Stats Review

by Marcell Vazquez-Chanlatte

Counting:

Mult Principle: $|A| = \prod_{i=0}^{n} m_i$ Counting n tasks(m_i) todo in son

Add Principle: $|A| = \sum_{i=0}^{n} m_i$ The number of ways to do n non-overlappi

Permutations: $P(n,k) = \frac{n!}{(n-k)!}$ Choose k ordered elements of A,

Combinations: $C(n,k) = \binom{n}{k} = \frac{P(n,k)}{P(k,k)} = \frac{n!}{(n-k)!k!}$ The number of unique non-orde

Counting events: $|X| = 2^n$ Size of the set(X) of all possible events in set A, wit

Probability Function:

Properties

(a) $0 \leq \mathbb{P}(X) \leq 1$

(b) $\mathbb{P}(\emptyset) = 0, \mathbb{P}(\Omega) = 1$, where $\emptyset$ is the empty set, and Ω is the problem space

(c) $\mathbb{P}(A \cap B) = \mathbb{P}(B)\mathbb{P}(A|B)$, Condition probability

(d) $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$

(e) $\mathbb{P}(A^c) = 1 - \mathbb{P}(A)$, Reason: $A \cap A^c = \emptyset \wedge \mathbb{P}(A \cup A^c) = \mathbb{P}(\Omega) \Rightarrow \mathbb{P}(A) + \mathbb{P}(A^c) = 1$

(f) Given: A has mutally disjoint, $\mathbb{P}(\cup_{i=0}^{n} A_i) = \sum_{i=0}^{n} \mathbb{P}(A_i)$

(g) $\mathbb{P}(A) = \sum_{i=0}^{n} \mathbb{P}(t_i)$, assuming $t_i \cap t_j = \emptyset$

(h) $A \subseteq B \rightarrow \mathbb{P}(A) \leq \mathbb{P}(B)$

(i) Independent iff: $\mathbb{P}(A \cap B) = \mathbb{P}(B)\mathbb{P}(B)$, alternatively written $\mathbb{P}(A|B) = \mathbb{P}(A)$ for $\mathbb{P}(B) \neq 0$

Random Variables:

Random Variable(f): $f : \Omega \to \mathbb{R}$
Prob mass function: $\mathbb{P}(X = a) : X \to [0, 1]$
Prob density function (1): $f(x)$ s.t. $\mathbb{P}(a \leq X \leq b) = \displaystyle\int_a^b f(x)dx,$
Prob density function (2): $int_{-\infty}^{\infty} f(x)dx = 1 = \mathbb{P}(\Omega)$
Cumdist function(Countable): $F(x) = \mathbb{P}(X \leq a) = \displaystyle\sum_{i=0}^{a} \mathbb{P}(X_i), a \in \Omega$
Cumdist func(Continuous): $F(x) = \mathbb{P}(X \leq a) = \displaystyle\int_{-\infty}^{a} \mathbb{P}(X_i)dx$
Cumdist/Prob density: $F(x) = \displaystyle\int_{-\infty}^{\infty} f(x)dx$
Prob density/Cumdist: $f(x) = \dfrac{d}{dx}F(x)$
Mean(μ) or Expected($\mathbb{E}$) (Countable): $\displaystyle\sum_{i=0}^{n} X_i\mathbb{P}(X_i)$
Mean(μ) or Expected($\mathbb{E}$) (Continuous): $\displaystyle\int_{-\infty}^{\infty} X_i\mathbb{P}(X_i)dx$
Variance(σ^2): $\mathbb{E}([X - \mu]^2) = \mathbb{E}(X^2) - \mu^2$
Standard Deviation(σ): $\sqrt{\sigma^2} = \sqrt{(}Var[X])$

Cumdist Properties (continuous):

(a) $0 \leq F(x) \leq 1$

(b) $\lim_{x\to-\infty} F(x) = 0 = \mathbb{P}(\emptyset) \wedge \lim_{x\to\infty} F(x) = 1 = \mathbb{P}(\Omega)$

(c) $\forall(x, y)x < y \to F(x) \leq F(y)$

Bernoulli trials: TODO Poisson Process/Continuous Trials: TODO

Distributions:

(a) $S =$ number of successes, $S : \mathbb{N} \to \mathbb{N}$

(b) $W =$ waiting time, $W : \mathbb{N} \to \mathbb{N}$

(c) $\mathbb{L} =$ life time, $\mathbb{L} : \mathbb{R} \to \mathbb{R}$

(d) Binomial Dist. (countable)

$$
\begin{aligned}
&|\Omega| = 2^n \\
&\mathbb{P}(S = k) = \binom{n}{k} p^k (1-p)^{m-k} \\
&\mu = np \\
&\sigma^2 = mp(1-p)
\end{aligned}
$$

(e) K-nomial(multinomial) Dist. (countable)

$$
\begin{aligned}
&|\Omega| = K^n \\
&\mathbb{P}(S = k) = \prod_{i=0}^{k} \binom{n - m_i}{m_i} p_i^{m_i}
\end{aligned}
$$

(f) Geometric Dist. (countable)

$$
\begin{aligned}
&\mathbb{P}(W = k) = p(1-p)^{(k-1)} \\
&\mathbb{P}(W \leq K) = 1 - (1-p)^k \\
&\mu = \frac{1}{p} \\
&\sigma^2 = \frac{1-p}{p^2}
\end{aligned}
$$

(g) Poisson Dist. (countable)

$$
\begin{aligned}
&\Lambda = t\lambda, \lambda = \text{ Longterm average rate} \\
&\mathbb{P}(S = k) = e^{-\Lambda} \frac{\Lambda^k}{k!} \\
&\mu = \Lambda \\
&\sigma^2 = \Lambda
\end{aligned}
$$

(h) Uniform Dist (continuous):

$$
p = \frac{1}{b-a} \tag{1}
$$

$$
f(x) = \begin{cases} p & : x \in [a,b] \\ 0 & : x \notin [a,b] \end{cases}
$$

(i) Linear Dist (continuous):

$$
C(b-a) + D(b-a)^2 = 1 \tag{2}
$$

$$
f(x) = \begin{cases} C + Dx & : x \in [a,b] \\ 0 & : x \notin [a,b] \end{cases}
$$

(j) Exponetial Dist. (continuous)

(k) Normal Dist. (continuous)