Machine learning

Huong Bui

November 17, 2021

1 Ex1

$$\mathcal{L}(w,b,a) = \frac{1}{2}\|w\|^2 - \sum_{n=1}^{N} a_n(t_n(w^T\phi(x_n)+b)-1) \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial w} = w - \sum_{n=1}^{N} a_n t_n \phi(x_n) = 0$$

$$\rightarrow w = \sum_{n=1}^{N} a_n t_n \phi(x_n) \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial b} = \sum_{n=1}^{N} a_n t_n \phi(x_n) = 0 \quad (3)$$

From (2) and (3) :

$$\mathcal{L}(w,b,a) = \frac{1}{2}\|w\|^2 - \sum_{n=1}^{N}(a_n t_n w^T\phi(x_n) + a_n t_n b - a_n)$$

$$= \frac{1}{2}\|w\|^2 - \sum_{n=1}^{N} a_n t_n w^T\phi(x_n) - \sum_{n=1}^{N} a_n t_n b + \sum_{n=1}^{N} a_n$$

$$= \frac{1}{2}\|w\|^2 - w^T\sum_{n=1}^{N} a_n t_n \phi(x_n) - b\sum_{n=1}^{N} a_n t_n + \sum_{n=1}^{N} a_n$$

$$= \frac{1}{2}\|w\|^2 - \|w\|^2 + \sum_{n=1}^{N} a_n$$

$$= \frac{-1}{2}\|w\|^2 + \sum_{n=1}^{N} a_n$$

$$= \frac{-1}{2}\sum_{n=1}^{N} a_n t_n \phi(x_n)^T \sum_{m=1}^{N} a_m t_m \phi(x_m) + \sum_{n=1}^{N} a_n$$

$$= \frac{-1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N} a_n a_m t_n t_m \phi(x_n)^T\phi(x_m) + \sum_{n=1}^{N} a_n$$

2 Ex2