illustrated in Fig. 4 does not hold since the CPPI portfolio is path-dependent and thus its payoff is no longer necessarily a simple power function of the risky asset value.

3.2.2. Omega and kappa bootstrapping on the S & P 500

We simulate OBPI and CPPI 1 year portfolio returns using moving block bootstrap as in Annaert et al. (2009). This procedure allows to take account for cross sectional correlation and serial dependence of returns within each block of the original data. To begin, we randomly draw a starting date with replacement. Next, we compute the OBPI and CPPI performance over the 252 days following the starting date. This procedure is then repeated 100,000 times. Omega and Kappa ratios are computed on this 100,000 OBPI and CPPI yearly returns.

Notice that on the black Monday of 1987, the S & P 500 experienced a drop of 20.47%. For the CPPI to resist to such an unfavorable event, the multiple should have been set to a value not greater than about 4.9. This is not the case in our simulations. Thus in some of our random draws, the CPPI exhibits an ending value below the insured amount reflecting the possibility of such an extreme event. It is the only day that is a problem since the second worst daily return of the sample is a drop of 9.03% during the financial turmoil of 2008. In this case, even a multiple of 10 allows the fund value to remain above its floor.

Table 8
OMEGA and Kappa of OBPI and CPPI.

	OBPI				OMEGA CPPI				KAPPA 2 CPPI			
p	Omega	$\kappa2$	$\kappa3$	$\kappa4$	$m = 5$	$m = 6$	$m = 7$	$m = 8$	$m = 5$	$m = 6$	$m = 7$	$m = 8$
	Threshold = 1%											
95	4.65	2.10	1.72	1.56	7.57	6.39	5.52	4.79	2.95	2.62	2.36	2.12
96	4.98	2.37	1.97	1.79	9.31	7.73	6.63	5.72	3.60	3.19	2.86	2.57
97	5.49	2.76	2.33	2.13	12.18	9.91	8.39	7.20	4.60	4.07	3.63	3.26
98	6.36	3.41	2.90	2.65	17.46	13.94	11.55	9.84	6.35	5.60	4.98	4.45
99	8.07	4.60	3.86	3.40	29.42	23.22	18.74	15.60	10.04	8.85	7.83	6.95
100	13.03	7.40	5.30	3.98	109.4	77.07	56.97	44.02	21.86	19.41	17.15	15.15
	Threshold = 2%											
95	3.57	1.51	1.25	1.13	5.14	4.45	3.91	3.45	1.99	1.79	1.61	1.45
96	3.71	1.65	1.39	1.26	5.98	5.13	4.46	3.91	2.33	2.09	1.88	1.69
97	3.94	1.85	1.57	1.44	7.22	6.11	5.28	4.59	2.83	2.53	2.27	2.04
98	4.30	2.15	1.84	1.70	9.23	7.67	6.57	5.66	3.60	3.20	2.86	2.57
99	4.90	2.62	2.26	2.06	12.78	10.46	8.82	7.56	4.91	4.36	3.89	3.48
100	6.16	3.52	2.93	2.51	25.39	19.82	15.94	13.19	7.51	6.71	5.99	5.35
	Threshold = 3%											
95	2.78	1.08	0.90	0.81	3.56	3.17	2.85	2.56	1.32	1.20	1.08	0.96
96	2.84	1.15	0.97	0.89	3.95	3.49	3.11	2.78	1.50	1.36	1.22	1.09
97	2.94	1.25	1.07	0.98	4.48	3.92	3.47	3.09	1.74	1.57	1.42	1.27
98	3.09	1.39	1.20	1.11	5.24	4.55	3.99	3.53	2.09	1.88	1.69	1.51
99	3.32	1.60	1.40	1.29	6.36	5.47	4.78	4.19	2.61	2.34	2.11	1.89
100	3.76	1.96	1.70	1.54	9.60	8.02	6.84	5.89	3.41	3.08	2.78	2.50
	Threshold = 4%											
95	2.21	0.75	0.63	0.57	2.50	2.30	2.12	1.94	0.83	0.76	0.69	0.61
96	2.22	0.78	0.66	0.61	2.67	2.44	2.24	2.04	0.92	0.84	0.76	0.67
97	2.25	0.83	0.71	0.65	2.88	2.62	2.39	2.17	1.03	0.94	0.85	0.75
98	2.31	0.89	0.77	0.72	3.15	2.85	2.59	2.34	1.18	1.07	0.97	0.86
99	2.40	0.98	0.86	0.80	3.50	3.16	2.86	2.58	1.38	1.26	1.14	1.01
100	2.57	1.14	1.00	0.93	4.50	4.00	3.57	3.18	1.64	1.51	1.37	1.23

Table 9
(continuation of Table 8) OMEGA and Kappa of OBPI and CPPI.

	OBPI				OMEGA CPPI				KAPPA 2 CPPI			
p	Omega	$\kappa2$	$\kappa3$	$\kappa4$	$m = 5$	$m = 6$	$m = 7$	$m = 8$	$m = 5$	$m = 6$	$m = 7$	$m = 8$
	Threshold = 1%											
95	4.65	2.10	1.72	1.56	2.18	2.01	1.85	1.70	1.85	1.74	1.63	1.51
96	4.98	2.37	1.97	1.79	2.63	2.42	2.23	2.04	2.22	2.08	1.95	1.81
97	5.49	2.76	2.33	2.13	3.31	3.05	2.80	2.57	2.77	2.61	2.44	2.26
98	6.36	3.41	2.90	2.65	4.48	4.12	3.78	3.46	3.72	3.50	3.27	3.03
99	8.07	4.60	3.86	3.40	6.89	6.34	5.82	5.32	5.65	5.31	4.97	4.62
100	13.03	7.40	5.30	3.98	14.39	13.33	12.25	11.20	11.57	10.94	10.27	9.55
	Threshold = 2%											
95	3.57	1.51	1.25	1.13	1.50	1.39	1.29	1.18	1.29	1.21	1.14	1.05
96	3.71	1.65	1.39	1.26	1.75	1.62	1.49	1.37	1.49	1.41	1.32	1.22
97	3.94	1.85	1.57	1.44	2.10	1.94	1.79	1.64	1.78	1.68	1.57	1.46
98	4.30	2.15	1.84	1.70	2.63	2.43	2.23	2.05	2.22	2.09	1.95	1.81
99	4.90	2.62	2.26	2.06	3.53	3.26	3.00	2.75	2.96	2.79	2.61	2.42
100	6.16	3.52	2.93	2.51	5.34	4.94	4.55	4.17	4.44	4.19	3.92	3.65
	Threshold = 3%											
95	2.78	1.08	0.90	0.81	1.01	0.95	0.88	0.80	0.88	0.83	0.78	0.72
96	2.84	1.15	0.97	0.89	1.15	1.07	0.99	0.90	0.99	0.94	0.88	0.81
97	2.94	1.25	1.07	0.98	1.33	1.23	1.14	1.04	1.14	1.08	1.01	0.93
98	3.09	1.39	1.20	1.11	1.58	1.47	1.35	1.24	1.36	1.28	1.20	1.11
99	3.32	1.60	1.40	1.29	1.96	1.82	1.68	1.54	1.68	1.58	1.48	1.37
100	3.76	1.96	1.70	1.54	2.57	2.39	2.21	2.03	2.19	2.07	1.95	1.81
	Threshold = 4%											
95	2.21	0.75	0.63	0.57	0.65	0.61	0.57	0.51	0.57	0.54	0.51	0.46
96	2.22	0.78	0.66	0.61	0.72	0.68	0.62	0.56	0.63	0.60	0.56	0.51
97	2.25	0.83	0.71	0.65	0.81	0.76	0.70	0.63	0.70	0.67	0.62	0.57
98	2.31	0.89	0.77	0.72	0.92	0.86	0.80	0.72	0.80	0.76	0.71	0.65
99	2.40	0.98	0.86	0.80	1.08	1.01	0.93	0.85	0.94	0.89	0.83	0.76
100	2.57	1.14	1.00	0.93	1.30	1.21	1.13	1.03	1.13	1.08	1.01	0.93

Consider typically the CPPI method for the following values of the multiple: $m = 5, 6, 7$ and 8. Several levels of insured percentage are also examined[19]: $p = 95\%, 96\%, 97\%, 98\%, 99\%$, and 100%.

The results for the OBPI and CPPI Omega and Kappa (for $l = 2,3$ and 4) are displayed in Tables 8 and 9. According to both the Omega and the Kappa criterion and ceteris paribus, the CPPI performs better for relatively low threshold levels. The CPPI also performs better for relatively high levels of the insured percentage p and/or for relatively low levels of multiple, m. To sum up, in most of the market circumstances, CPPI strategy is better rated than OBPI strategy by Omega as well as Kappa performance measures.[20]

4. Conclusion

In this paper, we analyze the performance of the two main portfolio insurance methods and search for the best method, according to a large class of performance measures. Since the payoffs of portfolio insurance strategies are convex with respect to the risky reference asset, their return distributions are clearly asymmetric. Thus, we need to introduce performance measures which take account of this feature, as opposed to traditional performance measures such as the Sharpe ratio. Additionally, we impose that these measures correspond as usual to "reward/risk" ratios. Due to the non-normality of the returns, we also have to use downside risk measures. The Kappa performance measures satisfy these two conditions and take all the moments of the returns distribution into account. They have also been intensively used to study assets with non-normally distributed returns, such as hedge funds. However, as for performance measures based on downside deviations, we have to choose carefully the threshold associated with the Kappa functions. It is exogenously defined but the loss threshold could also be defined by the investor's preferences. We emphasize the particular case of the Omega measure, since it is related to loss aversion. The evaluation of an investment using the Omega measure should take thresholds between 0% (above the guarantee in this paper) and the risk-free rate. Intuitively, this type of threshold corresponds to the notion of capital protection. We show that, for this criterion and more generally for Kappa performance measures, the CPPI method generally performs better than the OBPI strategy. We illustrate this property for two main cases: first, the risky log-return is assumed to be a Brownian motion with drift, which is the standard model; second, it is the sum of a Brownian motion and a compound Poisson process with jump sizes double-exponentially distributed. We also confirm this result by backtesting on the S & P 500 data. This result has important consequences, since now such methods are not only applied to equity markets but also to various financial instruments.

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[19] We are not able to go beyond 100% because the short term interest rate dropped to nearly 0% at some dates.
[20] The numbers underlined in the table are the ones for which the value for the OBPI is higher than that of the CPPI.

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