

$$1) \quad y = \text{sigmoid}(a) \Rightarrow \frac{dy}{da} = y(1-y)$$

$$\Rightarrow \frac{d}{da} = \left(\frac{1}{1+e^{-a}} \right)$$

$$\frac{d}{da} (1+e^{-a})^{-1} = -(1+e^{-a})^{-2} \cdot \frac{d}{da} \left(\frac{1}{1+e^{-a}} \right)$$

$$= \frac{d}{da}$$

$$\Rightarrow -(1+e^{-a})^{-2} \frac{d}{da} (1+e^{-a}) = -(1+e^{-a})^{-2} \left[\frac{d}{da} (1) + \frac{d}{da} (e^{-a}) \right]$$

$$= -(1+e^{-a}) \left[0 + \frac{d}{da} (e^{-a}) \right]$$

$$= -(1+e^{-a})^{-2} \left(e^{-a} \cdot \frac{d}{da} (-a) \right)$$

$$= -(1+e^{-a})^{-2} \left(e^{-a} \cdot \frac{d}{da} (a) \right)$$

$$= \cancel{-(1+e^{-a})^{-2}} (e^{-a} \cdot \cancel{(-1)})$$

$$= (1+e^{-a})^{-2} (e^{-a})$$

$$= \frac{e^{-a}}{(1+e^{-a})^2}$$

$$= \frac{e^{-a}}{(1+e^{-a})(1+e^{-a})}$$

$$= \frac{1}{1+e^{-a}} \cdot \frac{e^{-a}}{1+e^{-a}}$$

$$= \frac{1}{1+e^{-a}} \cdot \frac{e^{-a} + 1 - 1}{1+e^{-a}}$$

$$= \frac{1}{1+e^{-a}} \cdot \left(\frac{1+e^{-a}}{1+e^{-a}} - \frac{1}{1+e^{-a}} \right)$$

$$y = \left[\frac{1}{1+e^{-a}} \right] \cdot \left(1 - \left[\frac{1}{1+e^{-a}} \right] \right)$$

$$\frac{dy}{da} = y(1-y) \quad \text{or} \quad y = \frac{1}{1+e^{-a}}$$