homework 1

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September 2021

1. To evaluate a new test for detecting Hansen's disease, a group of people 5% of which are known to have Hansen's disease are tested. The test finds Hansen's disease among 98% of those with the disease and 3% of those who don't. What is the probability that someone testing positive for Hansen's disease under this new test actually has it?

Let: A: people are known to have Hansen's disease
B: people are known to have no Hansen's disease
H: people Hansen's disease
P(A) = 5% = 0.05
P(B) = 0.95
P(H | $A) = 0.98$
$P(H \mid B) = 0.03$
$Probability that someone testing positive for Hansen's disease under this new test is:$

$P(A \mid H) = \frac{P(A).P(H|A)}{P(H)} = \frac{P(A).P(H|A)}{P(A).P(H|A)+P(B).P(H|B)} = \frac{0.05x0.98}{0.05x0.98+0.95x0.03}$
$P(A \mid H) = 0.6323$

2. Proof the following distributions are normalized then calculate the mean and standard deviation of these distribution:
a) Univariate normal distribution.
b)(Optional) Multivariate normal distribution.

(a) Probability normal distribution when:

$$P(X) = \frac{1}{\sqrt{2\pi\sigma^2}} . \int_{-\infty}^{\infty} e^{\frac{-(x-\mu)^2}{2\sigma^2}} = 1$$

We have to proof: P(X) = 1
Set: $y = x - \mu \Rightarrow dy = dx$

$$P(X) = \frac{1}{\sqrt{2\pi\sigma^2}}.\int\limits_{-\infty}^{\infty} e^{\frac{-(x-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty} e^{\frac{-y^2}{2}} dy$$

We must show that:

$$\int\limits_{-\infty}^{\infty} e^{\frac{-y^2}{2}} dy = \sqrt{2\pi}$$

Let: I $= \int\limits_{-\infty}^{\infty} e^{\frac{-y^2}{2}} dy. Then$

$$\int\limits_{-\infty}^{\infty} e^{\frac{-y^2}{2}} dy \int\limits_{-\infty}^{\infty} e^{\frac{-x^2}{2}} dx = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} e^{\frac{-(x^2+y^2)}{2}} dydx$$

Let: x = rcos $\theta, y = rsin\theta, dydx = rd\theta dr.\ Because:\ cos^2\theta + sin^2\theta = 1. Then:$

$$I^2 = \int\limits_{0}^{\infty}\int\limits_{0}^{2\pi} e^{\frac{-r^2}{2}} rd\theta dr$$

$$= 2\pi \int\limits_{0}^{\infty} re^{\frac{-r^2}{2}} dr$$

$$= -2\pi e^{\frac{-r^2}{2}} \ |_0^\infty \ = 2\pi \ (e^\infty = 0)$$

$$\Rightarrow Univariate\ normal\ distribution\ normalization.$$