



MA493 MATHEMATICS PROJECT

Home Life and Social Habits of British Muggles

by

Hermione Granger Student ID: 6X102010XX1

Harry Potter Student ID: 6X102010XX2

Ron Weasley Student ID: 6X102010XX3

Advisor

Professor Albus Percival Wulfric Brian Dumbledore

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By : Hermione Granger
Harry Potter
Ron Weasley
Advisor : Professor Albus Percival Wulfric Brian Dumbledore, Ph.D.

Accepted by the Department of Mathematics, Faculty of Science, Srinakharinwirot University in Partial Fulfillment of the Requirements for the Bachelor's Degree of Science in Mathematics.

..... Program Chairperson
(Assistant Professor Pisuttawan Sripirom Sirininlakul, Ph.D.)

Committee

..... Advisor
(Professor Albus Percival Wulfric Brian Dumbledore, Ph.D.)

..... Examiner
(Examiner 1)

..... Examiner
(Examiner 2)

..... Examiner
(Examiner 3)

Abstract

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Cupcake ipsum dolor. Sit amet chocolate jelly tiramisu halvah croissant bear claw sugar plum. Jujubes sugar plum lemon drops pudding cake cheesecake biscuit cotton candy gummi bears. Gummi bears cake chocolate cake icing marshmallow. Apple pie cake cupcake pie halvah. Jelly cotton candy croissant jujubes. Ice cream chupa chups jujubes lollipop marzipan. Fruitcake halvah sweet.

Jujubes biscuit jelly-o halvah candy croissant jelly-o liquorice. Tootsie roll donut muffin muffin toffee donut cupcake halvah. Ice cream jelly beans soufflé. Cake chocolate bar bear claw ice cream jujubes liquorice. Toffee oat cake sweet. Brownie oat cake bear claw. Cake brownie jelly beans. Marshmallow pastry fruitcake fruitcake gummies chocolate. Tiramisu soufflé macaroon gummi bears cotton candy jelly-o chocolate bar caramels. Halvah liquorice jelly icing carrot cake jelly-o lemon drops.

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CHAPTER 1

Introduction

Tumeric twee banh mi ex, umami blue bottle cred. Lo-fi ut deserunt thundercats. Cray slow-carb laborum asymmetrical hammock, tempor drinking vinegar migas flannel. Do cliché wayfarers, selfies affogato aesthetic authentic culpa austin single-origin coffee. Fam before they sold out est whatever iPhone cred cold-pressed, flannel polaroid gastropub nisi. Viral dreamcatcher pour-over, minim jianbing occaecat small batch butcher tumeric deep v.

$$a^2 + b^2 = c^2 \tag{1.1}$$

Fam pop-up hammock kombucha coloring book. Hashtag gluten-free cornhole poke craft beer hot chicken. Deep v 8-bit coloring book pinterest dreamcatcher, raw denim street art adaptogen chillwave raclette salvia fashion axe. Fashion axe pabst shoreditch DIY yuccie aesthetic. Venmo laboris squid coloring book godard pop-up banh mi artisan. Nisi fam master cleanse ethical ut celiac, truffaut officia franzen dreamcatcher neutra in bushwick chillwave.

(1) interesting statement 1

(2) interesting statement 2

(3) interesting statement 3

CHAPTER 2

Preliminaries

In this chapter, basic definitions and propositions are defined. In this report, all Alexandrov spaces are assumed to have a finite Hausdorff dimension.

2.1 Basic definitions

First, we recall the notion of a strainer.

Definition 2.1.1. Let X be an Alexandrov space of curvature bounded below by c . Let $p \in X$. An m -strainer at p of quality δ and scale r is a collection $\{(a_i, b_i)\}_{i=1}^m$ of pairs of points such that $d(p, a_i) = d(p, b_i) = r$ and in terms of comparison angles,

$$\begin{aligned}\tilde{\angle}_p(a_i, b_i) &> \pi - \delta, \\ \tilde{\angle}_p(a_i, a_j) &> \frac{\pi}{2} - \delta, \\ \tilde{\angle}_p(a_i, b_j) &> \frac{\pi}{2} - \delta, \\ \tilde{\angle}_p(b_i, b_j) &> \frac{\pi}{2} - \delta,\end{aligned}\tag{2.1}$$

for all $i, j \in \{1, \dots, m\}$, $i \neq j$. The comparison angles are defined using comparison triangles in the model space of constant curvature c .

Definition 2.1.2. The *strainer number* of X is the supremum of numbers m such that there exists an m -strainer of quality $\frac{1}{100m}$ at some point and some scale.

2.2 Basic propositions and theorems

In this section, we view the card game \mathcal{SET} as a mathematical object. We identify all 81 cards in the pile as a vector space over a finite field. Some basic definitions and propositions are also given.

Proposition 2.2.1. *Let x_1 and x_2 be distinct elements of $(\mathbb{Z}_3)^4$. There is a unique $x_3 \in (\mathbb{Z}_3)^4$ such that $x_1 + x_2 + x_3 = 0$ and x_3 is not equal to x_1 and x_2 .*

Proof. Assume that x_1 and x_2 are distinct elements of $(\mathbb{Z}_3)^4$. Let $x_3 = -x_1 - x_2$. Then $x_1 + x_2 + x_3 = x_1 + x_2 + (-x_1 - x_2) = 0$. Suppose that there is another $x \in (\mathbb{Z}_3)^4$ such that $x_1 + x_2 + x = 0$. We have $x = -x_1 - x_2 = x_3$. Hence, x_3 is the only element such that $x_1 + x_2 + x_3 = 0$.

Suppose that $x_3 = x_1$. Then $x_1 = x_3 = -x_1 - x_2$, so $2x_1 = 2x_2$. Equivalently, $x_1 = x_2$ which is a contradiction. Next, suppose that $x_3 = x_2$. Then $x_2 = x_3 = -x_1 - x_2$, so $2x_1 = 2x_2$. Equivalently, $x_1 = x_2$ which is a contradiction. \square

CHAPTER 3

Main Results

3.1 Result 1

The following lemma is from [1].

Lemma 3.1.1. *For $x, y \in X$, $x = (x \triangleright^{-1} y) \triangleright y$.*

Proof. This can be obtain directly form the definition of quandles. □

Theorem 3.1.2. *The general solution to the differential equation*

$$\frac{dy}{dx} = \frac{x^2 + y^2}{xy} \quad (3.1)$$

is $y = \pm \sqrt{2x^2 \ln |x| + Cx^2}$.

Proof. This is a homogeneous first-order equation. Thus, we can solve the equation by substituting $y = xv$. By the product rule, $\frac{dy}{dx} = x \frac{dv}{dx} + v$. Substituting $\frac{dy}{dx} = x \frac{dv}{dx} + v$ and $y = xv$ into the given equation to have that

$$x \frac{dv}{dx} + v = \frac{x^2 + (xv)^2}{x \cdot xv} = \frac{1 + v^2}{v} = \frac{1}{v} + v. \quad (3.2)$$

Hence,

$$x \frac{dv}{dx} = \frac{1}{v}. \quad (3.3)$$

Then,

$$v \frac{dv}{dx} = \frac{1}{x}. \quad (3.4)$$

Integrate both sides to get that

$$\frac{1}{2}v^2 = \int \frac{1}{x} dx = \ln |x| + k \quad (3.5)$$

for some constant k . That is

$$v^2 = 2 \ln |x| + 2k. \quad (3.6)$$

Substitute $v = \frac{y}{x}$ to have that

$$\left(\frac{y}{x}\right)^2 = 2 \ln |x| + 2k. \quad (3.7)$$

Hence,

$$y^2 = 2x^2 \ln |x| + 2kx^2. \quad (3.8)$$

Therefore, the general solution of the given differential equation is

$$y = \pm \sqrt{2x^2 \ln |x| + Cx^2}$$

for some constant C .

□

Corollary 3.1.3. *The following is true.*

APPENDIX A

Some additional facts

Let $0 < \beta_1 < \beta_2 < \beta_3$ be new parameters. At scale \mathfrak{r}_p , we partition points in M as follows:

- A point p in M is a *3-stratum* point if $(\frac{1}{\mathfrak{r}_p}M, p)$ is β_3 -close to $(\mathbb{R}^3, 0)$ in the pointed Gromov-Hausdorff topology.
- A point p in M lies in the *2-stratum* if it does not lie in the 3-stratum and $(\frac{1}{\mathfrak{r}_p}M, p)$ is β_2 -close to $(\mathbb{R}^2 \times Y_p, (0, Y_p^*))$ in the pointed Gromov-Hausdorff topology, where Y_p is a point, a circle, an interval, or a half-line, and Y_p^* is a basepoint in Y_p .
- A point p in M lies in the *1-stratum* if it does not lie in the k -stratum for $k \in \{2, 3\}$ and $(\frac{1}{\mathfrak{r}_p}M, p)$ is β_1 -close to $(\mathbb{R} \times Y_p, (0, Y_p^*))$ in the pointed Gromov-Hausdorff topology, where Y_p is a 2-dimensional Alexandrov space.

Furthermore, if a point $p \in M$ is in the k -stratum, then at some scale comparable to \mathfrak{r}_p , M is close in the pointed C^K -topology to $N_p \simeq \mathbb{R}^k \times F_p$ where F_p is given in the following table.

k	F_p
3	S^1
2	S^2, T^2, D^2
1	$S^3/\Gamma, T^3/\Gamma, S^2 \times S^1, \mathbb{R}P^3 \# \mathbb{R}P^3, D^3, S^2 \times_{\mathbb{Z}_2} I, S^1 \times D^2, T^2 \times_{\mathbb{Z}_2} I$

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