Marcell Vazquez-Chanlatte

Anthony Louie, Eric Mills

F2

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1. **Graph Concepts**

   A. Find the size of the **minimum cut**, i.e. the smallest number of edges that you could remove to disconnect the graph into two separate connected components. Briefly justify your answer.

   2, cut ef and fd, then f. (There is no cut edge thus 1 is not an option.
   The graphs starts as 1 component so 0 is invalid)

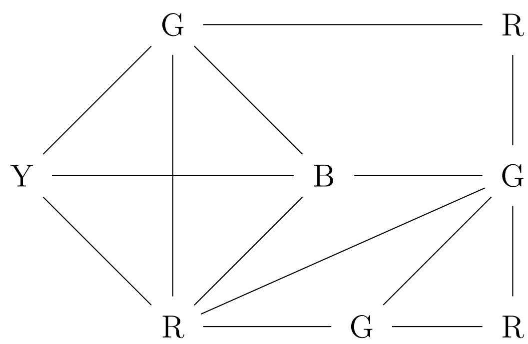
   B. What is the diameter of the graph? List two nodes with the greatest distance.

   3, a to c

   C. What is the chromatic number of the graph? Provide a proof.

   The chromatic number is 4. Here is a sample coloring

   G1:



   Claim: G1 cannot be colored with 3 colors

   (a) Notice that a complete graph with n nodes requires n colors because each vertex is connected. $n \in \mathbb{Z}$

   (b) Note that the subgraph connecting the vertices a, e, g, b is a complete graph.

   (c) Therefore the subgraph requires 4 colors

   (d) Thus, G1 must be colored with 4 or more colors...which is greater than 3 $\square$

2. **Induction**

A. Claim: For $n \in \mathbb{N}, P(n) : 5|(n^5 - n)$.

Base: $P(0)$

(a) Suppose $n = 0$, $n \in \mathbb{N}$
(b) Notice $P(0) : 5|(0^5 - 0)$
(c) Using the Def of Divisibility, $P(n) : n = 5m$, $m \in \mathbb{Z}$
(d) Consider $m = 0$
(e) Notice that $0 = 0 = 5 * 0$
(f) Thus $P(0)$ evaluates true.

Induction

(a) I.H. Suppose that for some $n \in \mathbb{N}$, $P(n) : 5|(n^5 - n) \rightarrow (n^5 - n) = 5m$. $m \in \mathbb{Z}$
(b) N.T.S. $P(n+1) : 5|(n+1)^5 - (n+1)$
(c) Notice that $(n+1)^5 = n^5 + 5n^4 + 10n^3 + 10n^2 + 5n + 1$
(d) Therefore $(n+1)^5 - (n+1) = (n^5 - n) + 5n^4 + 10n^3 + 10n^2 + 5n + 1 - 1$
(e) Simplifying $(n+1)^5 - (n+1) = (n^2 - n) + 5(n^4 + 2n^3 + 2n^2 + n)$
(f) Using the I.H. $(n+1)^5 - (n+1) = 5m + 5(n^4 + 2n^3 + 2n^2 + n) = 5(m + n^4 + 2n^3 + 2n^2 + n)$
(g) Applying the Def of Divisibility: $5|(n+1)^5 - (n+1)$
(h) $\square$

B. Claim: For $n \in \mathbb{N}, P(n) : \sum_{i=0}^{n} i \cdot i! = (n+1)! - 1.$

Base: $P(0)$

(a) Suppose $n = 0$ $n \in \mathbb{N}$

(b) Notice $\sum_{i=0}^{0} i \cdot i! = 0 * 0! = 0 * 1 = 0 = 1 - 1 = (0+1)! - 1$

(c) Thus $P(0)$ evaluates true.

Induction

(a) I.H. Suppose that some $n \in \mathbb{N}$, $P(n) \sum_{i=0}^{n} i \cdot i! = (n+1)! - 1$

(b) N.T.S $\sum_{i=0}^{n+1} i \cdot i! = (n+1+1)! - 1 = (n+2)! - 1.$

(c) Observe that $\sum_{i=0}^{n} i \cdot i! = \sum_{i=0}^{n} i \cdot i! + (n+1) * (n+1)!$

(d) Using the I.H. $\sum_{i=0}^{n+1} i \cdot i! = (n+1)! - 1 + (n+1) * (n+1)!$

(e) Simplifying $\sum_{i=0}^{n+1} i \cdot i! = (n+1)! * (n+1+1) - 1 = (n+2)! - 1$

(f) $\square$