Week 3

0.1 Joint Entropy in multiple RV's

$$H(X_1, \cdots, X_n) = \sum_{(x_1,\cdots,x_n)\in supp(P_{X_1,\cdots,X_n})} P(x_1, \cdots, x_n) \log_2 \frac{1}{P(x_1, \cdots, x_n)}$$

Where $P_{X,Y} := \mathcal{X} \times \mathcal{Y} \rightarrow$ Cartesian Product.
$\mathcal{X} \times \mathcal{Y} := \{(x, y) : x \in \mathcal{X}, y \in \mathcal{Y}\}$

0.2 Conditional Joint Entropy

$$H(X, Y) := \sum_{(u,v)\in supp(P_{U,V})} P_{U,V}(u, v) H(X, Y/U = u, V = v)$$

where,

$$H(X, Y/U = u, V = v) = \sum_{(x,y)\in supp(P_{X,Y/U=u,V=v})} P(x, y/u, v) \log_2 \frac{1}{P(x, y/u, v)}$$

This can be extended to any number of variables before and after the conditioning.
eg: $P(x_1, x_2, x_3/y_1, y_2) = P(x_1, x_2/x_3, y_1, y_2) + P(x_3/x_1, x_2, y_1, y_2)$.

1 Chain Rule for Joint Entropy

Lemma:

$$\begin{aligned} H(X_1, \cdots, X_n) = H(X_1) + H(X_2/X_1) + H(X_3/X_1, X_2) + \cdots \\ + H(X_n/X_1, \cdots, X_n) \end{aligned} \tag{1}$$

Proof:

$$\begin{aligned} P(x_1, \cdots, x_n) &= P(x_1)P(x_2, \cdots, x_n/x_1) \\ &= P(x_1)P(x_2/x_1)P(x_3, \cdots, x_n/x_1, x_2) \\ &= P(x_1)P(x_2/x_1)P(x_3/x_1, x_2)P(x_4, \cdots, x_n/x_1, x_2, x_3) \\ &= P(x_1)P(x_2/x_1)P(x_3/x_1, x_2) \cdots P(x_n/x_1, \cdots, x_{n-1}) \end{aligned}$$