MA4J1 Continuum Mechanics Notes

Thomas Hudson

2025-01-01

Table of contents

Preface

These notes are a guide to the main content of the fourth-year Maths module MA4J1 Continuum Mechanics taught at Warwick. They are intended to be essentially self–contained, but are based upon (my interpretation of) the content of chapters drawn from A First Course in Continuum Mechanics by Oscar Gonzalez and Andrew Stuart. This book is a good place to read more about the topics covered and to find various exercises to test your understanding of the content, but it is definitely not the only good book covering many of the topics we consider here. In particular, for those looking for additional reading, various additional books are recommended on the module Moodle page.

Aims and structure

The central aim of the module is to present a mathematical framework within which various continuum models of solids, liquids and gases can be described and derived, and to provide some examples of the Partial Differential Equation (PDE) models which result.

As background, we make use of various concepts from Linear Algebra, Analysis and Vector Calculus. In Chapters @ref(cha:tensor-algebra) and @ref(cha:tensor-calculus), we therefore recap various concepts and discuss some operations on these objects which may be new to you, and at the very least, may take a different perspective. Since some of these concepts will be taken as understood, it will be up to you to ensure that you are comfortable with any concepts which are not covered in detail in lectures. In particular, one of the techniques we use as a baseline throughout the module is converting back and forth between tensor notation and component notation in a particular coordinate system. You should ensure you get comfortable with doing this for yourself.

Chapter @ref(cha:mass-forces) introduces some of the important physical concepts central to this module, including mass density, force and stress fields. In particular, developing an understanding of *stress* is one of the keys to unlocking this module. Chapter @ref(cha:kinematics) introduces the study of kinematics, or the study of motion of continuous bodies. Here, we introduce the concept of *strain*, which is another central focus for the module. In Chapter @ref(cha:balance-laws), we combine these concepts to derive balance laws, which are the Partial Differential Equations which govern a continuum body's motion. As we will see, these balance laws can be formulated in *Eulerian* or *Lagrangian* form. The former formulation is

generally used for the study of fluids, while the latter is generally used for the study of solid materials.

There will doubtless be errors and typos in these notes. If you spot anything, please feel free to let me know via email.

Dr Thomas Hudson, Janury 2025