Diff Eq Notes

10 March 2012

Contents

1 Initial Definitions

- Definition:
  - DE is an equation that describes the properties of an unkown
- Ordinary DE:
  - describes functions of 1 variable
- Partial DE:
  - describes multivariable functions
- Notation:
  - independent variable: y
  - dependent variable: t

2 Operator Notation

Definition: $\frac{d^n}{dt^n} = D^n \rightarrow f^{(n)} = D^n(f)$

3 Linear Diff Equations

Definition: For an operator,L, the DE: L(y) = 0 is linear iff:

- $L(y_1+y_1) = L(y_1)+L(y_2)$
- $L(cy) = cL(y)$

4 Initial Value Problems

$$IVP = \begin{cases} DE \\ y_0 = C \end{cases}$$

5 Seperable DE

5.1 Definition

- Can be written as f(y)dy = g(t)dt
- Technique for Solving: $\int f(y)dy = \int g(t)dt$

5.2 Homogenous Equations

6 Exact Equations

- Ψ(x,y) = Ψ(f(x), y(x))
- $\Psi_x = \Psi_f\ f_x + \Psi_y\ y_x$
- Technique for Solving:
  - Suppose DE is of the form: M(x,y) + N(x,y) $y_x = 0$
  - If $M_y = N_x$, then DE is an Exact Eq, solve for Ψ

7 First Order Linear Eq

7.1 Integration Factors

7.2 Bernoulli's equations

7.3 Existence and Uniquness Theorem

7.3.1 Picard Iteration

7.3.2 Lipsichitz Condition

7.3.3 Uniform Convergence

- Weirstress M Test

7.3.4 Existence Theorem

7.3.5 Uniqueness Theorm

8 Autonomous Equations

9 Second Order Linear Eq

Definition:

9.1 Theorm: The general solution to Second Order Linear Eq

Claim: The general soln of eq1$\equiv [y'' + p(t)y' + g(t)y = 0]$ is y = c_1 y_1 +c_2 y_2

9.1.1 Proof:

- Q1:

  Given y_1 and y_2 are solutions, why is c_1 y_1+ c_2 y_2 a solution

  - Eq1 = D^2(y) + p(t)D(y)+q(t)y = 0
  - Eq1 = [D^2 + p(t)D + q(t)]y = 0
  - Let L = [D^2 + p(t)D + q(t)] $\rightarrow eq1\equiv$ L(y)=0
  - Notice the L is a linear operator and thus obeys the superposition principle
  - Thus $y = c_1y_1 + c_2y_2$ is a solution $\square$

- Q2:

  Given 2 indepent solutions y_1 and y_2 for the DE, $\forall$ IVP and its unique solution y, $\exists(c_1, c_2) \in \mathbb{C}^2$ s.t. $y = c_1y_1 + c_2y_2$

  - The Wronskian:

$$\text{W(f,g)(t)} = \begin{vmatrix} f_1(x) & f_2(x) & \cdots & f_n(x) \\ f_1'(x) & f_2'(x) & \cdots & f_n'(x) \\ \vdots & \vdots & \ddots & \vdots \\ f_1^{(n-1)}(x) & f_2^{(n-1)}(x) & \cdots & f_n^{(n-1)}(x) \end{vmatrix}, \qquad x \in I$$

- Sub Proof of Q2

  Consider IVP: $y'' + py' + qy = 0$

  * Take c_1 and c_2 s.t.: \$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} y_1 & y_2 \\ y_1' & y_2' \end{pmatrix}_{t=t_0} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$
  * Notice this is only solvable iff $W(y_1, y_2)_{t=t_0} \neq 0$
  * Theorm: If u and v solve $y'' + p(t)y' + g(t)y = 0$ then W(u,v)=0 for all t or W is never 0

9.2 Second Order Homogenous Eq

9.3 Complex Number Review