Exercise Chapter 1

Tobias Hugentobler

09.03.2023

Contents

Exercise 1

In this exercise, we consider 95%-confidence intervals for the true mean of a uniform distribution.

a.

Generate a sample of 30 observations from the standard uniform distribution and calculate a Student confidence interval for the true mean μ. Interpret it.

```
set.seed(145)

nObs = 30
alpha = 0.95
degreesOfFreedom = nObs-1

x = runif(nObs)
mu_hat <- mean(x)
se_hat <- sd(x) / sqrt(nObs)


conf_int = c(mu_hat - -1:1*qt(alpha/2, degreesOfFreedom))
names(conf_int) = c("lci ", "estimate", "uci")
conf_int
```

```
##       lci   estimate       uci
## 0.5091013 0.5723531 0.6356049
```

With confidence 0.95 we can say that the true mean is higher than 0.5091013 and lower than 0.6356049

b.

Calculate the Bootstrap estimate of the standard error and compare it with the usual estimate of the standard error. Plot a histogram of the Bootstrap replications.

c.

Use the `plot_stability()` function of the lecture notes to figure out after how many Bootstrap samples the Bootstrap estimate of the standard error would stabilize.

d.

Calculate a standard normal Bootstrap CI and a percentile Bootstrap CI for μ. Compare with the interval from 1a.

Exercise 2

Consider the two samples $y_1 = 1, 2, \ldots, 21$ and $y_2 = 1, 2, \ldots, 51$.

a.

Resample within groups to calculate a percentile Bootstrap CI for the true median difference $\theta = \text{Med}(y_2) - \text{Med}(y_1)$. Interpret the result.

b.

Calculate a standard normal Bootstrap CI for θ. Compare the two solutions.

Exercise 3

For the situation in Exercise 1, use simulation to estimate real coverage probabilities of the Student CI and the two types of Bootstrap CIs. What do you observe?

Exercise 4

Here, we study a test on Spearman's rank correlation. a. What is Spearman's rank correlation? b. Write a function `spearman_test2(x, y, B = 10000)` that calculates a one-sided permutation p value for the null hypothesis of no positive monotonic association. I.e., you want to show the alternative hypothesis that the true rank correlation is positive. c. Use a simulated example to compare with the corresponding p values from the "coin" package, and also using `stats::cor.test(x, y, method = "s", method = "greater")`.

Exercise 5

In the situation of Exercise 4: Use simulation to compare your approach with `stats::cor.test()` regarding… a. … Type 1 error? (Work with independent normal random variables) b. … power? (Work with dependent normal random variables). c. How do you interpret your result?