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HW0

1. **Piazza Setup**
   Done

2. (a) $\Pi_{k=2}^{n}(1-\frac{1}{k^2})$
      i. $\Pi_{k=2}^{n}(1-\frac{1}{k^2}) = \Pi_{k=2}^{n}(\frac{k^2-1}{k^2})$
      ii. $\frac{3}{4}, \frac{2}{3}, \frac{5}{8}, \frac{3}{5}$
      iii. $\frac{3}{4}, \frac{4}{6}, \frac{5}{8}, \frac{6}{10}$
      iv. $\Pi_{k=b2}^{n}(1-\frac{1}{k^2}) = \frac{n+1}{2n}$

   (b) $3^{1000} \mod 7$
      i. $3^1, 3^2, 3^3, 3^4, 3^5, 3^6, 3^7$
      ii. $3, 2, 6, 4, 5, 1, 3$
      iii. $3^{1000} \mod 7 = 1000 \mod 6 = 4$

   (c) $\sum_{r=1}^{\infty} \frac{1}{2}^r = \lim_{n->\infty} \frac{n^2-1}{n^2} = 1$

   (d) $\frac{\log_7 81}{\log_7 9} = \frac{4\log_7 3}{2\log_7 3} = 2$

   (e) $\log_2 4^{2n} = 2n\log_2 2^2 = 4n$

   (f) $\log_{17} 221 - \log_{17} 13 = \log_{17}\frac{221}{13} = 1$

3. Claim: $1 + \sum_{j=1}^{n} j!j = (n+1)!$

   (a) Let $n > 0$, $n \in \mathbb{N}$

   (b) Base: Suppose $n = 1$, $1 + \sum_{j=1}^{1} j!j = 1 + 1 = 2 = (1+1)!$

   (c) I.H : Suppose for some $n > 0$ that $1 + \sum_{j=1}^{n} j!j = (n+1)!$

   (d) Observe that, $1 + \sum_{j=1}^{n+1} j!j = 1 + (n+1)!(n+1) + \sum_{j=1}^{n} j!j$

   (e) Applying the I.H, $1 + (n+1)!(n+1) + \sum_{j=1}^{n} j!j = 1 + (n+1)!(n+1) + (n+1)!$

   (f) Simplifying, $1 + \sum_{j=1}^{n+1} j!j = 1 + (n+1)!(n+1+1)$
   $\square$

4. (a) $4^{\log_4 n}$ and $2n+1$
   i. Claim: $4^{log_4 n}\Theta(2n+1)$
   ii. Observe that $4^{log_4 n} = n$ due to properties of logs $n \in \mathbb{N}$
   iii. Sub Claim: $nO(2n+1)$
      A. Notice that $2n = n+n > n-1$ for $n > 1$
      B. Therefore, $2n+1 > n$
      C. Suppose that $C > 1$, $C \in \mathbb{N}$
      D. Notice that $C(2n+1) > n$ for $n > 1$
      E. Thus $nO(2n+1)$
   iv. Sub Claim: $(2n+1)O(n)$
      A. Observe that $2n < 10^{100}n - 1$ for $n > 1$
      B. Let C be 10^{100}, $C \in \mathbb{N}$
      C. Substituting $2n+1 < Cn$
      D. Applying Def of Big O, $(2n+1)O(n)$
   v. Therefore, Applying the Def of Big Theta,
   $\square$ $4^{log_4 n} = n\Theta(2n+1)$

   (b) Claim: n^2 is $\Omega(\sqrt{2}^{\log n})$
   i. Let n be an integer such that $n > 1$
   ii. Observe that $\log_2 n^2 = 2\log_2 n > \log_2 \sqrt{n} = \log_2 n^{\frac{1}{2}} = \frac{\log_2 n}{2}$
   iii. Thus, $2^{\log_2 n^2} = n^2 > \sqrt{2}^{\log_2 n} = 2^{log_2 \sqrt{n}}$
   iv. Let $C = 1$, $C \in \mathbb{Z}$
   v. Notice that $n^2 > C\sqrt{2}^{\log_2 n}$
   vi. Applying the Def of Big Ω, n^2 is $\Omega(\sqrt{2}^{\log_2 n})$

   (c) $\log n!$ is $O(n \log n)$
   (d) n^k is $O(c^n)$

5. (a) $T(n) = 5\log_2 n + 1$
   (b) $T(n) = (n-1)\sum_{x=1}^{n-1} \frac{1}{x}$
   (c) $T(n) = n(\log_2 n)^2$
   (d) Claim: $T(n) = T(\frac{n}{2}) + 5 = 5\log_2 n + 1$, $T(1) = 1$
   i. Base: $T(1) = 1 = 5 \cdot 0 + 1 = 5\log_2 0 + 1$
   ii. I.H : Suppose the for some $n \in \mathbb{Z}^+$, $T(n) = 5\log_2 n + 1$
   iii. Observe that $T(n)$ will only result integer results if given power's of 2 because $\frac{n}{2}$ will only recursively be divisible by to if all its factors are 2, i.e. $n = 2^x$
   iv. Therefore the next integer input after n is $2n$ with all inbetween results having an imply floor to make them equal to the n input.
   v. Note that $T(2n) = T(\frac{2n}{2}) + 5 = T(n) + 5$
   vi. Applying the I.H, $T(2n) = 5\log_2 n + 1 + 5 = 5(\log_2(n) + 1) + 1 = 5\log_2 n \log_2(2) + 1 = 5\log 2n + 1$
   $\square$

6. (a) Assuming n is even:
   i. $T(n) = T(\frac{n}{2}) + C$,$T(1) = D$
   ii. $T(n) = C \log_2 n + D$

   (b) Assuming n is even
   i. Assuming lines compute in C time...$T(n) = 2T(\frac{n}{2}) + C(n+1)$, $T(1) = 1$
   ii. $T(n) = C(2n-1) + n \log_2 n$

7. (a)

$$\begin{array}{lr}
x, n: & return \\
2, 12: & 2^{12} \\
2^2, 6: & 2^{12} \\
2^4, 3: & 2^4 \cdot 2^6 \\
2^8, 1: & 2^8 \cdot 1 \\
2^16, 0: & 1
\end{array}$$

   (b) it thats x^n

   (c) The algorithm takes essientally the same time for an even integer n and $n-1$ (odd) because $\frac{n-1}{2}$ is essientally flooring the result if n is odd and because its pretty hard to predict primes. Therefore
   $T(0) = D$, $T(1) = C + D$
   $T(n) = T(\frac{n}{2}) + C$

   (d) $T(n) = C \log_2(n) + D + C) = C(\log_2(n) + 1) + D$, $n > 1$