# 2.8 Exponential Growth and Decay

## Exponential Growth

Many systems exhibit exponential growth. These systems follow a model of the form $y = y_0 e^{kt}$, where $y_0$ represents the initial state of the system and $k$ is a positive constant known as the growth constant. Notice that in exponential growth, $y' = ky_0 e^{kt} = ky$. That is the rate of growth is proportional to the current value of the function.

**Rule: Exponential Growth Model**

Systems that exhibit **exponential growth** increase according to the mathematical model

$$y = y_0 e^{kt},$$

where $y_0$ represents the initial state of the system and $k > 0$ is a constant called the growth constant.

**Definition**

If a quantity grows exponentially, the **doubling time** is the amount of time it take the quantity to double. It is given by

$$\text{Doubling time} = \frac{\ln(2)}{k}.$$

## Exponential Decay Model

Exponential functions can also be used to model populations that shrink.

We say that such systems exhibit exponential decay, rather than exponential growth. The model is nearly the same, except there is a negative sign in the exponent. Thus, for some positive constant $k$, $y = y_0 e^{-kt}$.

As with exponential growth, there is a differential equation associated with exponential decay. We have

$$y' = -ky_0 e^{-kt} = -ky.$$

**Rule: Exponential Decay Model**

Systems that exhibit **exponential decay** behave according to the model

$$y = y_0 e^{-kt},$$

where $y_0$ represents the initial state of the system and $k > 0$ is a positive constant, called the decay constant.

**Definition**

If a quantity decays exponentially, the **half-life** is the amount of time it takes the quantity to be reduced by half. It is given by

$$\text{Half-life} = \frac{\ln(2)}{k}.$$