Row Reduction Activity 1

Name ________________
Date _________________

In case you have not already memorized this important procedure, please refer to the following steps, which detail how to row-reduce a matrix in a computationally consistent manner.

1. Starting from the first row, find the leftmost nonzero entry (pivot) in the matrix.
2. If there is no pivot in the first column, swap the first row with a row below it that has a nonzero entry in the first column.
3. Divide the pivot row by the pivot value so that the pivot becomes 1.
4. Use row operations to eliminate all entries below the pivot in the same column by subtracting a suitable multiple of the pivot row from each of the lower rows.
5. Move to the next column and repeat steps 1 to 4 until all columns have been processed or all remaining rows are zero rows.
6. Back-substitute to eliminate all entries above the pivots by subtracting a suitable multiple of the row above from each of the upper rows.
7. The resulting matrix should now be in reduced row echelon form.

According to our detailed research, which we did so you don't have to, the $O(n)$ runtime complexity of this algorithm is $O(n^3)$ (simple answer) or $O(r \times c^2)$ (more complex answer). Thus, for the 6x6 matrix that we'll be reducing today, you will probably only have to execute 372 operations!

1. Row-reduce the following matrix. Show your work, including which rows you operated upon. Make sure to check your answer afterwards. Feel free to request extra scrap paper, you will need it.

$$\begin{bmatrix} -7 & 8 & 9 & 1 & 3 & 0 \\ 9 & -2 & -4 & 3 & -4 & -7 \\ -8 & -3 & 4 & 7 & -4 & -7 \\ -1 & 1 & -2 & 5 & 6 & -9 \\ 1 & 3 & -6 & -2 & 5 & -6 \\ -4 & 0 & -8 & 1 & -2 & 8 \end{bmatrix}$$

Once you finish, you will discover an incredible fact about linear algebra! What is it?

Bonus

Show that the level of fatigue experienced after row reducing a 6 x 6 matrix, f, is less than or equal to the sum of the fatigue experienced during a double period of AP Physics C and a single period of AP Microeconomics, F. Use the property that matrix multiplication can be seen as a linear combination of the columns. You will need to find the determinant of the matrix with more eigenvalues than the number of hours of sleep a Stuyvesant freshman gets.