Week 5

1 Some comments on engineering quantitites

1.1 Achievability

There exists some scheme using which we can show that the length of compression, rate or any other quantity of interest happens to be equal to be L or that value which is achievable.

1.2 Converse

No scheme exists which can improve upon some value. (can be upper or lower bound depending on situation)

Eg: Suppose in a running race, the fastest speed a human can run is say 10 m/s. Then

- 'Achievable': There exists some person who can run 10 m/s.
- 'Converse': There exists no person who can run more than 10 m/s.
- 'Matching Converse': No human can run at a speed $10 + \epsilon$, for any $\epsilon > 0$.

2 Channel Coding

Say we are inputting $x \in \mathcal{X}$, $(x_1, \cdots, x_n)$, in a channel and we are getting $y \in \mathcal{Y}$, $(y_1, \cdots, y_n)$, with $\mathcal{Y}$ as output alphabet.

If multiple x_i are mapping to a single y, it signifies a noisy channel as we would be unable to decode accurately.

To make this channel one-one (and therefore ensure correct decoding), we omit some sequences (n-length vectors in $\mathcal{X}^n$) from the set of all transmittable sequences.

This subset of transmittable sequences is called as the 'channel code' (or simply code). Denoted generally by $\mathscr{C}$. Note that, $\mathscr{C} \subseteq \mathcal{X}^n$.

Each vector in $\mathscr{C}$ is called a codeword. Number of bits required to represent $|\mathscr{C}|$ codewords

$$= \log_2 |\mathscr{C}| \text{ bits}$$

$$\text{Rate of the code } \mathscr{C} = \frac{\log_2 |\mathscr{C}|}{n} \text{ bits per channel use (bpcu or b/cu)}$$

Intuitively, higher the rate, more the chance of many-one kind of system and higher the chance of error.

2.1 Probabilistically noisy channel

Also called a random channel or random noise.

For $X = x \in \mathcal{X}$, there will be a probability distribution on the output random variable Y. The coditional distribution on Y given $X = x$,

$$P_{Y/X=x} = \{P(Y = y/X = x) : y \in \mathcal{Y}\}$$

These distributions $P_{Y/X}(y/x)$ $\forall x$ completely characterize or describe the random channel.

Now, we need to think about how to calculate $P(error)$. $X_i's$ are given as input to the random channel which are not independent. Then $(Y_1, \cdots, Y_n)$ is given out as output of the random channel, this is then sent into the decoder which then gives out $\hat{X}$, which is an estimate for X.

When $\hat{X} \neq X$, it is called an error event.

$$P(\text{Decoding error}) = P(\hat{X} \neq X)$$