Classification and Prediction

——Classification by Decision Tree——

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Classification and Prediction

- Basic Concepts
- Issues Regarding Classification and Prediction
- Decision Tree
- Bayesian Classification
- Neural Networks
- Support Vector Machine
- K-Nearest Neighbor
- Associative classification
- Classification Accuracy

Training Dataset

- This follows an example from Quinlan's ID3

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

Training Dataset

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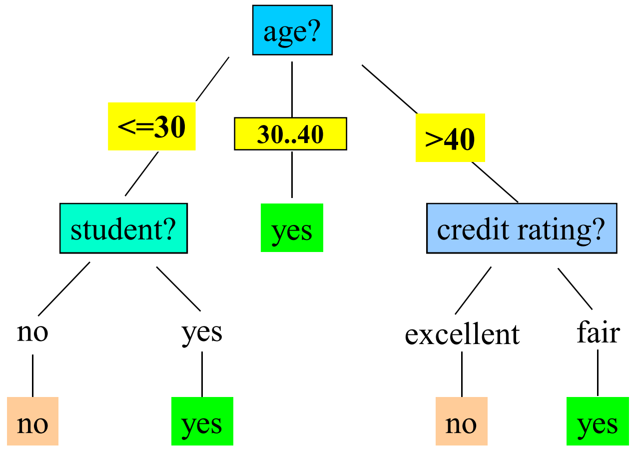
<=30	high	no	excellent	no
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
<=30	medium	yes	excellent	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
>40	medium	yes	fair	yes
>40	medium	no	excellent	no
31…40	high	no	fair	yes
31…40	low	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes

Output: A Decision Tree for "buys_computer"



Algorithm for Decision Tree Induction

- Basic algorithm (a greedy algorithm)
  - Tree is constructed in a top-down recursive divide-and-conquer manner
  - At start, all the training examples are at the root
  - Attributes are categorical (if continuous-valued, they are discretized in advance)
  - Examples are partitioned recursively based on selected attributes
  - Test attributes are selected on the basis of a heuristic or statistical measure (e.g., information gain)
- Conditions for stopping partitioning
  - All samples for a given node belong to the same class
  - There are no remaining attributes for further partitioning – majority voting is employed for classifying the leaf
  - There are no samples left

Attribute Selection Measure: Information Gain (ID3/C4.5)

- Select the attribute with the highest information gain(信息增益)
- S contains s_i tuples of class C_i for i = {1, …, m}
- **information** measures info required to classify any arbitrary tuple

$$I(s_1,s_2,\ldots,s_m) = -\sum_{i=1}^{m} \frac{s_i}{s} \log_2 \frac{s_i}{s}$$

- **Entropy（熵）** of attribute A with values $\{a_1,a_2,\ldots,a_v\}$

$$E(A)=\sum_{j=1}^{v} \frac{s_{1j}+\ldots+s_{mj}}{s} I(s_{1j},\ldots,s_{mj})$$

- **information gained（信息增益）** by branching on attribute A

$$Gain(A) = I(s_1, s_2, \ldots, s_m) - E(A)$$

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Attribute Selection: Information Gain

- Class P: buys_computer = "yes"
- Class N: buys_computer = "no"
- I(p, n) = I(9, 5) =0.940
- Compute the entropy for age:

age	p_i	n_i	$I(p_i, n_i)$
<=30	2	3	0.971
30…40	4	0	0
>40	3	2	0.971

age	income	student	credit_rating	buys_computer
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<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
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>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
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<=30	medium	yes	excellent	yes
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$$E(age) = \frac{5}{14} I(2,3) + \frac{4}{14} I(4,0) + \frac{5}{14} I(3,2) = 0.694$$

$\frac{5}{14} I(2,3)$ **means "age <=30" has 5 out of 14 samples, with 2 yes'es and 3 no's. Hence**

$$Gain(age) = I(p,n) - E(age) = 0.246$$

Similarly,

$$Gain(income) = 0.029$$
$$Gain(student) = 0.151$$
$$Gain(credit\_rating) = 0.048$$

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Computing Information-Gain for Continuous-Value Attributes

- **Let attribute A be a continuous-valued attribute**
- **Must determine the *best split point* for A**
  - **Sort the value A in increasing order**
  - **Typically, the midpoint between each pair of adjacent values is considered as a possible *split point***
    - **$(a_i+a_{i+1})/2$ is the midpoint between the values of a_i and a_{i+1}**
  - **The point with the *minimum expected information requirement* for A is selected as the split-point for A**
- **Split:**
  - **D1 is the set of tuples in D satisfying A ≤ split-point, and D2 is the set of tuples in D satisfying A > split-point**

Gain Ratio for Attribute Selection (C4.5)

- **Information gain measure is biased towards attributes with a large number of values**
- **C4.5 (a successor of ID3) uses gain ratio to overcome the problem (normalization to information gain)**

$$SplitInfo_A(D) = -\sum_{j=1}^{v} \frac{|D_j|}{|D|} \times \log_2\left(\frac{|D_j|}{|D|}\right)$$

  - **GainRatio(A) = Gain(A)/SplitInfo(A)**
- **Ex.** $SplitInfo_A(D) = -\frac{4}{14} \times \log_2\left(\frac{4}{14}\right) - \frac{6}{14} \times \log_2\left(\frac{6}{14}\right) - \frac{4}{14} \times \log_2\left(\frac{4}{14}\right) = 0.926$
  - **gain_ratio(income) = 0.029/0.926 = 0.031**
- **The attribute with the maximum gain ratio is selected as the splitting attribute**

Gini index (CART, IBM IntelligentMiner)

- **If a data set *D* contains examples from *n* classes, gini index, *gini*(*D*) is defined as**

$$gini(D) = 1 - \sum_{j=1}^{n} p_j^2$$

where p_j is the relative frequency of class *j* in *D*

- **If a data set *D* is split on A into two subsets D_1 and D_2, the *gini* index *gini*(*D*) is defined as**

$$gini_A(D) = \frac{|D_1|}{|D|} gini(D_1) + \frac{|D_2|}{|D|} gini(D_2)$$

- **Reduction in Impurity:**

$$\Delta gini(A) = gini(D) - gini_A(D)$$

- **The attribute provides the smallest $gini_{split}(D)$ (or the largest reduction in impurity) is chosen to split the node (*need to enumerate all the possible splitting points for each attribute*)**

Gini index (CART, IBM Intelligent Miner)

- **Ex. D has 9 tuples in buys_computer = "yes" and 5 in "no"**

$$gini(D) = 1 - \left(\frac{9}{14}\right)^2 - \left(\frac{5}{14}\right)^2 = 0.459$$

- **Suppose the attribute income partitions D into 10 in D_1: {low, medium} and 4 in D_2**

$$gini_{income \in \{low, medium\}}(D) = \left(\frac{10}{14}\right) Gini(D_1) + \left(\frac{4}{14}\right) Gini(D_2)$$

but $gini_{\{medium,high\}}$ is 0.30 and thus the best since it is the lowest

- **All attributes are assumed continuous-valued**
- **May need other tools, e.g., clustering, to get the possible split values**
- **Can be modified for categorical attributes**

Comparing Attribute Selection Measures

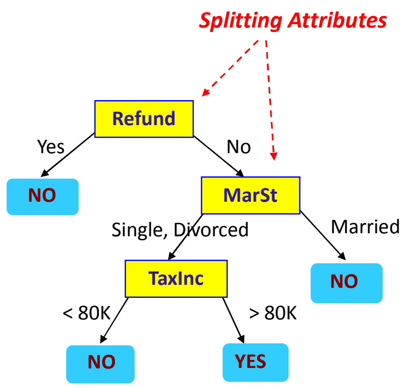
- The three measures, in general, return good results but
  - Information gain:
    - biased towards multivalued attributes
  - Gain ratio:
    - tends to prefer unbalanced splits in which one partition is much smaller than the others
  - Gini index:
    - biased to multivalued attributes
    - has difficulty when the number of classes is large
    - tends to favor tests that result in equal-sized partitions and purity in both partitions

Review the Decision Tree Task

categorical categorical continuous class

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Training Data



Model: Decision Tree

Review the Decision Tree Task

categorical categorical continuous class

Tid	Refund	Marital Status	Taxable Income	Cheat
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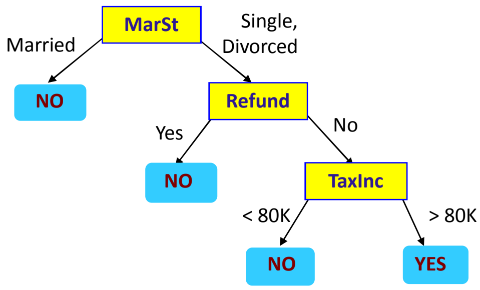

There could be more than one tree that fits the same data!

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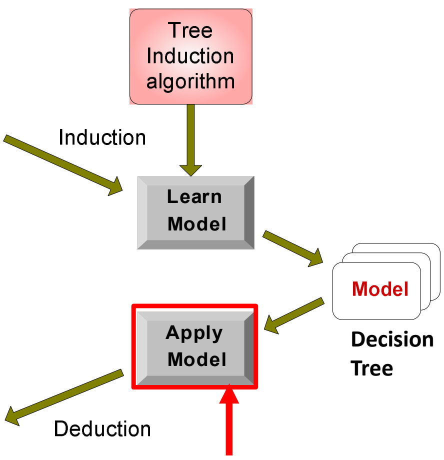
Review the Decision Tree Task

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

Training Set

Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set



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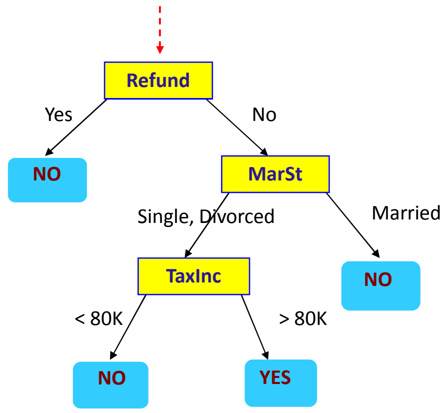
Review the Decision Tree Task

Start from the root of tree.



Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?

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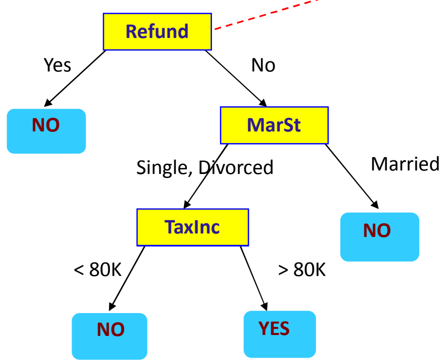
Review the Decision Tree Task

Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



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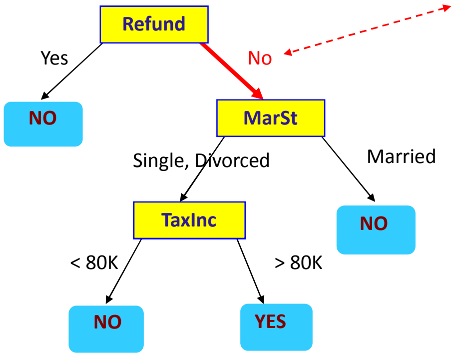
Review the Decision Tree Task

Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



Review the Decision Tree Task

Test Data

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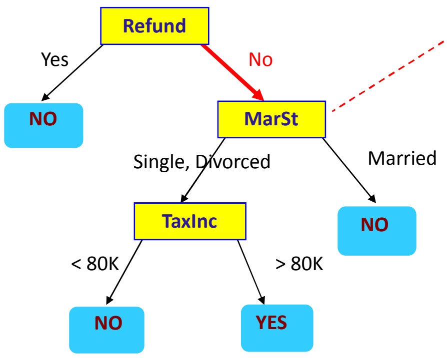

Review the Decision Tree Task

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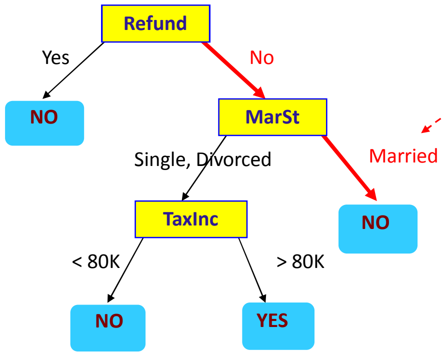

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Review the Decision Tree Task

Test Data

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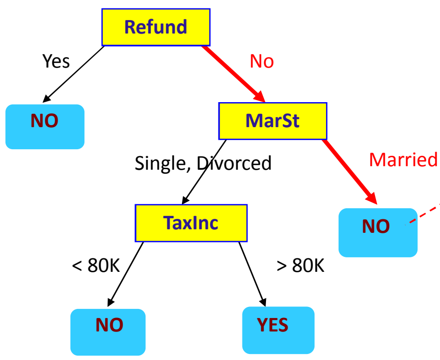
No	Married	80K	?



Assign Cheat to “No”

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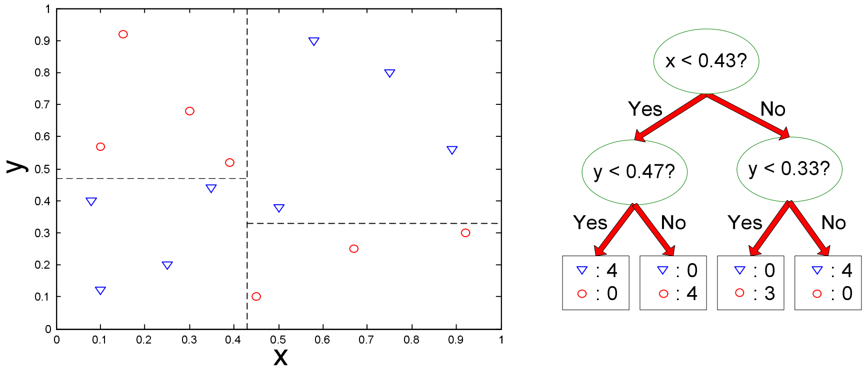
Decision Boundary



- Border line between two neighboring regions of different classes is known as decision boundary
- Decision boundary is parallel to axes because test condition involves a single attribute at-a-time

Other Attribute Selection Measures

- CHAID: a popular decision tree algorithm, measure based on χ^2 test for independence
- C-SEP: performs better than info. gain and gini index in certain cases
- G-statistics: has a close approximation to χ^2 distribution
- MDL (Minimal Description Length) principle (i.e., the simplest solution is preferred):
  - The best tree as the one that requires the fewest number of bits to both (1) encode the tree, and (2) encode the exceptions to the tree
- Multivariate splits (partition based on multiple variable combinations)
  - CART: finds multivariate splits based on a linear comb. of attrs.
- Which attribute selection measure is the best?
  - Most give good results, none is significantly superior than others

Random Forest (Breiman 2001)

- **Random Forest:**
  - Each classifier in the ensemble is a *decision tree* classifier and is generated using a random selection of attributes at each node to determine the split
  - During classification, each tree votes and the most popular class is returned
- **Two Methods to construct Random Forest**
  - Forest-RI (*random input selection*): Randomly select, at each node, F attributes as candidates for the split at the node. The CART methodology is used to grow the trees to maximum size
  - Forest-RC (*random linear combinations*): Creates new attributes (or features) that are a linear combination of the existing attributes (reduces the correlation between individual classifiers)
- **Comparable in accuracy to Adaboost, but more robust to errors and outliers**
- **Insensitive to the number of attributes selected for consideration at each split, and faster than bagging or boosting**

Extracting Classification Rules from Trees

- **Represent the knowledge in the form of IF-THEN rules**
- **One rule is created for each path from the root to a leaf**
- **Each attribute-value pair along a path forms a conjunction**
- **The leaf node holds the class prediction**
- **Rules are easier for humans to understand**
- **Example**

IF *age* = "<=30" AND *student* = "*no*" THEN *buys_computer* = "*no*"
IF *age* = "<=30" AND *student* = "*yes*" THEN *buys_computer* = "*yes*"
IF *age* = "31…40" THEN *buys_computer* = "*yes*"
IF *age* = ">40" AND *credit_rating* = "*excellent*" THEN *buys_computer* = "*yes*"
IF *age* = "<=30" AND *credit_rating* = "*fair*" THEN *buys_computer* = "*no*"

Overfitting and Tree Pruning

- **Overfitting**: An induced tree may overfit the training data
  - Too many branches, some may reflect anomalies due to noise or outliers
  - Poor accuracy for unseen samples
- **Two approaches to avoid overfitting**
  - **Prepruning**: *Halt tree construction early*-do not split a node if this would result in the goodness measure falling below a threshold
    - Difficult to choose an appropriate threshold
  - **Postpruning**: *Remove branches* from a "fully grown" tree—get a sequence of progressively pruned trees
    - Use a set of data different from the training data to decide which is the "best pruned tree"

Enhancements to basic decision tree induction

- **Allow for continuous-valued attributes**
  - **Dynamically define new discrete-valued attributes that partition the continuous attribute value into a discrete set of intervals**
- **Handle missing attribute values**
  - **Assign the most common value of the attribute**
  - **Assign probability to each of the possible values**
- **Attribute construction**
  - **Create new attributes based on existing ones that are sparsely represented**
  - **This reduces fragmentation(碎片), repetition（重复）, and replication（复制）**

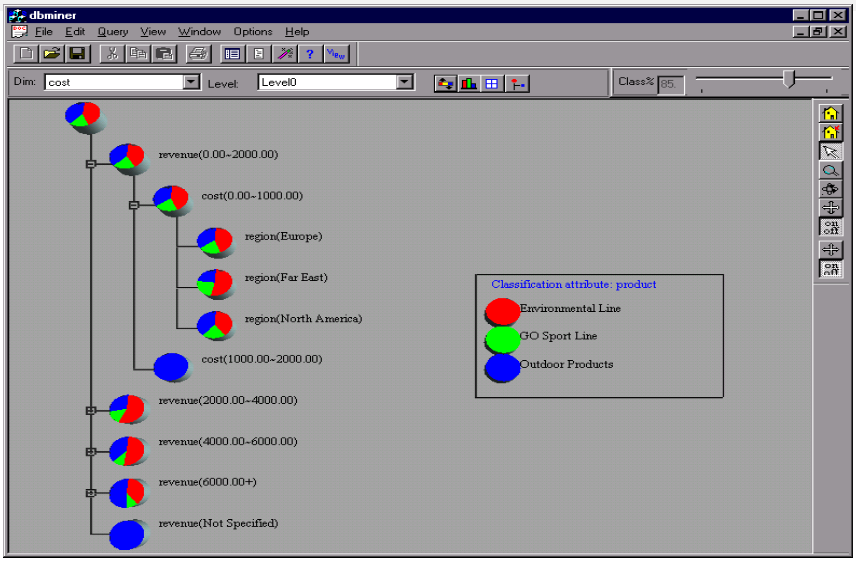
Classification in Large Databases

- Classification—a classical problem extensively studied by statisticians and machine learning researchers
- Scalability: Classifying data sets with millions of examples and hundreds of attributes with reasonable speed
- Why decision tree induction in data mining?
  - relatively faster learning speed (than other classification methods)
  - convertible to simple and easy to understand classification rules
  - can use SQL queries for accessing databases
  - comparable classification accuracy with other methods

Scalable Decision Tree Induction Methods in Data Mining Studies

- SLIQ (EDBT'96 — Mehta et al.)
  - builds an index for each attribute and only class list and the current attribute list reside in memory
- SPRINT (VLDB'96 — J. Shafer et al.)
  - constructs an attribute list data structure
- PUBLIC (VLDB'98 — Rastogi & Shim)
  - integrates tree splitting and tree pruning: stop growing the tree earlier
- RainForest (VLDB'98 — Gehrke, Ramakrishnan & Ganti)
  - separates the scalability aspects from the criteria that determine the quality of the tree
  - builds an AVC-list (attribute, value, class label)

Presentation of Classification Results



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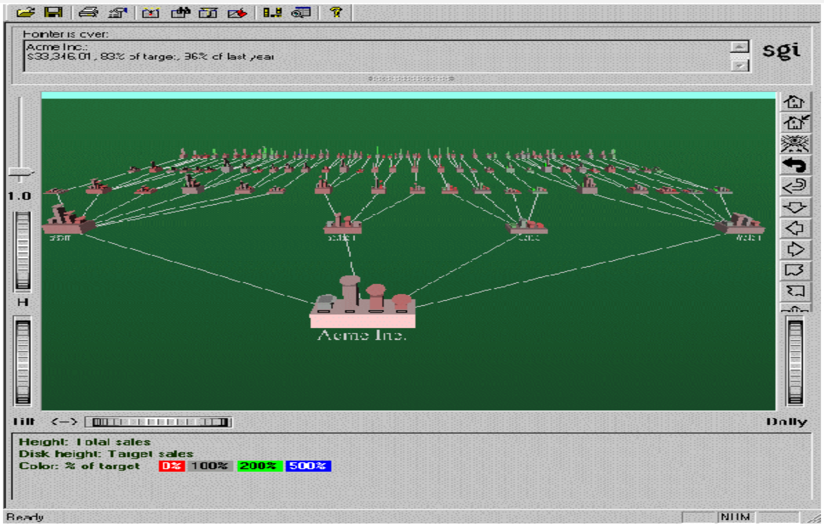
Visualization of a Decision Tree in SGI/MineSet 3.0



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Thanks !