Calc 3 Review

$\tau = 2\pi$

Part 1

Lines and Surfaces

$$\text{Line: } \vec{r}(t) = \vec{r}_0 + t * \vec{v}$$
$$\text{Line: } \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$
$$\text{Line Segment: } \vec{r}(t) = (\vec{1} - t)\vec{r}_0 + t * \vec{r}_1 \text{ for } 0 \leq t \leq 1$$
$$\text{Plane: } \vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$$
$$\text{Elliptic Paraboloid: } \frac{z}{c} = \frac{x}{a}^2 + \frac{y}{b}^2$$
$$\text{Hyperbolic Paraboloid: } \frac{z}{c} = \frac{x}{a}^2 - \frac{y}{b}^2$$
$$\text{Ellipsoid: } \frac{x}{a}^2 + \frac{y}{b}^2 + \frac{z}{c}^2 = 1$$
$$\text{Cone: } \frac{x}{a}^2 + \frac{y}{b}^2 - \frac{z}{c}^2 = 0$$
$$\text{Hyperboloid of 1 sheet: } \frac{x}{a}^2 + \frac{y}{b}^2 - \frac{z}{c}^2 = 1$$
$$\text{Hyperboloid of 2 sheets: } \frac{x}{a}^2 + \frac{y}{b}^2 - \frac{z}{c}^2 = -1$$

Vectors

$$|\vec{r}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$
$$\vec{a} \cdot \vec{b} = a_1 * b_1 + b_2 * b_3$$
$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| * \cos\theta, 0 \leq \theta \leq \tau$$
$$comp_a\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$$
$$proj_a\vec{b} = comp_a\vec{b} * \frac{\vec{a}}{|\vec{a}|}$$
$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$
$$\text{Distance from point to a line} = comp_{\vec{n}}\vec{b}$$
$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| * \sin\theta$$

Vector Functions

$$\text{Arc Length} = \int_a^b |r(\vec{t})| dt \vec{v} = \vec{r}, \vec{a} = \vec{r}, v = |\vec{v}|$$

$$\vec{T} = \vec{v}/|\vec{v}|$$

$$\vec{B} = \vec{v} \times \vec{a}$$

$$\vec{B} = \vec{T} \times \vec{N}$$

$$\vec{N} = \vec{N} \times \vec{B}$$

$$a_T = comp_T \vec{a} = \vec{a} \cdot \vec{T}$$

$$\vec{w} = \vec{a} a_T * \vec{T}$$

$$\vec{w} = a_N \vec{N}$$

$$a_N = |\vec{w}|$$

$$\vec{N} = \frac{\vec{w}}{a_N}$$

$$\kappa = \frac{|\vec{a} \times \vec{v}|}{|\vec{v}|^3}$$

Part 2 Multivariable Differentiation

$$f_x(x, y) = f_x = \frac{\partial f}{\partial x}$$

$$f_{xy} = f_{yx}$$

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$$

$$\text{Linear Approx:} f(x, y) \approx f(a, b) + dz$$

$$\text{Chain Rule:} \frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$\text{Implicit Diff: } \frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$

$$\text{Gradient: } \boldsymbol{\nabla} f =< f_x, f_y, f_z >$$

$$\text{Normal of Tangent Plane: } \vec{n} = \boldsymbol{\nabla} f$$

$$\text{Directional Derivative: } D_{\vec{u}} f = \boldsymbol{\nabla} f \cdot \vec{u}$$

$$\text{2nd Derivative Test: } D = f_{xx} * f_{yy} - f_{xy}^2$$

(a) $D > 0$ and $f_{xx} > 0 \rightarrow f(a, b)$ is a local min

(b) $D > 0$ and $f_{xx} < 0 \rightarrow f(a, b)$ is a local max

(c) $D < 0 \rightarrow f(a, b)$ is a saddle point

$$\text{Lagrange Multipliers: } \boldsymbol{\nabla} f = \lambda \boldsymbol{\nabla} g + \mu \boldsymbol{\nabla} h$$

$$g = k$$

$$h = c$$

Part 3 Multiple Integrals:

$$\text{Double Integral: } \int\int_R f(x,y)$$
$$\text{Triple Integral: } \int\int\int_R f(x,y,z)$$

$$\int\int_D dA = Area(D)$$
$$\int\int\int_B dV = Volume(B)$$
$$\text{Interated Integral: } \int\int_R f(x,y)dA = \int_a^b \int_c^d f(x,y)dydx = \int_c^d \int_a^b f(x,y)dxdy$$
$$\text{Double Integral (Constant Bounds): } \int\int_R g(x)h(y)dA = \int_a^b *g(x)dx \int_c^d h(y)dy$$
$$\text{Iterated Integral: } \int\int\int_B f(x,y,z)dV = \int_r^s \int_c^d \int_a^b f(x,y,z)dxdydz$$
$$\text{Triple Integral (Constant Bounds): } \int\int\int_B f(x)g(y)h(z)dV = \int_a^b *f(x)dx \int_c^d g(y)dy \int_r^s h(z)dz$$

Change of Variable:

$$\text{2D Jacobian: } J = |\nabla x(u,v) \times \nabla y(u,v)| = \left\| \begin{matrix} \nabla x(u,v) \\ \nabla y(u,v) \end{matrix} \right\|$$
$$\text{3D Jacobian: } J = \left\| \begin{matrix} \nabla x(u,v,w) \\ \nabla y(u,v,w) \\ \nabla z(u,v,w) \end{matrix} \right\|$$
$$dA = dydxdz = Jdudvdw$$
$$\text{Polar: } r^2 = x^2 + y^2$$
$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$
$$J = r$$
$$\text{Spherical: } \rho^2 = x^2 + y^2 + z^2$$
$$x = \rho\sin\phi\cos\theta$$
$$y = \rho\sin\phi\sin\theta$$
$$z = \rho\cos\phi$$
$$J = \rho^2\sin(\phi)$$

Applications of Multiple Integrals

$$\text{Mass: } m = \int\int\int_E \rho(x, y, z) dV$$

$$\text{Moment: } M_{vw} = \int\int\int_E u\rho(u, v, w) dV$$

$$\text{Center of Mass Cord: } \bar{u} = \frac{M_{vw}}{m}$$

$$\text{Center of Mass: } c_m = (\bar{x}, \bar{y}, \bar{z})$$

Part 4 Vector Fields

Line integral:

$$\int_C \vec{F}\cdot d\vec{r} = \int_C F\cdot \vec{T}ds$$

$$\int_C f(x,y)ds = \int_a^b f(x(t),y(t))\sqrt{\frac{dx}{dt}^2 + \frac{dy}{dt}^2}dt$$

$$\int_C \vec{F}\cdot d\vec{r} = \int_C Pdx + Qdy + Rdz \text{ where } \vec{F} = P\vec{i} + Q\vec{j} + R\vec{k}$$

Vector Operations 2:

Curl: $curl\vec{F} = \boldsymbol{\nabla}\times\vec{F}$

$$curl(\boldsymbol{\nabla} f) = \boldsymbol{\nabla}\times\boldsymbol{\nabla} f = 0$$

Divergence: $div\vec{F} = \boldsymbol{\nabla}\cdot\vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$

$$div(curl(\vec{F})) = \boldsymbol{\nabla}\cdot\boldsymbol{\nabla}\times\vec{F} = 0$$

Parametric surface: $\vec{\vec{r}}(u,v) =< x(u,v), y(u,v), z(u,v) >$

Surface Area:

$$\vec{N} = \vec{r}_u \times \vec{r}_v$$

$$A(S) = \int\int_D |\vec{N}|dA$$

$$A(S) = \int\int_D \sqrt{1 + z_x^2 + z_y^2}dA$$

Surface Integrals:

$$\int\int f(x,y,z)dS = f(\vec{r}(u,v))|\vec{N}|dA$$

$$\int\int f(x,y,z)dS = \int\int_D f(x,y,g(x,y))\sqrt{1 + z_x^2 + z_y^2}dA$$

$$\vec{n} = \frac{N}{|N|}$$

$$\int\int_S \vec{F}\cdot d\vec{S} = \int\int_S \vec{F}\cdot\vec{n}dS$$

$$\int\int_S \vec{F}\cdot d\vec{S} = \int\int_D \vec{F}\cdot\vec{N}dA$$

$$\int\int_S \vec{F}\cdot d\vec{S} = \int\int_D (-Pg_x - Qg_y + R)dA$$

Conservative Vector Fields

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \text{ and D is an open simply-connected region}$$

$$\vec{F}(x, y, z) = \boldsymbol{\nabla} f$$

$$\oint \vec{F} \cdot \vec{dr} = 0$$

$$\int_C \boldsymbol{\nabla} f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$$

Green's Theorem:

$$\int_C Pdx + Qdy = \int \int_D (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})dA$$

$$\oint_C \vec{F} \cdot d\vec{r} = \int \int_D (\boldsymbol{\nabla} \times \vec{F}) \cdot \vec{k} dA$$

$$\oint_C \vec{F} \cdot \vec{n} ds = \int \int_D \boldsymbol{\nabla} \cdot \vec{F}(x, y) dA$$

Stoke's Theorem

$$\int_C \vec{F} \cdot d\vec{r} = \int \int_S \boldsymbol{\nabla} \times \vec{F} \cdot d\vec{S}$$

Divergence Theorem

$$\int \int_S \vec{F} \cdot d\vec{S} = \int \int \int_E \boldsymbol{\nabla} \cdot \vec{F} dV$$