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F2

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1. **Set-valued functions**

   $f : \mathbb{N} \to \mathbb{P}(\mathbb{N})$, $f(n) = \{p \in \mathbb{N} : n|p\}$.

   (a) $f(m) = f(p) \cap f(q)$
   Notice that q can be thought of as a multiple of p. Therefore only the elements of $f(p)$ that have a multiple of a q as the multiple of p are selected. This happens to be defined as an intersection.

   (b) Observe that this is merely an extension of part a with multiple sets. Thus the same logic applys recursively.

   Lets define $T : \mathbb{N} \to \mathbb{P}(\mathbb{N})$ to be
   $T(1) = f(p_1)$
   $T(n) = f(p_n) \cap T(n-1)$

   Therefore $f(m) = T(m)$

   (c) Notice that if $f(p)$ is a subset of $f(q)$ then all the multiples of p are in $f(q)$.
   Consider $p = q * k$ for some $p, q, k \in \mathbb{N}$
   $f(q)$ would then contain multiples of p.
   Therefore $f(p)$ is a subset of $f(q)$ for $p = q * k$

   (d) $m = p * q$ see part a for reasoning.

2. **Set-valued functions and partitions**

(a) $M : V \times \mathbb{N} \to \mathbb{P}(V)$ by $M(x, n) = \{y \in V \mid \text{ there is a path of length } n \text{ from } x \text{ to } y\}$

$M(c, 0)$	$\{c\}$
$M(c, 1)$	$\{a, d, f, k\}$
$M(c, 2)$	$\{e, b, f, h, k\}$
$M(c, 3)$	$\{e, b, g, h\}$
$M(c, 4)$	$\{a, b, d, g, h\}$
$M(c, 5)$	$\{b, d, e, f, h\}$

(b) No, because if a node is reachable using to diferent path lengths, to instances of that node will show up in the set....violating the def of a Partition.

(c) $Q(x, 0) = \{x\}$

For any $n \geq 1$, $Q(x, n) = \{y \in V \mid y \text{ is a neighbor of } p \text{ for some } p \in Q(x, n-1)\}$

They are not equal because $Q(x, n)$ generates a walk instead of a path (Due to not blocking previously visited nodes). Therefore it may revisit a node which $P(x, n)$ clearly cannot.