Remarks on VLR

AAD

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1 Introduction

Variable load risk (VLR) is one part in the financial management of a load serving contract. For one temporal period (usually taken to be one hour), unit VLR in $/MWh is defined as

$$VLR = (T - P)(\frac{L}{L_{wn}} - 1) \tag{1}$$

where T is the tariff price, P is the energy price of this period, L is the load in this period, and L_{wn} is the weather normal load.

Given that both P and L are random variables, the mean VLR can be shown to be

$$E[VLR] = -\frac{\mathrm{Cov}(L, P)}{E[L]} = -\frac{\sigma_P \sigma_L}{E[L]}\rho \tag{2}$$

where σ_P, σ_L is the standard deviation of the price and load respectively, ρ is the correlation between load and price.

VLR mean does not depend on the tariff price T and will equal zero if the correlation between load and price is zero. Also, as price volatility increases, VLR mean also increases.

Note that a low VLR mean does not imply a small financial risk. So how to price a random variable with a small mean but large variance? Is the concept of VLR delta $\Delta = dVLR/dP$ useful in managing VLR risk? Below, we discuss cases that shed more insight into these questions.

2 Price as a deterministic function of load

In general, the joint distribution of price and load cannot be expressed in closed analytical form. To gain more insight we consider in this section the case when price is a deterministic function of load only. We discuss relaxing this constraint in the next section.

Let's denote the relationship between load and price by the function $P = S(L)$. We can think of function $S(L)$ as representating the generation stack.

VLR as a function of market price P is a concave function with two zeros. One zero is trivial and happens when market price equals the tariff price T. To find the other zero we recognize that the stack function S is invertible, so there will be a price for which $S^{-1}(P) = L_{wn}$. This price will be the second zero of the VLR function. In general, this second zero is close to the market price P.

It is reasonable to expect that for various load ranges, there is a linear relationship btween price and load. For simplicity, take the load to be a normally distributed random variable. First, consider the linear model

$$\begin{aligned} L &= \mathcal{N}(L_{wn}, \epsilon_L) && (3) \\ P &= aL + b, && (4) \end{aligned}$$

and an adjusted linear model with quadratic tail. For moderate loads, there is a linear relationship between load and price, but for higher loads, this linear relationship becomes quadratic. This quadratic tail model is more realistic for dynamic of power prices.

$$\begin{aligned} L &= \mathcal{N}(L_{wn}, \epsilon_L) && (5) \\ P &= aL + b + c(L - L_*)^2 \mathrm{max(L - L_*, 0)}. && (6) \end{aligned}$$

Let's look at a concrete example. With tariff $T = 70$ \$/MWh, $L_{wn} = 600$ MWh. Take $\epsilon_L = 30$ MWh, which corresponds to 0.05% of the L_{wn} value. Using $a = 0.04$ and $b = 4$, the price distribution of the linear model is $\mathcal{N}(28, 1.2^2)$. The quadratic tail model is $P = 0.04L + 4 + 0.02 * \max(L - L_*, 0)(L - L_*)^2$ with $L_* = 665$.

For the linear model, $VLR(P)$ is a quadratic. It has zeros at the tariff price T and at $E[P]$. The VLR delta is trivially computed to be

$$\Delta = \frac{d\mathrm{VLR}}{dP} = \frac{a}{2E[L]}(P_* - P) \quad (7)$$

where P_* is the price for the vertex of the VLR function.

This formula indicates that as prices are below the vertex price P_* and increasing, VLR delta decreases. And as prices increase above the vertex price P_* delta becomes more and more negative. This is exactly a short Gamma Γ position. The $\Gamma = -a/E[L]$ is independent of price level.

The

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
VLR_0	-0.65	-0.08	-0.02	-0.06	-0.01	-0.00
VLR_1	-7.39	-1.48	-0.09	-0.06	1.45	6.25
VLR_2	-7.39	-1.48	- 0.10	-0.10	1.44	3.91

Table 1: Summary VLR distribution. VLR_0 is calculated using F instead of the tariff price T, VLR_1 is calculated using the linear model and the actual tariff price T, VLR_2 is calculated using the quadratic model. As expected, the VLR mean for VLR_0 and VLR_1 are equal.

3 Zero correlation

4 Quadratic model

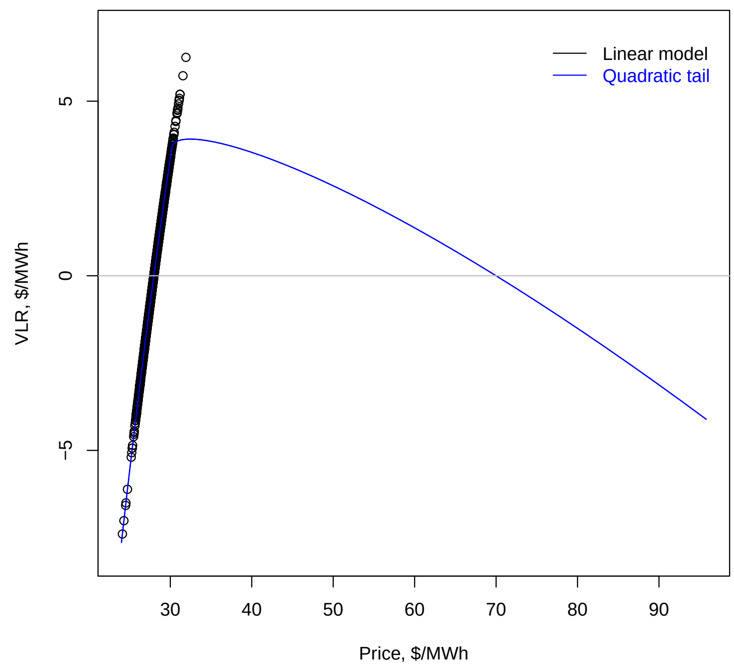

Figure 1: VLR as a function of price when prices are a deterministic function of load. Hard to see in the figure, but the linear model is a quadratic, with the second zero at the tarrif price of \$70/MWh. The prices for the linear model shown as points, correspond to 1000 values of load $\mathcal{N}(600, 30)$ by the linear transformation $P = 0.04 * L + 4$. The quadratic tail model (blue) has prices equal $P = 0.04L + 4 + 0.02 * \max(L - L_*, 0)(L - L_*)^2$ with $L_* = 665$. The vertex price for the linear model is $P_* = 49$, half way between $E[P]$ and tariff T.