Sentiment, Implied Volatility Slope, and Risk-Neutral Skewness

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Abstract

Empirical studies show that sentiment, implied volatility slope and risk-neutral skewness all predict future stock returns. High sentiment and steeper implied volatility curve typically predict negative future returns. However, evidence of return predictability from risk-neutral skewness are mixed. This paper proposes a theoretical model that explains and reconciles these empirical findings. We show that up to a sentiment threshold, implied volatility slope becomes steeper when sentiment is higher. The threshold is large enough to contain most empirical situations. The relationship between risk-neutral skewness and sentiment (or implied volatility slope) can be either positive or negative depending on market conditions. Our model helps explain the difference between implied volatility curves of index options and stock options. It generates implied volatility curves that are consistent to empirical data.

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1 Introduction

Extensive empirical literature shows that sentiment, slope of option implied volatility, and risk-neutral skewness all predict stock returns. Sentiment is a biased belief on asset fundamentals. Investor sentiment is able to predict future stock returns because it causes temporary overreaction in stock prices and such overreaction is typically followed by stock price reversals. Implied volatility slope is measured as the gap of implied volatility at different strike prices. Implied volatility slope is able to predict future stock returns because the slope embeds information about forward-looking risk and such risk is going to be compensated by stock returns if forward-looking risk is fully or partly realised. Risk-neutral skewness measures the asymmetry of risk-neutral density. It is extracted from option prices and is model-independent. Risk-neutral skewness may contain additional information which is not captured by implied volatility. Several empirical studies on risk-neutral skewness and its stock return predictability provide conflicting results and the current literature does not offer convincing reconciliation between these conflicting findings.

Although sentiment, implied volatility slope, and risk-neutral skewness are important financial concepts and are investigated widely in the literature, there is no theoretical framework which unifies these three together. In this paper, we propose a theoretical model to integrate these three concepts through sentiment-driven option trading and reconcile empirical findings around them.

In stock market, Brown and Cliff (2005) and Baker and Wurgler (2006) show that sentiment leads to overreaction and this is followed by stock price reversal in the future. Stambaugh et al. (2012) find that bullish sentiment results in overreaction and price reversal but bearish sentiment does not necessarily lead to overreaction. Greenwood and Shleifer (2014) show that individual and institutional investors' sentiment levels are highly correlated, and both negatively predict future stock returns. Overall, empirical findings tend to have a consensus that investor sentiment negatively predict returns.

Bridging options market and stock market, one strand of studies documents the relation between implied volatility slope and future stock returns. Bates (1991) finds that implied volatility slope of index options was extraordinarily steep before the 1987 stock market crash. Yan (2011) uses stock option implied volatility slope as a proxy for downside risk and finds that volatility slope negatively predicts cross-section stock returns. Some research attributes implied volatility slope's return predictability to informed trading. Xing et al. (2010) find steeper implied volatility slope before negative earnings shocks. Jin et al. (2012) show that around price sensitive events the return predictability of implied volatility slope is stronger. This strand of studies supports that steepening of implied volatility slope negatively predict returns.

Another strand of studies examines stock return predictability through skewness of risk-neutral density. Stilger et al. (2016) find that risk-neutral skewness positively predict future returns. However, Conrad et al. (2013) find the opposite result. Xing et al. (2010) find that risk-neutral skewness has no predictability for future stock returns after controlling implied volatility slope and they also find weak correlation between risk-neutral skewness and implied volatility slope. Bali and Murray (2013) summarise these contradictory results but they provide no explicit solution to reconcile them.

Empirical findings in above two strands of studies seem to reject the one-to-one mapping relationship between implied volatility slope and risk-neutral skewness (Bakshi et al., 2003). From the theoretical point of view, we need a unifying theory that generates return predictability from both sentiment and implied volatility slope, consistent to myriad empirical findings. At the same time, the theory should provide both positive and negative relations between risk-neutral skewness and sentiment (or implied volatility slope), accommodating conflicting results on return predictability from risk-neutral skewness.

In this paper, we propose a model in which sentiment traders and rational traders jointly determine the market price of options. Sentiment traders have a poor understanding of future risk and price the options with irrationally low implied volatility when past returns are high. Rational traders expect that high risk is associated with high returns and price the options accordingly. The market option price is a mixture of two Black-Scholes components and the mixing weights are proportions of sentiment and rational traders. In this model, we are able to obtain implied volatility slope and risk-neutral skewness in closed-forms and analyse their relations with sentiment. We show that implied volatility slope is steeper when sentiment is higher, up to a certain threshold. This sentiment threshold is large enough to contain most empirical situations. We also show that the relation between risk-neutral skewness and sentiment (or implied volatility slope) is not monotonic (a.k.a. not a one-to-one correspondence) and is dependent on market conditions. Our model can also generate implied volatility curves that match the shapes observed in different types of options (e.g. index options vs. stock options; options under different maturities).

Our model is different from previous sentiment-based models (e.g. Barberis et al. (1998); Barberis et al. (2015)) in several ways. First, previous models focus on sentiment investors' biased estimation of future cash flows (dividends). Our model focuses on sentiment investors' underestimation of future risk. Second, previous models focus on how sentiment is formed by extrapolating past returns and how subsequent price reversal happens. Our model takes the relation between high sentiment and subsequent stock price reversal as a *priori*. Last, previous models explain how sentiment drives stock prices. Our model explains how sentiment drives implied volatility slope and risk-neutral skewness. In our model, the return predictability from implied volatility slope or risk-neutral skewness roots in the change of sentiment.

In Section 2, we present the theoretical model and main results. In Section 3, we discuss the empirical implications of our results. Section 4 concludes. All proofs are in Appendix.

2 Model

We analyse an options market in which there are two types of traders: sentiment traders and rational traders. Sentiment traders extrapolate past stock returns to future returns (Barberis et al., 2015) and misperceive risk (volatility) associated with returns (Yu and Yuan, 2011). On the contrary, rational traders have realistic belief on return-risk tradeoff (Yu and Yuan, 2011) and price in high risk when returns are high.

The underlying stock price dynamic is as follows:

$$\frac{dX_t}{X_t} = \mu_t dt + \sigma_t d\omega_t,$$

where X_t is the stock price and ω_t is the standard one-dimensional Brownian motion; μ_t and σ_t are drift and diffusion coefficients respectively. Rational traders believe in positive μ_t and σ_t relation but sentiment traders tend to ignore such relation. The traders' belief on drift term coefficient μ_t does not impact option pricing so we focus our analysis on σ_t. We denote S as sentiment traders and R as rational traders. For sentiment traders $\sigma_t = \sigma_S$ and for rational traders $\sigma_t = \sigma_R$. For simplicity, we assume σ_S and σ_R are both constant.

The empirical findings on sentiment-related anomalies are much more pronounced during high sentiment periods than low sentiment periods (e.g. Stambaugh et al. (2012); Chang et al. (2015); Seo and Kim (2015)). When market sentiment is high, Yu and Yuan (2011) find that the positive mean-variance relation weakens. They argue that such weakening is due to the ignorance of sentiment traders on realistic mean-variance tradeoff. Goyal and Saretto (2009) find that option implied volatility tends to be lower than historical volatility when stocks perform well. Combining these studies, we assume that $\sigma_R > \sigma_S$, which means sentiment traders underestimate future risk.

The options market is populated by sentiment traders in proportion θ and rational traders in proportion $1-\theta$. The market price of option is O ($O = C$ for call options and $O = P$ for put options), a mixture of sentiment and rational traders' valuations:

$$O_M = \theta\, O(\sigma_S) + (1-\theta)\, O(\sigma_R), \tag{1}$$

where $\mathrm{O}(\sigma_S)$ $\big(\mathrm{O}(\sigma_R)\big)$ is sentiment traders' (rational traders') valuation. This market price model is similar to Brown and Cliff (2005, 2004).

All market participants use Black-Scholes option pricing model. Call option price is $C(\sigma) = X_t\,\Phi[d_+(\sigma)] - Ke^{-rT}\Phi[d_-(\sigma)]$ and put option price is $P(\sigma) = Ke^{-rT}\Phi[-d_-(\sigma)] - X_t\,\Phi[-d_+(\sigma)]$, where T denotes the expiry of the option, r is the risk-free rate, K is the strike price and Φ denotes the standard normal cumulative distribution function.[1] The implied volatility σ_{IV} is derived backward from the market price using Black-Scholes formula: $O_M = O(\sigma_{IV})$.

To analyse the change of implied volatility slope when sentiment level changes, here we focus on two model parameters: θ and K. From Equation (1), we explicitly derive implied volatility slope (denoted as $IVS(O,\theta,K)$). Proposition 1 presents our result.

Proposition 1 *The slope of option implied volatility curve is:*

$$\begin{aligned} & IVS(P,\theta,K) = IVS(C,\theta,K) \\ & = \frac{e^{-rT}}{\sqrt{T}} \times \frac{-\theta\Big(\Phi\big[d_-(\sigma_S)\big] - \Phi\big[d_-(\sigma_R)\big]\Big) + \Big(\Phi\big[d_-(\sigma_{IV})\big] - \Phi\big[d_-(\sigma_R)\big]\Big)}{\phi\big[d_+(\sigma_{IV})\big]} \end{aligned} \tag{2}$$

Proof: See Appendix.

In our model, put and call options with the same parameters (θ, K, T, r) have the same option price. The closed-form result obtained in Proposition 1 allows us to conduct detailed analysis on implied volatility slope and its change.

Lemma 1 shows that with the coexistence of two groups of traders ($\theta \in (0,1)$), the implied volatility curve is not flat and implied volatility slope has typical sign

[1] $d_\pm(\sigma) = \frac{\ln \frac{X_t\, e^{rT}}{K} \pm \frac{\sigma^2}{2}T}{\sigma\sqrt{T}}$.

as observed in the market.

Lemma 1 *a)* $IVS < 0$ *for out-of-the-money (OTM) put options and in-the-money (ITM) call options* $(K < X_t\, e^{rT})$ *when* $\theta \in (0, 1)$*;*

b) $IVS > 0$ *for ITM put options and OTM call options* $(K > X_t\, e^{rT})$ *when* $\theta \in (0, 1)$*;*

c) $IVS = 0$ *for at-the-money (ATM) put and call options* $(K = X_t\, e^{rT})$*;*

d) $IVS = 0$ *when* $\theta = 0$ *or* $\theta = 1$*.*

Proof: See Appendix.

[Insert Figure 1 here.]

Figure 1 illustrates implied volatility curves at different sentiment level θ. The parameters are: $r = 6\%$, $T = 60/360$, $\sigma_S = 20\%$ and $\sigma_R = 40\%$. These parameter values are also used by subsequent figures in this paper, unless the values are stated otherwise. When $\theta = 0\%$ or 100%, the market is dominated by only rational traders or sentiment traders, implied volatility curve is flat. When θ increases from 0% to 70%, implied volatility curve is steeper at each scaled strike price level K/X_t. In Figure 1, implied volatilities are scaled by ATM implied volatility for better illustration.[2]

Proposition 2 shows that within a range of θ, implied volatility curve is steeper as sentiment increases.

[2] Some empirical studies (Toft and Prucyk (1997),Vagnani (2009)) scale implied volatility slope at different strike prices by ATM implied volatility. Dennis and Mayhew (2002) criticize such practice and argue that scaling distorts implied volatility slope, especially when ATM implied volatility is low. Our results in this paper are independent of volatility scaling and unscaled volatility curves have similar patterns to scaled curves.

Proposition 2 *For OTM puts (ITM calls), when* $\sigma_{IV} < \sqrt{\frac{2ln\frac{X_te^{rT}}{K}}{T}}$*, implied volatility slope will be more negative as* θ *increases in the range* $(0, \theta^*)$*. For ITM puts (OTM calls), when* $\sigma_{IV} < \sqrt{-\frac{2ln\frac{X_te^{rT}}{K}}{T}}$*, implied volatility slope will be more positive as* θ *increases in the range* $(0, \theta^*)$*.*

$$\begin{aligned} \theta^* &= \frac{P(\sigma^*) - P(\sigma_R)}{P(\sigma_S) - P(\sigma_R)} \\ \sigma^* &= \sqrt{\frac{ln(X_te^{rT}/K)/T}{\frac{Ke^{-rT}\left(\Phi[d_-(\sigma_S)] - \Phi[d_-(\sigma_R)]\right)}{P(\sigma_R)-P(\sigma_S)} - \frac{1}{2}}} \end{aligned} \tag{3}$$

Proof: See Appendix.

The constraint $\sigma_{IV} < \sqrt{\frac{2ln\frac{X_te^{rT}}{K}}{T}}$ or $\sigma_{IV} < \sqrt{-\frac{2ln\frac{X_te^{rT}}{K}}{T}}$ is satisfied by typical empirical studies. According to Xing et al. (2010), the average implied volatility of short-term OTM puts is 54.35%. For OTM 2-month put option with $T = 60/360$, the constraint requires that implied volatility σ_{IV} shouldn't exceed 85.76% ($K/X_t = 0.95$) and 164% ($K/X_t = 0.80$). The range $K/X_t \in (0.80, 0.95)$ is typical in empirical studies (e.g. Xing et al. (2010); Lin and Lu (2015)).

To further analyze the constraint, we discuss two limit cases. First, ATM options ($X_te^{rT} = K$) is a limit case and the right-hand-side of the constraint is zero. The condition becomes more relaxed when options are deep out-of-the-money or deep in-the-money. Second, $T \to \infty$ is the other limit case, because $\lim_{T\to\infty} \sqrt{\frac{2ln\frac{X_te^{rT}}{K}}{T}} = \lim_{T\to\infty} \sqrt{\frac{2ln\frac{X_t}{K}}{T} + 2r} = \sqrt{2r}$. Empirically, when maturity T increases, the implied volatility (σ_{IV}) decreases and the constraint can be satisfied by most cases.

Proposition 2 gives a threshold point θ^*, below which the implied volatility curve is steeper when sentiment is higher. The implied volatility slope change driven by sentiment is one of the key contributions of our paper. Proposition 2 provides a sufficient condition for this important interaction between implied volatility slope and sentiment.

In previous studies (e.g. Vagnani (2009); Yan (2011)), analytical results depend on Taylor approximation around ATM point and numerical results or verification depend on Monte-Carlo simulation. Our results do not rely on either approximation or simulation and can cover a large range of implied volatility curve.

Figure 2 plots θ^* when the ratio of σ_R/σ_S changes. The figure allows us to examine the relation between sentiment threshold value θ^* and rational/sentiment belief gap.

[Insert Figure 2 here.]

Figure 2 shows that when rational/sentiment belief gap increases, the threshold value θ^* increases. When sentiment is high, rational/sentiment belief gap typically increases together with θ. Figure 2 shows that our theoretical result in Proposition 2 has potential to cover a large range of sentiment levels. We will discuss more about θ^* in Section 3.

In our model, risk-neutral skewness can also be obtained analytically. Proposition 3 presents the result.

Proposition 3 *Risk-neutral density implied by the market option price is a mixture of two lognormals with parameters* $(lnX_0 + (r - \frac{1}{2}\sigma_S^2)T, \sigma_S^2 T)$ *and* $(lnX_0 + (r - \frac{1}{2}\sigma_R^2)T, \sigma_R^2 T)$*. The mixture weights are* θ *and* $1-\theta$ *respectively.*

The corresponding risk-neutral skewness (RNS) is:

$$RNS = \frac{\theta\left(e^{3\sigma_S^2 T} - e^{3\sigma_R^2 T} + 3e^{\sigma_R^2 T} - 3e^{\sigma_S^2 T}\right) + e^{3\sigma_R^2 T} - 3e^{\sigma_R^2 T} + 2}{\left(\theta\left(e^{\sigma_S^2 T} - e^{\sigma_R^2 T}\right) + e^{\sigma_R^2 T} - 1\right)^{\frac{3}{2}}} \tag{4}$$

Proof: See Appendix.

Bakshi et al. (2003) use an approximation of density function to derive a linear relationship between implied volatility (implied volatility slope) and risk-neutral skewness. Our results on risk-neutral skewness and the relationship between risk-neutral skewness and implied volatility slope do not require approximation and can be used to uncover the nonlinear and non-monotonic relationship which may help to reconcile previous conflicting empirical findings.

In the next section, we use the closed-form results (2), (3) and (4) to analyse empirical implications of our model.

3 Empirical Implications

3.1 Sentiment and steepness of implied volatility slope

Our model reconciles empirical findings that high sentiment and steep implied volatility slope both negatively predict stock returns. In our model, θ is the main parameter that captures the sentiment-driven trading in options market and it drives the relation between sentiment and implied volatility slope in our theoretical results. When sentiment is higher, implied volatility slope is steeper, and the return predictability of both sentiment and implied volatility slope comes from the subsequent stock price reversal.

We discuss two empirical links that help bridge our theoretical model and options market reality. The first empirical link is that sentiment driven option traders engage in both call and put options trading. This means empirically $\theta \neq 0$ across call and put options. Lakonishok et al. (2006) and Mahani and Poteshman (2008) find that in financial bubble or before earnings announcement, optimistic option traders buy calls and write puts mainly for speculative purpose. Lakonishok et al. (2006) find that discount customers are more likely to increase their long positions in stocks through options trading (buying calls and writing puts) during financial bubbles. Mahani and Poteshman (2008) classify the option traders into firm proprietary traders, customers of full-service brokers, and customers of discount brokers. They find that the firm proprietary traders are the most sophisticated and the customers of discount brokers are driven by speculation and sentiment.

The second empirical link is that the threshold θ^* is large enough to cover most empirical findings. According to the option open interest data in Lakonishok et al. (2006) and Mahani and Poteshman (2008), approximately 20% of option traders are sophisticated traders. In Lemmon and Ni (2014), about 30% of option traders are sentiment option traders. Combining these studies, sentiment traders are about 30% to 80% of all options market participants.

Figure 3 plots the slope of OTM puts ($K = 0.8X_t$) against θ under three combinations of σ_S and σ_R. Combinations $\sigma_S = 20\%$ or 30% and $\sigma_R = 50\%$ allow us to compare implied volatility slope when sentiment traders' belief changes. Combinations $\sigma_S = 30\%$ and $\sigma_R = 40\%$ or 50% allow us to compare implied volatility slope when rational traders' belief changes. The value range of OTM puts' slopes in general match empirical data (e.g. Bollen and Whaley (2004)).

[Insert Figure 3 here.]

Figure 3 shows that θ^* is a conservative threshold. For example, when $\sigma_S = 20\%$ and $\sigma_R = 50\%$, $\theta^* = 65.36\%$, while implied volatility slope is monotonously decreasing in θ up to a point $\theta > 90\%$. $\theta^* = 56.56\%$ when sentiment traders' belief and rational traders' belief are close to each other ($\sigma_S = 30\%$ and $\sigma_R = 40\%$). Even at this level, $(0, \theta^*)$ still covers a large sentiment range.

Figure 3 also shows that when the gap between sentiment traders' belief and rational traders' belief becomes larger, implied volatility slope is steeper. This result reinforces the relation between sentiment and implied volatility slope because belief gap tends to be large when sentiment is high. The result is also consistent with the finding in Figure 2.

As an interesting phenomenon, Figure 3 also shows that when sentiment level is extremely high, implied volatility slope of OTM put becomes flatten again. Too much participation of sentiment traders (e.g. in financial bubbles) flattens implied volatility slope rather than steepens it. This result reconciles Bates (1991)'s finding that before the 1987 stock market crash, OTM put prices went up dramatically and then went down 2 months immediately before the crash.

We note that empirical literature does not provide consistent measure of implied volatility slope. Researchers tend to justify their choice of measure by emphasizing research topics and measurement error concerns. Empirical research in unifying various implied volatility slope measures or comparing performance of different empirical measures is still empty. Table 1 lists several recent empirical papers and their different measures of implied volatility slope.

[Insert Table 1 here.]

Our theoretical results cover a large range of OTM (and ITM) options and can be treated as complementary to previous theoretical results for ATM options (e.g. Yan (2011)). In our results, the choice of implied volatility slope measure is not as sensitive as previous studies.

3.2 Return predictability of risk-neutral skewness

Previous studies show that risk-neutral skewness, extracted from option prices, can predict stock returns. However, empirical evidence on the return predictability of risk-neutral skewness are mixed (Xing et al., 2010; Conrad et al., 2013; Stilger et al., 2016). Our model, through the angle of sentiment-driven option trading, is able to reconcile these conflicting empirical evidence.

Figure 4 plots risk-neutral skewness and implied volatility slope for OTM put ($K = 0.8X_0$) when belief gap is small ($\sigma_S = 30\%$ and $\sigma_R = 40\%$) in Panel (a) and when belief gap is large ($\sigma_S = 20\%$ and $\sigma_R = 50\%$) in Panel (b).

[Insert Figure 4 here.]

In previous studies, Conrad et al. (2013) find that risk-neutral skewness negatively predicts future stock returns; while Stilger et al. (2016) have the opposite finding. Xing et al. (2010) find that the correlation between risk-neutral skewness and implied volatility slope is low and there is no return predictability power from risk-neutral skewness when implied volatility slope is controlled in regression. In Panel (a), risk-neutral skewness and implied volatility slope are positively related up to $\theta^*(= 56.56\%)$. If high sentiment and steeper implied volatility slope negatively predict future returns, the positive relation between risk-neutral skewness and implied volatility slope supports Stilger et al. (2016)'s finding. In Panel (b), risk-neutral skewness and implied volatility slope are negatively related up to $\theta^*(= 65.36\%)$, lending support on Conrad et al. (2013)'s finding. Both panels show that the relation between risk-neutral skewness and implied volatility slope is not a one-to-one correspondence. Depending on the sentiment level and belief gap, risk-neutral skewness and implied volatility slope can have either positive or negative relation. This reconciles findings in Xing et al. (2010).

A key assumption in Bakshi et al. (2003)'s one-to-one RNS - IVS correspondence proof is that the risk-neutral density is uni-modal and a density approximation can be successfully applied. To make sure the rejection to the one-to-one correspondence is not driven by the violation of the uni-modal density assumption, we show a pair of typical risk-neutral densities in Figure 5.

[Insert Figure 5 here.]

Figure 5 illustrates the risk-neutral densities under two situations matching Panels (a) and (b) parameters in Figure 4: 1) $\sigma_S = 30\%$ and $\sigma_R = 40\%$; and 2) $\sigma_S = 20\%$ and $\sigma_R = 50\%$. For both situations $\theta = 50\%$, and this is one of the cases that reject one-to-one correspondence. Figure 5 shows that both risk-neutral densities are indeed uni-modal and thus reinforces our results in Figure 4.

For robustness, we also check the relation between risk-neutral skewness and implied volatility slope measured by OTM put implied volatility- OTM call implied volatility, the results are similar to what we obtain in Figure 4. Since a large belief gap and a high θ is observed when the sentiment level is high (Yu and Yuan, 2011), we also examine the relation between risk-neutral skewness and implied volatility slope when belief gap (σ_R/σ_S) is increasing in θ. The results are also similar. Details can be obtained from the authors.

3.3 Index options vs. stock options: when hedging demand matters

Empirical studies find that traders trade index options and stock options from different motives. Option traders use index options to hedge and use individual stock options to speculate. Bollen and Whaley (2004) find that index put options are mainly traded by institutional investors, to hedge against market declines. And Garleanu et al. (2009) find that institutional and non-institutional traders together hold a net long positions in OTM index put options due to hedging demand. The hedging demand of OTM index put is exemplified by its implied volatility curve's shift from smile to skew after the 1987 stock market crash (Dumas et al., 1998). On the contrary, Lakonishok et al. (2006) show that traders tend to hold naked positions in stock options and speculate on the movements of stock prices.

These two types of options have different implied volatility shape due to traders' different motives. Bollen and Whaley (2004) show that index options typically have a monotonically declining implied volatility curve, while stock option's implied volatility curve is much flatter and generally exhibit a smile pattern.

To incorporate hedging demand into our model, we allow θ to change across strike prices for index put options.

[Insert Figure 6 here.]

Figure 6 illustrates the difference between index put option implied volatility curves and stock put option implied volatility curves. For index put option, θ increases linearly on strike price K ($\theta = 20\%$ when $K = 0.80X_t$ and $\theta = 80\%$ when $K = 1.20X_t$). In this setting, rational investors (with hedging demand) hold a larger proportion of the OTM index put options and sentiment investors (with speculation motive) hold a smaller proportion. For stock put option, $\theta = 30\%$ over all levels of K. Figure 6 shows that hedging demand changes the shape of implied volatility curve. Index put option's implied volatility decreases when strike price increases. Stock option exhibits typical volatility smile. The implied volatility curve of stock option is flatter than the curve of index option. These results are consistent to the empirical findings in Bollen and Whaley (2004).

Han (2007) find that negative view on market from institutional investors comes at the same time with a steeper implied volatility slope of index options. When σ_R becomes larger (changing from 20% to 25%), institutional (rational) traders have more fear on market decline. In this situation, our result shows that the implied volatility slope is steeper, which reconciles the finding of Han (2007).

3.4 Implied volatility curve under different maturities

Empirical evidence show that the implied volatility curve is flatter for options with longer maturities (Rubinstein, 1985; Bakshi et al., 1997; Branger and Schlag, 2004). Two types of theoretical models - the jump diffusion model and the stochastic volatility model - fail to generate empirical curve pattern when maturity is long. Das and Sundaram (1999) find that the jump diffusion model' implied volatility curve is too flat and the stochastic volatility model's implied volatility curve is too steep.

Figure 7 compares the implied volatility curves generated by our model to the curves generated by the jump diffusion model and the stochastic volatility model.

[**Insert Figure 7 here.**]

For the jump diffusion model and the stochastic volatility model, we use the data from Das and Sundaram (1999). For our model, we present three curves with the following key parameters: T=1-month, $\theta = 60\%$; T=12-month, $\theta = 60\%$ and T=12-month, $\theta = 30\%$. Figure 7 shows that our model generates flatter implied volatility curve when option maturity increases. This is consistent to empirical finding. We note that rational traders (sentiment traders) may trade more long-maturity (short-maturity) options and our model can also produce flatter long-maturity curve when θ decreases in T.

For long maturity (T=12-month), our model produces a curve which is between the jump diffusion model (too flat) and stochastic volatility model (too steep). Although our model is parsimonious, it can generate implied volatility curves that match various shapes observed in options market.

4 Conclusion

Our paper proposes a new theoretical model to unify empirical findings on sentiment, implied volatility slope and risk-neutral skewness. Empirical evidence on return predictability from sentiment and implied volatility slope are well-established. However, the evidence on return predictability from risk-neutral skewness are mixed. Our model is distinguished from previous sentiment-based models. It helps explain the interaction between sentiment, implied volatility slope and risk-neutral skewness. It shows that high sentiment from option traders generates steeper implied volatility slope under a wide spectrum of situations. It also shows that the relation between risk-neutral skewness and implied volatility slope (or sentiment) is not monotonic. Our model explains why both high sentiment and steeper implied volatility slope, through their interaction, negatively predict future stock returns. And through the interaction between sentiment and risk-neutral skewness, our model also explains why risk-neutral skewness does not have consistent return predictability. Our model is parsimonious and it can generate implied volatility curves that match various shapes observed in options market.

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Appendix A

Proof of Proposition 1:

Use the constraint $P(\sigma_{IV}) = \theta\, P(\sigma_S) + (1-\theta)\, P(\sigma_R)$ and define $U = P(\sigma_{IV}) - \theta\, P(\sigma_S) - (1-\theta)\, P(\sigma_R)$. Apply implicit function, we obtain implied volatility slope as:

$$\begin{aligned}
IVS(P,\theta,K) &= \frac{d\sigma_{IV}}{d\frac{K}{X_t}} \\
&= -X_t \frac{\frac{\partial U}{\partial K}}{\frac{\partial U}{\partial \sigma_{IV}}} \\
&= -X_t \frac{\theta\frac{\partial P(\sigma_S)}{\partial K} + (1-\theta)\frac{\partial P(\sigma_R)}{\partial K} - \frac{\partial P(\sigma_{IV})}{\partial K}}{\theta\frac{\partial P(\sigma_S)}{\partial \sigma_{IV}} + (1-\theta)\frac{\partial P(\sigma_R)}{\partial \sigma_{IV}} - \frac{\partial P(\sigma_{IV})}{\partial \sigma_{IV}}} \\
&= X_t \frac{\theta\frac{\partial P(\sigma_S)}{\partial K} + (1-\theta)\frac{\partial P(\sigma_R)}{\partial K} - \frac{\partial P(\sigma_{IV})}{\partial K}}{\frac{\partial P(\sigma_{IV})}{\partial \sigma_{IV}}}
\end{aligned}$$

Then use the results $\frac{\partial P(\sigma)}{\partial K} = e^{-rT}\Phi[-d_-]$ and $\frac{\partial P(\sigma)}{\partial \sigma} = X_t\phi[d_+]\sqrt{T}$, we get:

$$\begin{aligned}
IVS = X_t &\frac{\theta\frac{\partial P(\sigma_S)}{\partial K} + (1-\theta)\frac{\partial P(\sigma_R)}{\partial K} - \frac{\partial P(\sigma_{IV})}{\partial K}}{\frac{\partial P(\sigma_{IV})}{\partial \sigma_{IV}}} \\
= &\frac{\theta e^{-rT}\Phi\big[-d_-(\sigma_S)\big] + (1-\theta)e^{-rT}\Phi\big[-d_-(\sigma_R)\big] - e^{-rT}\Phi\big[-d_-(\sigma_{IV})\big]}{\phi\big[d_+(\sigma_{IV})\big]\sqrt{T}} \\
= &\frac{\theta e^{-rT}\Big(1-\Phi\big[d_-(\sigma_S)\big]\Big) + (1-\theta)e^{-rT}\Big(1-\Phi\big[d_-(\sigma_R)\big]\Big) - e^{-rT}\Big(1-\Phi\big[d_-(\sigma_{IV})\big]\Big)}{\phi\big[d_+(\sigma_{IV})\big]\sqrt{T}} \\
= &\frac{e^{-rT}}{\sqrt{T}} \times \frac{-\theta\Big(\Phi\big[d_-(\sigma_S)\big] - \Phi\big[d_-(\sigma_R)\big]\Big) + \Big(\Phi\big[d_-(\sigma_{IV})\big] - \Phi\big[d_-(\sigma_R)\big]\Big)}{\phi\big[d_+(\sigma_{IV})\big]}
\end{aligned}$$

Proof of Lemma 1:

The sign of implied volatility slope can be determined by examining the numerator and the denominator of Equation (2). The denominator is positive. We drop the positive constant e^{-rT} in the numerator and denote the rest of the numerator as Υ. The sign of implied volatility slope only depends on Υ.

$$\Upsilon = -\theta\Big(\Phi\big[d_-(\sigma_S)\big] - \Phi\big[d_-(\sigma_R)\big]\Big) + \Big(\Phi\big[d_-(\sigma_{IV})\big] - \Phi\big[d_-(\sigma_R)\big]\Big)$$

When $\theta = 0$, $\sigma_{IV} = \sigma_R$. When $\theta = 1$, $\sigma_{IV} = \sigma_S$. Hence, Υ will be zero at the end points of θ.

Taking the derivative of Υ on θ:

$$\begin{aligned}
\frac{d\,\Upsilon}{d\theta} &= -\Big(\Phi\big[d_-(\sigma_S)\big] - \Phi\big[d_-(\sigma_R)\big]\Big) + \frac{d\Phi\big[d_-(\sigma_{IV})\big]}{d\theta} \\
&= -\Big(\Phi\big[d_-(\sigma_S)\big] - \Phi\big[d_-(\sigma_R)\big]\Big) + \phi\big[d_-(\sigma_{IV})\big]\frac{d\,d_-(\sigma_{IV})}{d\sigma_{IV}}\frac{d\sigma_{IV}}{d\theta} \\
&= -\Big(\Phi\big[d_-(\sigma_S)\big] - \Phi\big[d_-(\sigma_R)\big]\Big) + \phi\big[d_-(\sigma_{IV})\big]\Big(\frac{ln\frac{X_t\,e^{rT}}{K}}{\sqrt{T}\sigma_{IV}^2} + \frac{\sqrt{T}}{2}\Big)\frac{P(\sigma_R) - P(\sigma_S)}{X_t\sqrt{T}\phi\big[d_+(\sigma_{IV})\big]} \\
&= -\Big(\Phi\big[d_-(\sigma_S)\big] - \Phi\big[d_-(\sigma_R)\big]\Big) + \frac{e^{rT}}{K}\Big(\frac{ln\frac{X_t\,e^{rT}}{K}}{\sigma_{IV}^2 T} + \frac{1}{2}\Big)\big(P(\sigma_R) - P(\sigma_S)\big)
\end{aligned} \tag{A.1}$$

Taking second derivative of Υ on θ:

$$\begin{aligned}
\frac{d^2\,\Upsilon}{d\theta^2} &= \frac{d\,\frac{d\,\Upsilon}{d\theta}}{d\theta} \\
&= \frac{d\,\frac{d\,\Upsilon}{d\theta}}{d\sigma_{IV}}\frac{d\sigma_{IV}}{d\theta} \\
&= \frac{d\left(\frac{e^{rT}}{K}\big(\frac{ln\frac{X_t\,e^{rT}}{K}}{\sigma_{IV}^2 T} + \frac{1}{2}\big)\big(P(\sigma_R) - P(\sigma_S)\big)\right)}{d\sigma_{IV}} \times \frac{P(\sigma_S) - P(\sigma_R)}{X_t\,\sqrt{T}\phi\big[d_+(\sigma_{IV})\big]} \\
&= \frac{e^{rT}}{K}\Big(\frac{ln\frac{X_t\,e^{rT}}{K}}{\sigma_{IV}^3 T}\Big)\frac{\big(P(\sigma_S) - P(\sigma_R)\big)^2}{X_t\,\sqrt{T}\phi\big[d_+(\sigma_{IV})\big]}
\end{aligned}$$

The sign of second derivative depends only on the relative value of $X_t\,e^{rT}$ and K. For OTM put options (ITM call options), $K < X_t\,e^{rT}$, the second derivative will be positive and Υ will always be convex when $\theta \in (0, 1)$. Given that the value of Υ at the end points ($\theta = 0\,or\,\theta = 1$) is zero, Υ will always be negative in between. Therefore, $IVS < 0$ on the OTM side. Similarly, for ATM put options (ATM call options), $IVS = 0$; and for ITM put options (OTM call options) $IVS > 0$.

We note that Sircar et al. (1999) provide similar results without explicitly deriving implied volatility slope formula.

Proof of Proposition 2:

Implied volatility slope has the following expression:

$$IVS = \frac{e^{-rT}}{\sqrt{T}} \times \frac{-\theta\Big(\Phi\big[d_-(\sigma_S)\big] - \Phi\big[d_-(\sigma_R)\big]\Big) + \Big(\Phi\big[d_-(\sigma_{IV})\big] - \Phi\big[d_-(\sigma_R)\big]\Big)}{\phi\big[d_+(\sigma_{IV})\big]}$$

Drop the constant $\sqrt{T}$, the rest of the denominator is:

$$\begin{aligned}
\frac{d\,\phi\big[d_+(\sigma_{IV})\big]}{d\,\theta} &= \phi\big[d_+(\sigma_{IV})\big] \times (-d_+) \times \frac{d\,d_+}{d\,\theta} \\
&= \phi\big[d_+(\sigma_{IV})\big] \times (-d_+) \times \frac{d\,d_+}{d\,\sigma_{IV}} \times \frac{d\,\sigma_{IV}}{d\,\theta} \\
&= \phi\big[d_+(\sigma_{IV})\big] \times (-d_+) \times \Big(-\frac{ln\frac{X_t\,e^{rT}}{K}}{\sqrt{T}\sigma_{IV}^2} + \frac{\sqrt{T}}{2}\Big) \times \frac{d\,\sigma_{IV}}{d\,\theta} \\
&= \phi\big[d_+(\sigma_{IV})\big] \times (-d_+) \times \Big(-\frac{ln\frac{X_t\,e^{rT}}{K}}{\sqrt{T}\sigma_{IV}^2} + \frac{\sqrt{T}}{2}\Big) \times \frac{P(\sigma_S) - P(\sigma_R)}{X_t\sqrt{T}\phi\big[d_+(\sigma_{IV})\big]} \\
&= (-d_+) \times \Big(-\frac{ln\frac{X_t\,e^{rT}}{K}}{\sqrt{T}\sigma_{IV}^2} + \frac{\sqrt{T}}{2}\Big) \times \frac{P(\sigma_S) - P(\sigma_R)}{X_t\sqrt{T}}
\end{aligned} \tag{A.2}$$

Since $\sigma_S < \sigma_R$, $P(\sigma_S) < P(\sigma_R)$. For OTM put, $d_+ = \frac{ln\frac{X_t\,e^{rT}}{K}}{\sqrt{T}\sigma_{IV}} + \frac{\sigma_{IV}\sqrt{T}}{2}$ is positive. The sign of the denominator is determined by the second term in Equation (A.2). When $\sigma_{IV} < \sqrt{\frac{2ln\frac{X_t\,e^{rT}}{K}}{T}}$, $-\frac{ln\frac{X_t\,e^{rT}}{K}}{\sqrt{T}\sigma_{IV}^2} + \frac{\sqrt{T}}{2} < 0$ and the sign of Equation $\big($A.2$\big)$ is negative.

From the proof of Lemma 1, Υ is convex for OTM put options. Since the values at the end points ($\theta = 0$ or 1) are zero, the first derivative of Υ will increase from a negative number to a positive number. We obtain the threshold value θ^* and its corresponding σ^* by setting the first derivative of Υ (Equation (A.1)) to zero. The implied volatility at the threshold point is:

$$\sigma^* = \sqrt{\frac{ln(X_t\, e^{rT}/K)/T}{\frac{Ke^{-rT}\left(\Phi[d_-(\sigma_S)]-\Phi[d_-(\sigma_R)]\right)}{P(\sigma_R)-P(\sigma_S)} - \frac{1}{2}}}$$

The threshold sentiment level θ^* is:

$$\theta^* = \frac{P(\sigma^*) - P(\sigma_R)}{P(\sigma_S) - P(\sigma_R)}$$

Υ is negative and decreasing on θ when $\theta \in (0, \theta^*)$. Combining results on the denominator and numerator, implied volatility slope of OTM put option is more negative (steeper) when θ increases from 0 to θ^*. The same conclusion applies to ITM call because of the put-call parity.

The proof for OTM call (ITM put) is similar.

Proof of Proposition 3:

Option price is the conditional expectation of future payoff under risk neutral density (denoted by f). From the constraint $O(\sigma_{IV}) = \theta\, O(\sigma_S) + (1-\theta)\, O(\sigma_R)$, risk-neutral density implied by the market option price is a weighted average of two lognormal distributions:

$$\begin{aligned} f_{IV}(X) &= \theta f_S(X) + (1-\theta) f_R(X) \\ &= \theta \frac{\phi[d_-(\sigma_S)]}{X\sigma_S\sqrt{T}} + (1-\theta)\, \frac{\phi[d_-(\sigma_R)]}{X\sigma_R\sqrt{T}} \end{aligned}$$

The parameters for these two lognormals are $\big(lnX_0 + (r - \frac{1}{2}\sigma_S^2)T, \sigma_S^2 T\big)$ and $\big(lnX_0 + (r - \frac{1}{2}\sigma_R^2)T, \sigma_R^2 T\big)$ respectively.

Risk-neutral skewness is:

$$RNS = E\Big[\big(\frac{X - E(X)}{\sigma}\big)^3\Big] = \frac{E\Big[\big(X - E(X)\big)^3\Big]}{(\sigma^2)^{\frac{3}{2}}} = \frac{E\Big[\big(X - E(X)\big)^3\Big]}{\big(E[X^2] - E[X]^2\big)^{\frac{3}{2}}} \tag{A.3}$$

For a lognormal random variable with parameters (μ, σ), its n-th moment is $e^{n\mu + \frac{1}{2}n^2\sigma^2}$.

Using this result, the first moment of X can be calculated as:

$$
\begin{aligned}
E[X] &= \theta \int_0^{\infty} X f_S(X)\,\mathrm{d}X + (1-\theta) \int_0^{\infty} X f_R(X)\,\mathrm{d}X \\
&= \theta\, E_S(X) + (1-\theta)\, E_R(X) \\
&= \theta\, e^{lnX_0 + (r - \frac{1}{2}\sigma_S^2)T + \frac{1}{2}\sigma_S^2 T} + (1-\theta)\, e^{lnX_0 + (r - \frac{1}{2}\sigma_R^2)T + \frac{1}{2}\sigma_R^2 T} \qquad \text{(A.4)} \\
&= \theta\, e^{lnX_0 + rT} + (1-\theta)\, e^{lnX_0 + rT} \\
&= e^{lnX_0 + rT}
\end{aligned}
$$

Similarly, the second moment and the third moment are:

$$
E[X^2] = \theta\, e^{2lnX_0 + 2rT + \sigma_S^2 T} + (1-\theta)\, e^{2lnX_0 + 2rT + \sigma_R^2 T} \qquad \text{(A.5)}
$$

$$
E[X^3] = \theta\, e^{3lnX_0 + 3rT + 3\sigma_S^2 T} + (1-\theta)\, e^{3lnX_0 + 3rT + 3\sigma_R^2 T} \qquad \text{(A.6)}
$$

Substitute Equation (A.4), (A.5) and (A.6) into Equation (A.3):

$$
\begin{aligned}
RNS &= \frac{E\left[\left(X - E(X)\right)^3\right]}{\left(E[X^2] - E[X]^2\right)^{\frac{3}{2}}} \\
&= \frac{\theta\left(e^{3\sigma_S^2 T} - e^{3\sigma_R^2 T} + 3e^{\sigma_R^2 T} - 3e^{\sigma_S^2 T}\right) + e^{3\sigma_R^2 T} - 3e^{\sigma_R^2 T} + 2}{\left(\theta\left(e^{\sigma_S^2 T} - e^{\sigma_R^2 T}\right) + e^{\sigma_R^2 T} - 1\right)^{\frac{3}{2}}}
\end{aligned}
$$

Appendix B Table and Figures

Table 1: Implied volatility slope measures

This table lists several recent empirical papers and their measures of implied volatility slope.

Author (Year)	**Implied Volatility Slope Measure**
Chen, Lung, and Wang (2009)	(OTM put-ATM put)/VIX
Xing, Zhang, and Zhao (2010)	OTM put-ATM call
Yan (2011)	ATM put-ATM call
Jin, Livnat, and Zhang (2012)	IV spread for matched call and put; OTM put-ATM call
Lemmon and Ni (2014)	OTM call-OTM put

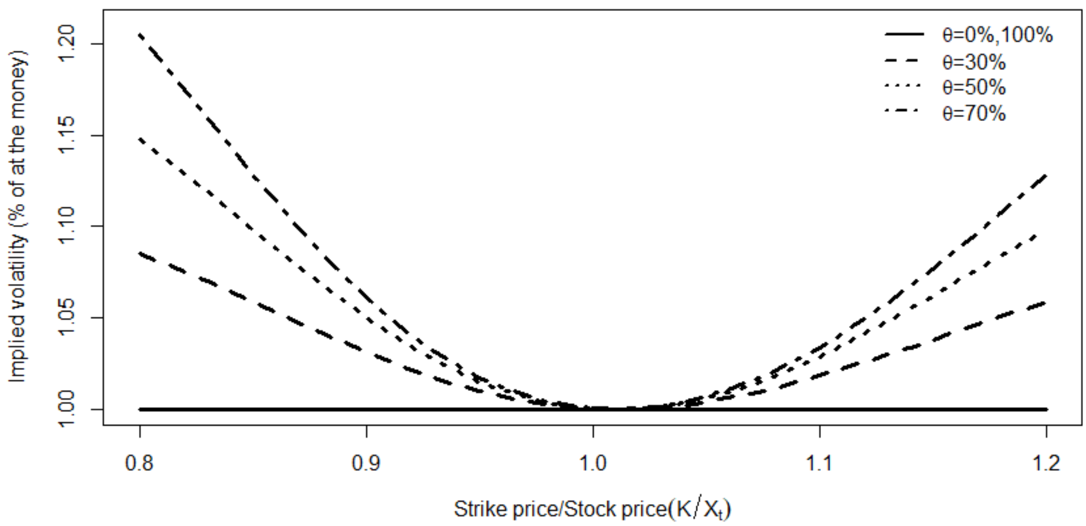


Figure 1: Implied volatility curves

This figure illustrates the implied volatility curves under different levels of θ ($\theta = 0\%, 30\%, 50\%, 70\%, 100\%$). Other parameters are $r = 6\%$, $T = 60/360$, $\sigma_S = 20\%$ and $\sigma_R = 40\%$.

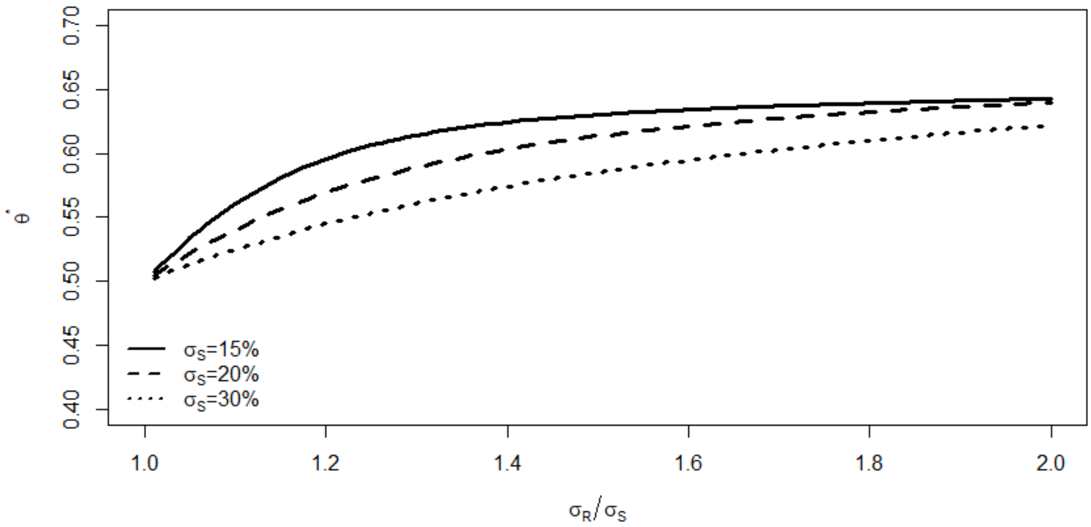


Figure 2: Threshold sentiment level θ^*

This figure illustrates the relation between the threshold sentiment level θ^* and belief gap σ_R/σ_S.

Other parameters are $r = 6\%$ and $T = 60/360$.

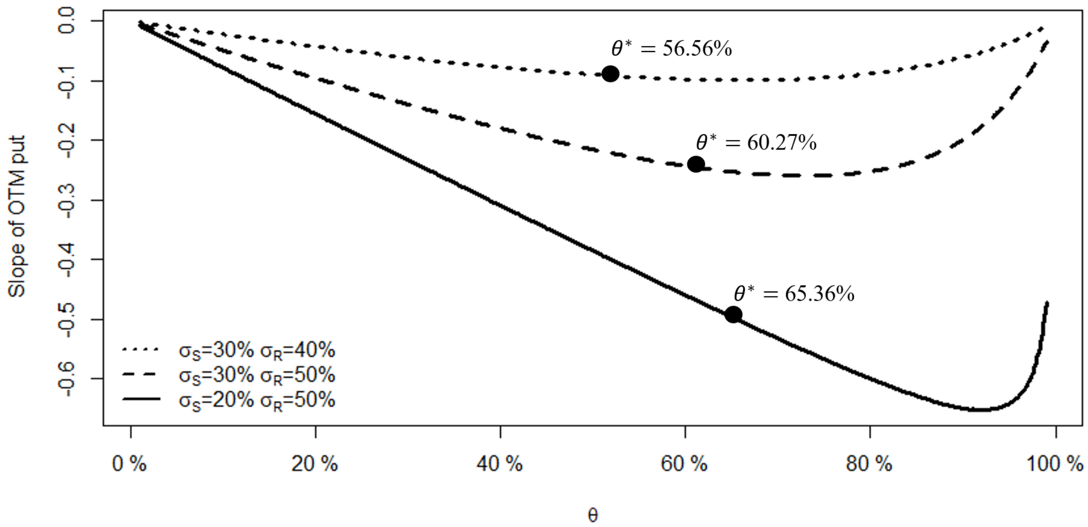
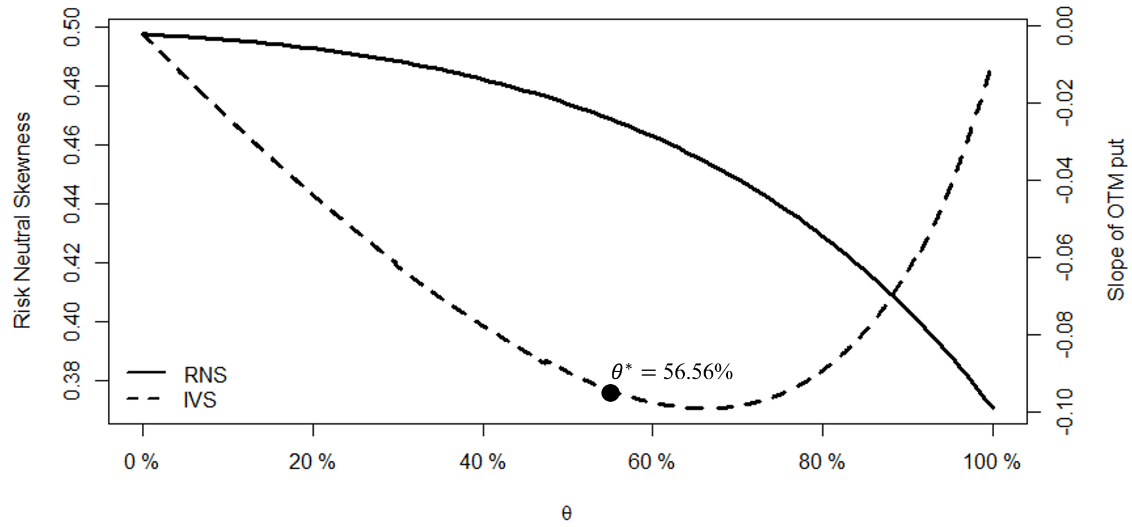
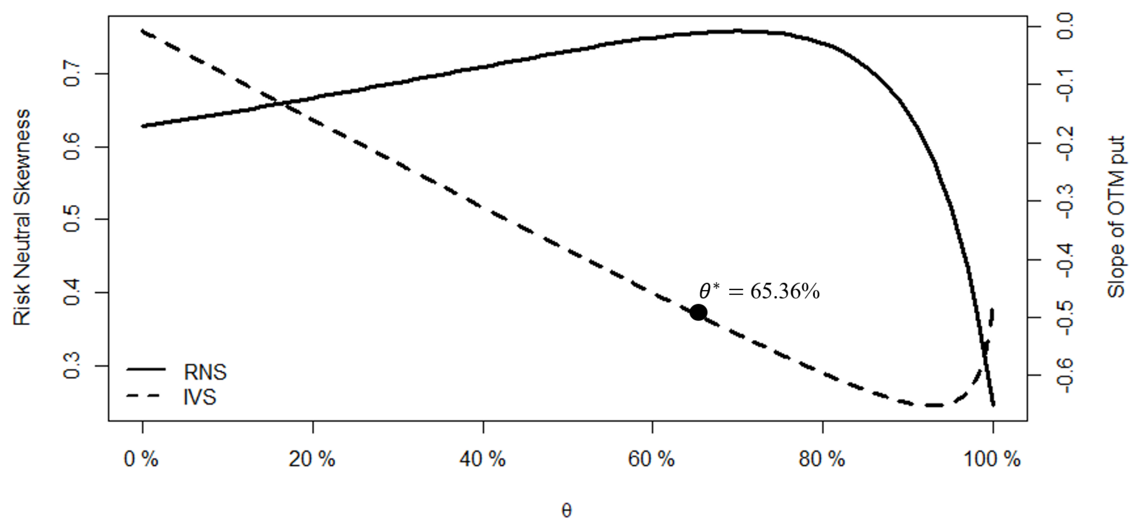


Figure 3: Implied volatility slope and sentiment threshold θ^*

This figure illustrates OTM put ($K = 0.8X_0$) slope under different sentiments level θ and different belief gaps. The belief gap pairs are $\sigma_S = 30\%$ and $\sigma_R = 40\%$, $\sigma_S = 30\%$ and $\sigma_R = 50\%$, $\sigma_S = 20\%$ and $\sigma_R = 50\%$. Other parameters are $r = 6\%$ and $T = 60/360$.



(a) Risk-neutral skewness and implied volatility slope when $\sigma_S = 30\%$ and $\sigma_R = 40\%$



(b) risk-neutral skewness and implied volatility slope when $\sigma_S = 20\%$ and $\sigma_R = 50\%$

Figure 4: Risk-neutral skewness, implied volatility slope, and sentiment

Panel (a) plots the risk-neutral skewness and implied volatility slope ($K = 0.8X_0$) when the belief gap is small ($\sigma_S = 30\%$ and $\sigma_R = 40\%$); Panel (b) plots the risk-neutral skewness and implied volatility slope when the belief gap is large ($\sigma_S = 20\%$ and $\sigma_R = 50\%$). Other parameters are $r = 6\%$ and $T = 60/360$.

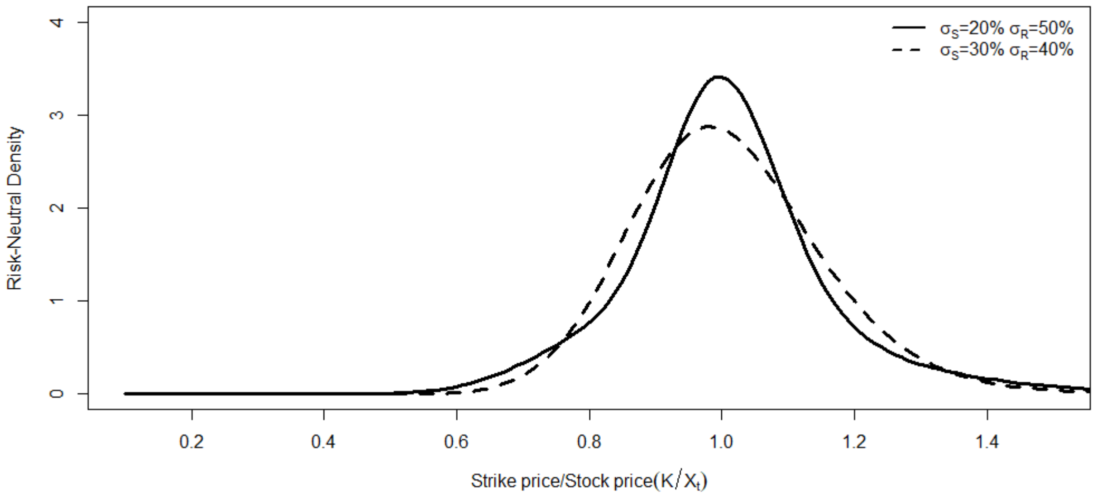


Figure 5: Risk-neutral densities from the mixture model

This figure plots the risk neutral densities of the lognormal mixture when $\sigma_S = 20\%$ and $\sigma_R = 50\%$ and when $\sigma_S = 30\%$ and $\sigma_R = 40\%$. Other parameters are $\theta = 50\%$, $r = 6\%$ and $T = 60/360$.

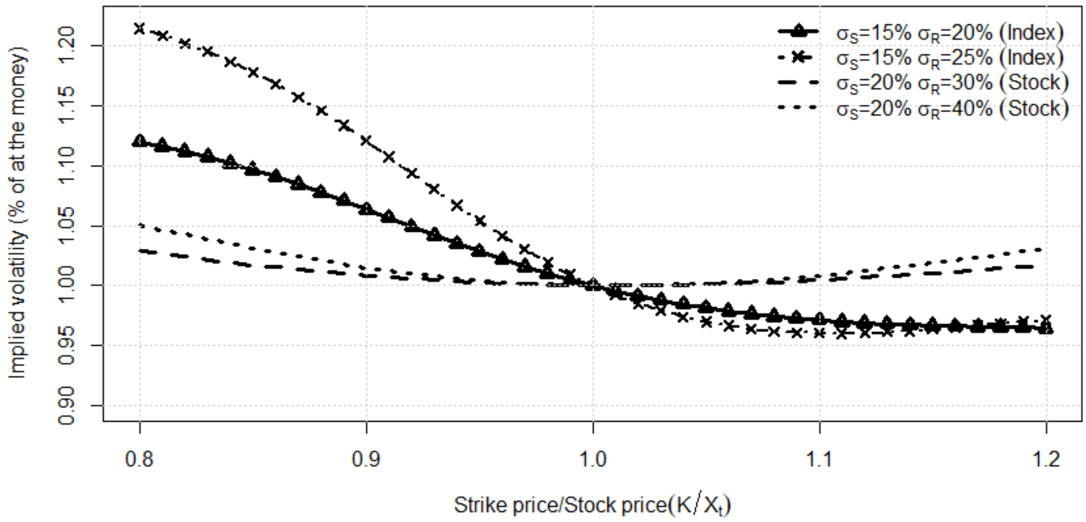


Figure 6: Implied volatility curves of index and stock options

This figure illustrates the implied volatility curves of index options and stock options under different belief gaps. The belief gap pairs for the index options are $\sigma_S = 15\%$ and $\sigma_R = 20\%$, $\sigma_S = 15\%$ and $\sigma_R = 25\%$. The belief gap pairs for the stock options are $\sigma_S = 20\%$ and $\sigma_R = 30\%$, $\sigma_S = 20\%$ and $\sigma_R = 40\%$. θ for index options increases linearly on K from 20% when $K = 0.80X_t$ to 80% when $K = 1.20X_t$. $\theta = 30\%$ for stock options. Other parameters are $r = 6\%$ and $T = 60/360$.

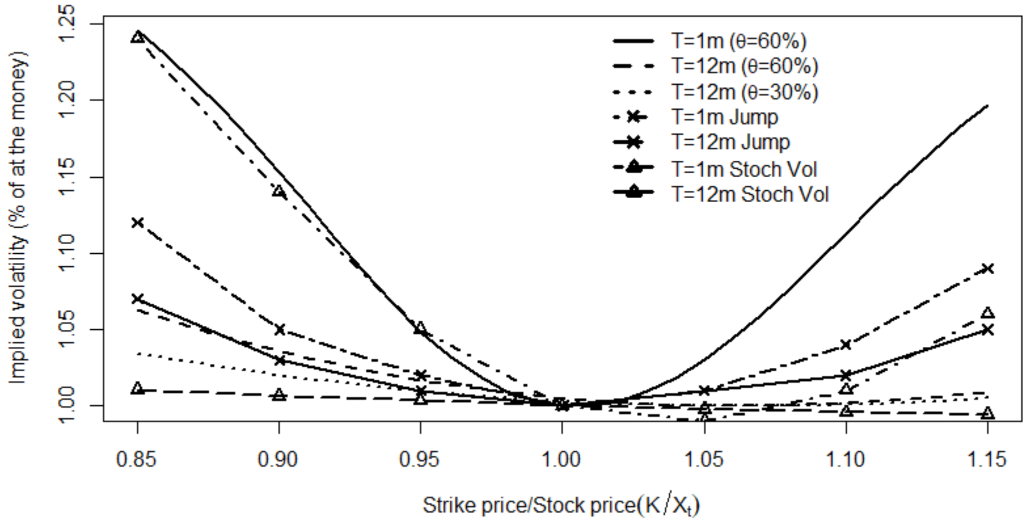


Figure 7: Implied volatility curves under different maturities

This figure illustrates the implied volatility curves under different maturities (1-month and 12-month). Other parameters are $r = 6\%$, $\sigma_S = 15\%$ and $\sigma_R = 30\%$. $\theta = 60\%$ for the 1-month curve, and $\theta = 30\%$ or 60% for the 12-month curve. The data on jump diffusion model and stochastic volatility model is obtained from the study of Das and Sundaram (1999).