Geov112 - Exercise Set 2 (Deadline: 14 September 2017)

Note: These exercises cover parts of both Chapter 2 and Chapter 3 of McMahon

Learning Outcomes for Exercise Set 2
In this exercise set you will learn:
1. How to define and work with vectors and matrices in Matlab

2. In particular, you will learn how to:
a. define vectors in Matlab,
b. do computations with vectors (addition, subtraction, multiplication with a scalar, component wise multiplication etc.),
c. create new vectors in Matlab using either 'linspace(a,b,n)' or [a:dx:b], compute the inner product of two vectors, their cross product etc.

3. In particular you will also learn how to:
a. define matrices in Matlab and
b. do computations with matrices in Matlab

4. You will learn how to make plots of functions of one variable with simple applications in geophysics

5. You will learn what polar coordinates are, how they are used, what the relationship is between polar coordinates and Cartesian coordinates and plot circles in Matlab using polar coordinates

6. You will learn how to numerically integrate and differentiate a function of one variable

1. Do exercises 4-7 at the end of Chapter 1 in McMahon.

2. **Scalar and Array Variables**

   (a) Redo exercise I2b (exercise 2b of the first exercise set) but now use arrays (or vectors; please note that McMahon uses the term 'arrays' both for vectors and matrices. We will do this too; from the context it should be clear whether we are dealing with a vector or matrix).

   (b) Redo exercise I3, but now use arrays.

   (c) Redo exercise I5 by using arrays.

3. **Creating and Manipulating Arrays.**

   (a) Explain in your own words the main differences and similarities between the `linspace(min,max,n)` and [min:dx:max] commands.

   (b) Create an array with 313 elements between -4 and 9.

   (c) Create a new array by extracting the third and ninth element of the array in (b).

   (d) Create a new array by extracting elements 101 to elements 200 from the array in (b).

   (e) Create a new array by extracting elements 12-23 and elements 52-63 from the array in (b) and then adding these two arrays..

   (f) Add 7 to the first array in (e) and subtract 2 from the second array in (e).

(g) Multiply the two arrays in (f) component wise.

(h) Create an array between -2000 and 2000 with a spacing of 50. Explain which of the two methods to generate arrays (see 3(a)) you use and why.

(i) Compute the number of elements in the array created in (h) using Matlab.

(j) Create an array that contains the values of $\cos(x)$ for x between 0 and π, consisting of 101 points.

(k) Indicate which of the arrays created in this exercise are row vectors and which arrays are column vectors (this is not a Matlab exercise obviously, so use comments in your Matlab script to answer this question).

4. Do exercises 1-4 at the end of Chapter 2 in McMahon (and call these exercises 4a, 4b etc.).

5. Do exercises 1-3 at the end of Chapter 3 in McMahon (and call these exercises 5a, 5b and 5c).

6. **Plotting**

   (a) Plot in one figure the sin function between 0 and π in four different ways: with 10, 20, 40 and 80 points. Plot both the points and the lines. Use different colors for the lines and different symbols (from now on give all plots a title that corresponds to number of the exercises).

   (b) Repeat this exercise by plotting each line in a different plot using the `subplot` command. Don't forget the title.

7. **Plotting of a seismic velocity model.**

   (a) In the first exercise set we introduced a simple linear seismic velocity function $v = az + b$. Compute the values of this function at 51 values between 0 meter and 1000 meter and plot the velocity as a function of depth (use $a = 0.25$ and $b = 1900$). Give the correct labels to the x-axis and y-axis. Also give the figure a title.

   (b) The model in the previous exercise is very smooth. Often models in seismics/seismology have discontinuities. Assume that the velocities contain a jump at each multiple of 20 meter and are constant everywhere else. Plot the resulting model. A useful Matlab command for this exercise is `stairs`.

   (c) Repeat the previous exercises for the five main layers of the whole Earth (estimate very roughly the average P- and S-velocities in these layers by using Figure 8.1 in Fowler).

   (d) Plot a histogram of the volumes of the 5 layers (look up how to plot histograms using `help hist` or `help bar`). So you will get a plot with five bars. Each bar has a length that is proportional to the volume of the corresponding layer.

8. **Polar coordinates**

   (a) If you are given the cartesian coordinates (x, y) of a point in 2D, then explain how the polar coordinates of this point can be computed.

   (b) If you have the polar coordinates (r, ϕ) of a point, then explain how the cartesian coordinates can be computed.

(c) (Note: In this exercise and 8g you should NOT use Matlab.) $a = (0, 4)$, $b = (-2, -1)$, $c = (5, -5)$. Give the polar coordinates of each of these vectors.

(d) Matlab has its own commands to compute polar coordinates from cartesian coordinates and vice versa. Find these commands and use them to compute the polar coordinates of a, b and c.

(e) Use Matlab to compute the lengths (not the number of elements but the length of the line segement representing the vector) of the vectors a, b and c.

(f) Use Matlab to compute the inner products (a, b), (a, c) and (b, c).

(g) Check your answer of the two previous exercises by computing the lengths and inner products yourself.

(h) Use Matlab and the dot product to compute the angle between a and b.

(i) Do the same for the angle between a and c.

(j) Do the same for the angle between b and c.

(k) Compute the cartesian coordinates of 5 points on the unit circle (the unit circle is the circle centred at the origin with radius equal to one). You may choose these 5 points yourself.

(l) Plot the unit circle in the same figure as the previous exercise (hint: use `linspace` to create an array of angles, then use this array to compute points on the circle in cartesian coordinates).

(m) Use polar coordinates to make a plot of 2 other circles centred at the origin, but now with radii 0.5 and 3. Again use the same figure.

(n) Plot a circle with radius 3 and centred at (2,2) and another circle with radius 1.5 centred at (0,3) in the same figure.

9. **Matrices in Matlab**

   (a) Use the vectors in exercise 2b to create a 2 by 3 matrix. Call this matrix A. Also create the matrix B in McMahon on page 30.

   (b) Compute the sum of the two matrices.

   (c) Apply component wise multiplication to the two matrices.

   (d) Divide the elements of matrix A by the elements of matrix B.

   (e) Divide the elements of matrix B by the elements of matrix A.

   (f) Compute the transpose of matrix A.

   (g) Compute the size of matrix B.

   (h) Add 10π to matrix B and call this matrix C.

   (i) Subtract 6 times matrix B from C.

10. Do exercises 5-9 at the end of Chapter 2 in McMahon.

11. **Numerical Integration and Differentiation.** We now know how to create and plot functions in Matlab. The next step is to apply the basic operations of calculus (integration and differentiation) to these functions. This is what we will do in this exercise. In Matlab (and any other programming language) a function is only defined at a number of discrete points. In this case integration and differentiation are called 'numerical integration' and 'numerical differentiation'. Integration using the rules of calculus is often called 'analytical integration'. Likewise, differentiation in calculus is often likewise called 'analytical differentation'. We will also use these terms in this course.

(a) What is the geometrical interpretation of the integral $\int_a^b f(x)dx$ of a function f over an interval $[a, b]$?

(b) Compute $\int_0^{2\pi} \sin(x)dx$.

(c) Apply numerical integration to the arrays created in exercise 6a.

(d) Compare the analytical and numerical integrations and explain the differences.

(e) What is the geometrical interpretation of the derivative $f'(x)$ of a function f at a point x?

(f) What is the derivative of the sin function?

(g) Compute the numerical derivatives of the arrays from exercise 6a.

(h) Plot the numerical derivatives of the previous exercise.

(i) Compare the numerical derivatives with the analytical derivative and explain your result.