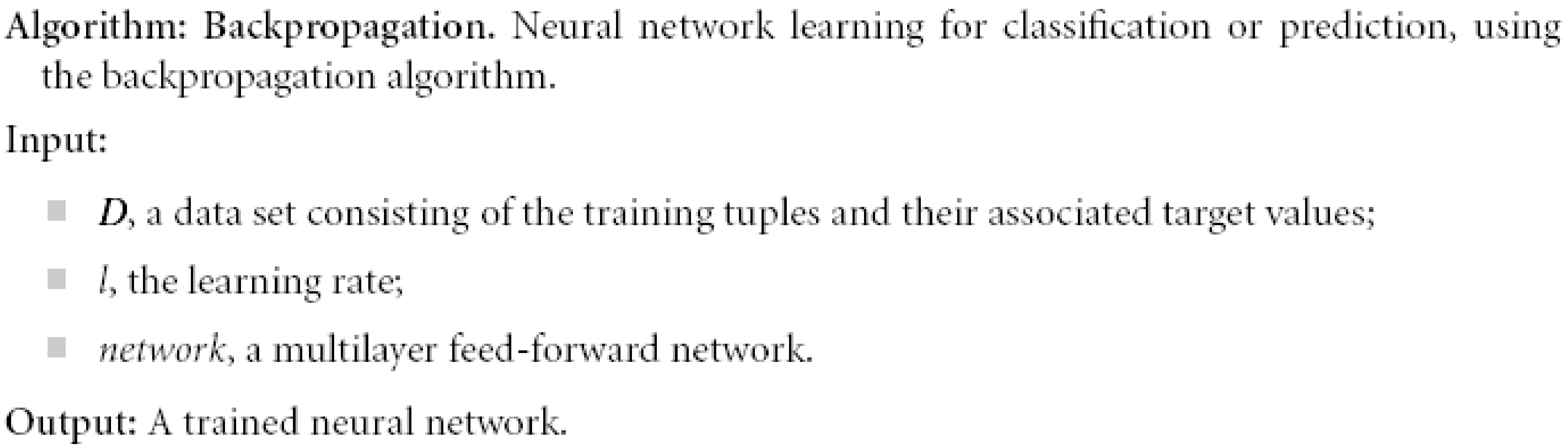
+ Algorithm with Backpropagation



Algorithm: Backpropagation. Neural network learning for classification or prediction, using the backpropagation algorithm.

Input:

- *D*, a data set consisting of the training tuples and their associated target values;
- *l*, the learning rate;
- *network*, a multilayer feed-forward network.

Output: A trained neural network.

Introduction Tools for Decision Making

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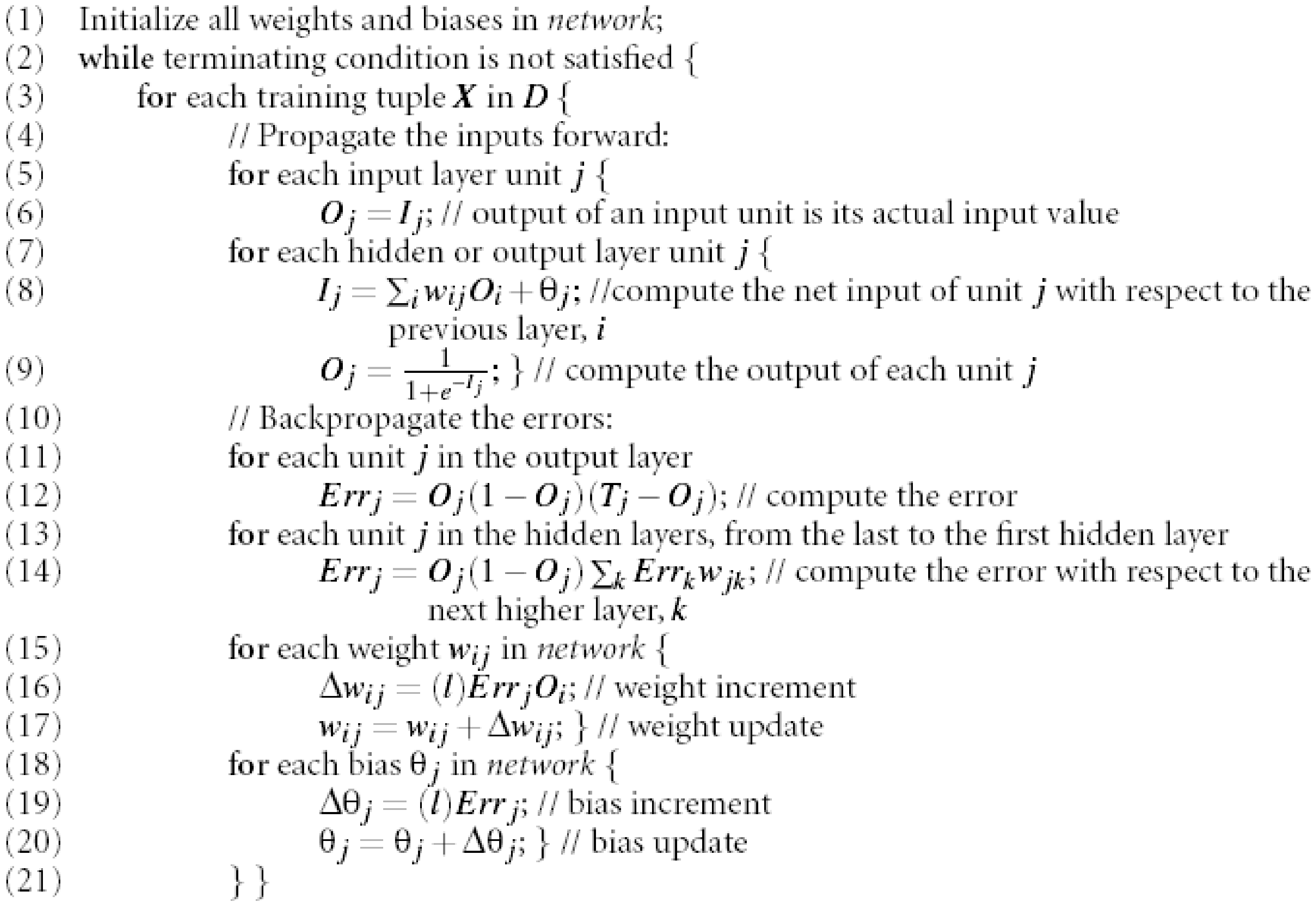

```
Initialize all weights and biases in network;
while terminating condition is not satisfied {
    for each training tuple $X$ in $D$ {
        // Propagate the inputs forward:
        for each input layer unit $j$ {
            $O_j = I_j$; // output of an input unit is its actual input value
        for each hidden or output layer unit $j$ {
            $I_j = \sum_i w_{ij} O_i + \theta_j$; //compute the net input of unit $j$ with respect to the
                previous layer, $i$
            $O_j = \frac{1}{1+e^{-I_j}}$; } // compute the output of each unit $j$
        // Backpropagate the errors:
        for each unit $j$ in the output layer
            $Err_j = O_j(1 - O_j)(T_j - O_j)$; // compute the error
        for each unit $j$ in the hidden layers, from the last to the first hidden layer
            $Err_j = O_j(1 - O_j) \sum_k Err_k w_{jk}$; // compute the error with respect to the
                next higher layer, $k$
        for each weight $w_{ij}$ in network {
            $\Delta w_{ij} = (l) Err_j O_i$; // weight increment
            $w_{ij} = w_{ij} + \Delta w_{ij}$; } // weight update
        for each bias $\theta_j$ in network {
            $\Delta \theta_j = (l) Err_j$; // bias increment
            $\theta_j = \theta_j + \Delta \theta_j$; } // bias update
        } }
```

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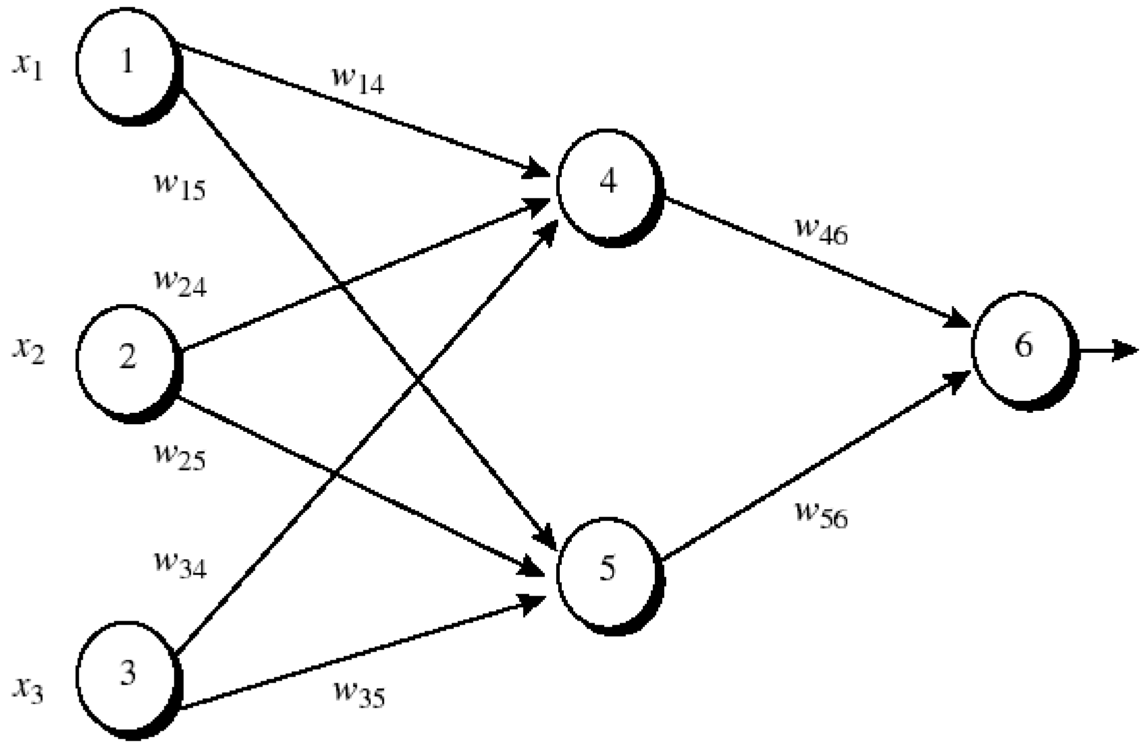
+ Example



Initial input, weight, and bias values.

x_1	x_2	x_3	w_{14}	w_{15}	w_{24}	w_{25}	w_{34}	w_{35}	w_{46}	w_{56}	θ_4	θ_5	θ_6
1	0	1	0.2	−0.3	0.4	0.1	−0.5	0.2	−0.3	−0.2	−0.4	0.2	0.1

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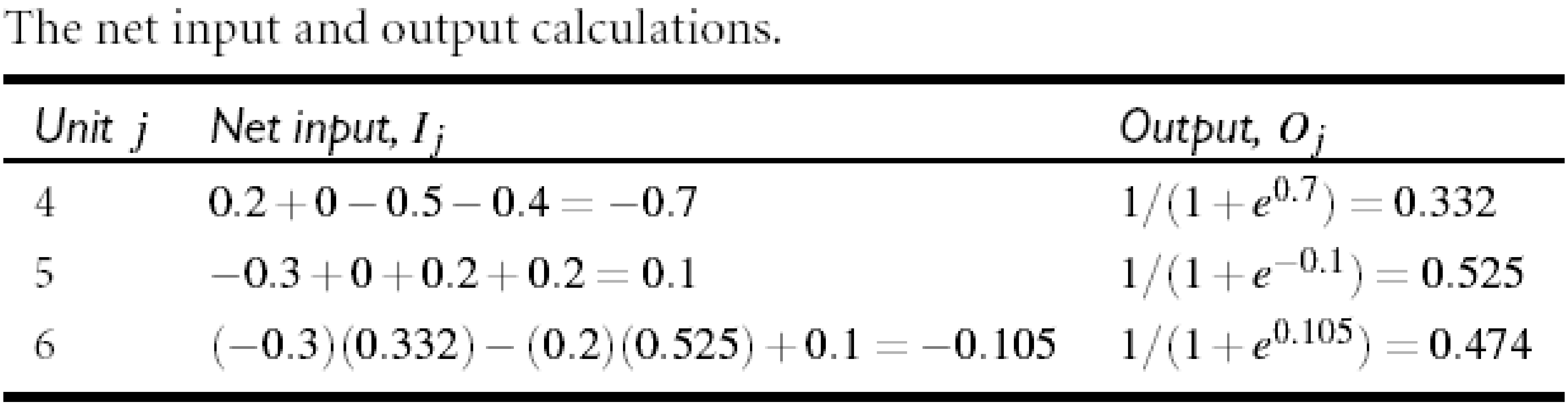
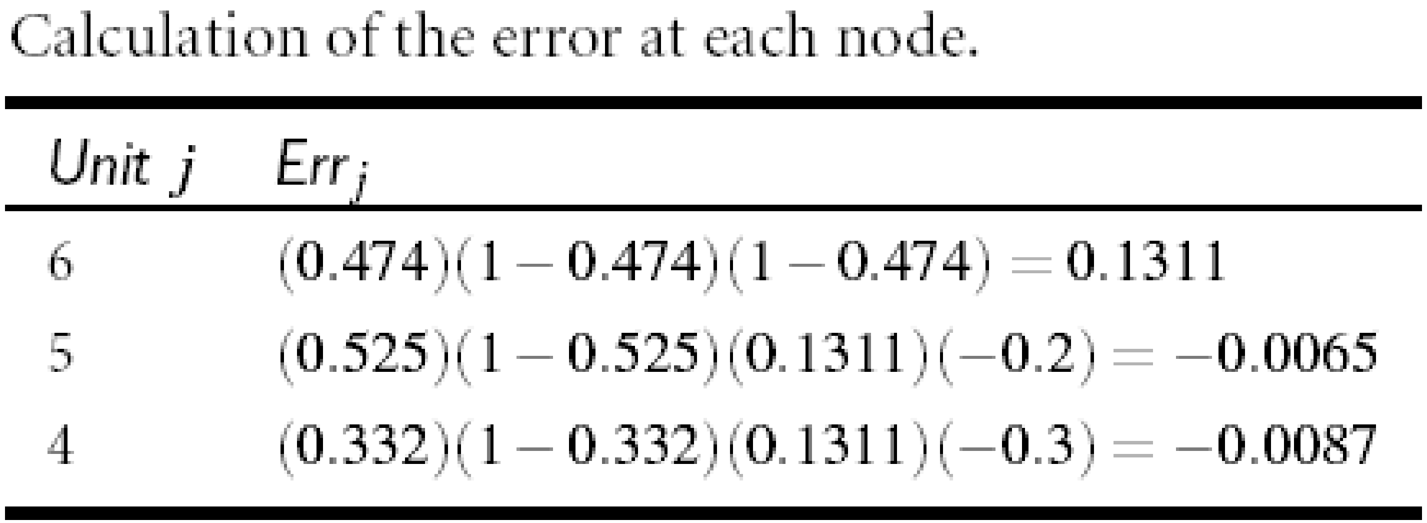


The net input and output calculations.

Unit j	Net input, I_j	Output, O_j
4	$0.2 + 0 - 0.5 - 0.4 = -0.7$	$1/(1 + e^{0.7}) = 0.332$
5	$-0.3 + 0 + 0.2 + 0.2 = 0.1$	$1/(1 + e^{-0.1}) = 0.525$
6	$(-0.3)(0.332) - (0.2)(0.525) + 0.1 = -0.105$	$1/(1 + e^{0.105}) = 0.474$



Calculation of the error at each node.

Unit j	Err_j
6	$(0.474)(1 - 0.474)(1 - 0.474) = 0.1311$
5	$(0.525)(1 - 0.525)(0.1311)(-0.2) = -0.0065$
4	$(0.332)(1 - 0.332)(0.1311)(-0.3) = -0.0087$

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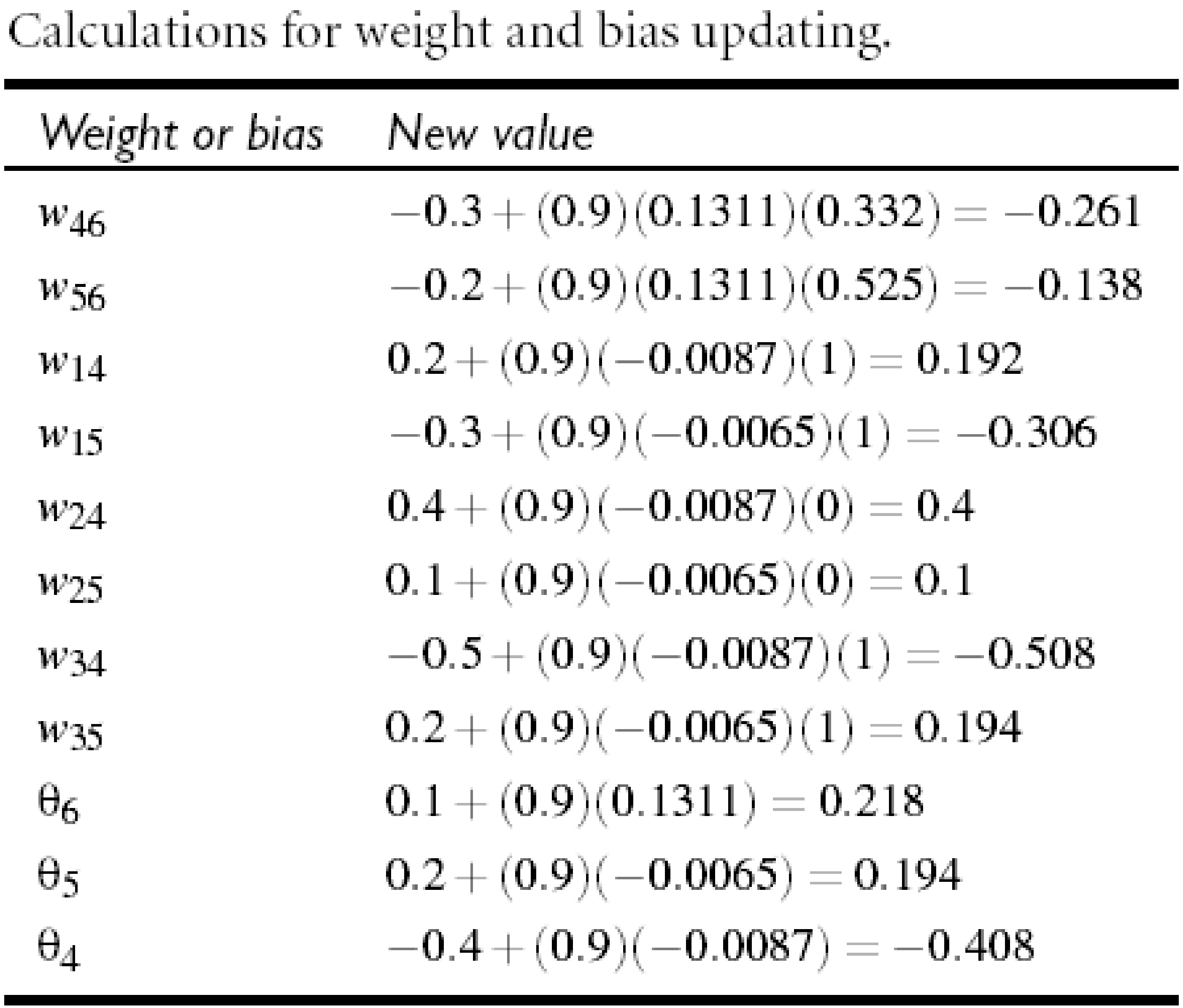

Calculations for weight and bias updating.

Weight or bias	*New value*
w_{46}	$-0.3+(0.9)(0.1311)(0.332)=-0.261$
w_{56}	$-0.2+(0.9)(0.1311)(0.525)=-0.138$
w_{14}	$0.2+(0.9)(-0.0087)(1)=0.192$
w_{15}	$-0.3+(0.9)(-0.0065)(1)=-0.306$
w_{24}	$0.4+(0.9)(-0.0087)(0)=0.4$
w_{25}	$0.1+(0.9)(-0.0065)(0)=0.1$
w_{34}	$-0.5+(0.9)(-0.0087)(1)=-0.508$
w_{35}	$0.2+(0.9)(-0.0065)(1)=0.194$
θ_6	$0.1+(0.9)(0.1311)=0.218$
θ_5	$0.2+(0.9)(-0.0065)=0.194$
θ_4	$-0.4+(0.9)(-0.0087)=-0.408$