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Invited Review

Optimization for dynamic ride-sharing: A review

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ABSTRACT

Dynamic ride-share systems aim to bring together travelers with similar itineraries and time schedules on short-notice. These systems may provide significant societal and environmental benefits by reducing the number of cars used for personal travel and improving the utilization of available seat capacity. Effective and efficient optimization technology that matches drivers and riders in real-time is one of the necessary components for a successful dynamic ride-share system. We systematically outline the optimization challenges that arise when developing technology to support ride-sharing and survey the related operations research models in the academic literature. We hope that this paper will encourage more research by the transportation science and logistics community in this exciting, emerging area of public transportation.

1. Introduction

Finite oil supplies, rising gas prices, traffic congestion, and environmental concerns have recently increased the interest in services that allow people to use personal automobiles more wisely. The demand for ride-sharing services, which aim to bring together travelers with similar itineraries and time schedules, has increased sharply in recent years (Saranow, 2006). Ride-share providers across the globe are offering online notice boards for potential carpoolers, whether for daily commutes or for one-time trips to festivals, concerts, or sports events. Some online services, such as Nuride, provide incentives like restaurant coupons, gift certificates, or retail sales discounts to participants. Ride-sharing has generated much interest, and recent media coverage can be found in the Wall Street Journal (Saranow, 2006), Newsweek (Levy, 2007), Business Week (Walters, 2007), ABC News (Bell, 2007), The NY Times (Wiedenkeller, 2008), among many others.

Private car occupancy rates (the number of travelers per vehicle trip) are relatively low; average car occupancies in Europe range from 1.8 for leisure trips to 1.1 for commuters (EEA, 2005). Similar occupancy rates are also found in the US (Santos et al., 2011). The large demand for automobile transportation at peak-hours together with low occupancies leads to traffic congestion in many urban areas. The annual cost of congestion in the US in terms of lost hours and wasted fuel was estimated to be $78 billion in 2007 (Schrank and Lomax, 2007). Private automobile usage is also the dominant transportation mode producing carbon dioxide emissions (Hensher, 2008). Vehicle emissions give rise to problems both on a local and global scale. Locally, the health effects of air pollution represent a serious problem in many of the most densely populated regions worldwide (Brunekreef and Holgate, 2002; Kunzli et al., 2000). Globally, carbon dioxide emissions are associated with climate change and global warming.

Effective usage of empty car seats by ride-sharing may represent an important opportunity to increase occupancy rates, and could substantially increase the efficiency of urban transportation systems, potentially reducing traffic congestion, fuel consumption, and pollution. Moreover, ride-sharing allows users to share car-related expenses, which can be substantial. While ride-sharing is not a new idea, recent technological advances should increase its popularity, as we will explain. Certainly, ride-sharing must be easy, safe, flexible, efficient and economical before it will be adopted more widely.

To broaden its appeal, ride-sharing must be able to compete with one of the greatest advantages of private car usage: immediate access to door-to-door transportation. Technological advances, both hardware and software, are key enablers. In the US, the number of smartphone subscribers using the mobile Internet has grown 45 percent since 2010 (Nielsen, 2011), with approximately 40% of the mobile subscribers using a smartphone in 2011 (Smith, 2011). Similar estimates apply to the leading European markets (comScore, 2011). The growing ubiquity of Internet-enabled mobile devices partially enables practical *dynamic* ride-sharing (Hartwig and Buchmann, 2007; Hartmann, 2008).

By dynamic ride-sharing, we refer to a system where an automated system made available by a ride-share provider matches up drivers and riders on very short notice or even en-route. Recent

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startups like Carticipate, EnergeticX, Avego, and Flinc offer dynamic ride-sharing applications that allow drivers with spare seats to connect to people wanting to share a ride. They provide applications that run on (location-aware) Internet-enabled mobile phones. To ease the fear of sharing a ride with a potential stranger, these services use reputation systems (see *e.g.*, PickupPal) or can be linked with social network tools like Facebook (see *e.g.*, GoLoco and Zimride).

The ability of a dynamic ride-share provider to successfully establish ride-shares on short notice depends on the characteristics of the environment in terms of participant geographic density, traffic patterns, and the available roadway and transit infrastructure. Hall and Qureshi (1997) analyze the likelihood that a person will be successful in finding a ride-match, given a pool size of potential ride matches. Using probabilistic analysis, they conclude that in theory ride-sharing is viable since a congested freeway corridor should offer sufficient potential ride-matches. The authors also observe that there are many obstacles, primarily in terms of communication, so that the chance of finding a ride match in practice may in fact be small. Fortunately, technological advances have greatly reduced this communication obstacle.

Although the enabling technology is available, ride-sharing success stories are still in short supply. The development of algorithmic approaches for optimally matching drivers and riders in real-time may only play a small role in the ultimate success of ride-sharing, but it is central to the concept. Therefore, we believe that the time is right for a systematic overview of the issues in ride-sharing and the relevant optimization models that support the matching of riders and drivers in real-time. Since the operations research community has only recently started to address the related optimization challenges, we not only focus on the literature that specifically considers ride-share optimization but also on optimization approaches in other areas of transportation that share similar features. By introducing and formally defining dynamic ride-sharing problems, and by illustrating and outlining the optimization challenges that arise when developing technology to support ride-sharing, we hope to encourage more research by the transportation science and logistics community in this exciting, emerging area of public transportation.

The remainder of the paper is structured as follows. In Section 2, we explain and characterize the dynamic ride-sharing concept and introduce several relevant planning issues that arise in this context. In Section 3, we present a more formal definition of the basic ride-sharing problem and its various variants and survey the available literature. In Section 4 we discuss dynamic ride-sharing problems. In Section 5, we present the multi-modal version of the ride-sharing problem. Finally, in Section 6, we summarize our main insights and discuss directions for future research. Throughout the paper we also provide an overview of related problems in passenger and freight transportation optimization.

2. Problem characteristics

2.1. Features of dynamic ride-sharing

In this paper, we use the term dynamic ride-sharing to describe an automated system that facilitates drivers and riders to share one-time trips close to their desired departure times. The concept is also known as real-time ride-sharing, ad hoc ride-sharing, and instant ride-sharing. We characterize this concept by the following features:

- **Dynamic** The ride-share can be established on short-notice, which can range from a few minutes to a few hours before departure time. The growing use of Internet-enabled mobile phones allows people to offer and request trips whenever they want, wherever they are. Thus, communication technology is a key enabler to dynamic, on-demand ride-sharing.
- **Independent** The drivers which provide the rides are independent private entities. This is different from most traditional forms of passenger transportation where a central organization owns vehicles and/or employs drivers.
- **Cost-sharing** The variable trip-related costs are reallocated among the ride-share participants in a way that makes it beneficial for them to participate from the perspective of cost reduction. The variable trip cost minimally includes fuel expense, but may also take into account wear and tear on vehicles, parking costs or road fees such as tolls.
- **Non-recurring trips** Dynamic ride-sharing focuses on single, non-recurring trips. This distinguishes it from traditional carpooling or vanpooling, both of which require a long-term commitment among two or more people to travel together on recurring trips for a particular purpose, often for traveling to work. Single-trip ride-sharing is more flexible because it does not require rigid time schedules or itineraries over time.
- **Prearranged** The trips are prearranged which means that the participants agree to share a ride in advance, typically while they are not yet at the same location. This is different from the spontaneous, so-called casual ride-sharing (see *e.g.*, Kelley, 2007) in which riders and drivers establish a ride-share on the spot, similar to hitch-hiking or hailing a taxi on the side of the street. In casual ride-sharing, drivers and riders line up at established locations to share rides to other established locations to take advantage of high occupancy vehicle lane time-savings or toll savings. The main limitation of casual ride-sharing is the inflexibility of its routes, which does not allow door-to-door transportation.
- **Automated matching** To establish ride-shares in a way that requires minimal effort from the participants, ride matching should be automated in a dynamic setting. This means that a system helps riders and drivers to find suitable matches and facilitates the communication between participants. We do not include in our definition simple (online) notice boards where riders and drivers can post desired or planned trips and choose to contact potential ride-share partners themselves.

Note that ride-sharing is different from *car-sharing*, a service where people rent cars for short periods of time, often by the hour. Similar to ride-sharing, it aims reduce car usage and increase mobility. Car-sharing gives people self-serve access to a network of vehicles stationed around the transportation network. The optimization challenges that arise are quite different from those that arise with ride-sharing and involve,among others, depot location, fleet assignment, and vehicle redistribution to correct for short-term demand imbalances between stations (see e.g. Correia and Antunes, 2012; Nair and Miller-Hooks, 2010; Kek et al., 2009).

2.2. The ride-share process

To facilitate a discussion on the planning issues in dynamic ride-sharing, we briefly explain the process of Avego, an Ireland-based software company that currently offers a dynamic ride-share application for Internet-enabled mobile phones. The service that they offer is quite generic and similar to that of other existing ride-share providers such as Carticipate, Piggyback, and EnergeticX.

With the Avego Shared Transport software application, users can offer a ride as a *driver* or a request for transportation as a *rider*. To facilitate easy trip specification, the application lets users store and select pre-defined locations such as home, work, and the grocery store. With a GPS-enabled phone, users can set their current location as the origin of their trip, even en-route. If a ride-share match is established, Avego proposes the arrangement to the participants. If the driver and the rider agree on the proposed arrangement, the driver picks up the rider at the agreed time and location. Avego sends the driver the rider's photo and personal identification number, which allows him to verify the rider's identity.

Avego will guide the driver to an appropriate pickup location and from thereon to the rider's destination via the incorporated navigation system. When the driver is in range for the pickup, the application will notify the rider in real-time. Avego automatically assesses a trip fee to the rider, of which the company receives a fixed percentage.

2.3. Ride-sharing system objectives

Ride-sharing allows participants with cars to save on travel-related expenses by sharing trip costs and enhance the mobility of system users without cars at their disposal. A ride-share provider, either private or public, helps people to establish ride-shares on short-notice by automatically matching up drivers and riders. If the system is private and operated for profit, the provider may generate revenues by commissions or advertisement. For example, a ride-share provider may charge a commission per successful ride-share, either a fixed fee or proportional to the trip cost. Public systems may have a societal objective, such as the reduction of pollution and congestion. The objectives of the ride-share provider and ride-share users are mostly in line. Given these overall system objectives, most studies on ride-share consider one (or a combination) of the following specific objectives when determining ride-share matches:

- *Minimize system-wide vehicle-miles (M).* The system-wide vehicle-miles represent the total vehicle-miles driven by all participants traveling to their destinations, either in a ride-share or driving alone. This objective is important from a societal point of view since it helps to reduce pollution (emissions) and congestion. This objective is also compatible with *minimizing total travel costs*, which is an important consideration for the participating drivers and riders and the revenues of the ride-share provider.
- *Minimize the system-wide travel time (T).* The travel time is the time spent in the vehicle while actually traveling between origin and destination. From a societal perspective, this is an important measure since vehicle emissions not only relate to vehicle-miles but also to vehicle speeds. Obviously, time is also an important convenience consideration for the participants.
- *Maximize the number of participants (P).* This objective maximizes the number of satisfied drivers and riders in the system. This objective may be beneficial for a private ride-share provider whose revenues are linked to the number of successful ride-share arrangements. Moreover, the matching success-rate may also be an important performance indicator for users of a particular ride-share service, and a high success rate may spur larger participant pools in the future.

2.4. Constraints on matches

When determining matches between drivers and riders in a ride-share system, a number of constraints on the feasibility of matches must be observed. The timing of rides is probably the most important consideration since time tends to be a more constraining factor than the availability of spare seats.

Both riders and drivers must provide information on their time schedule preferences. Many of the currently available and proposed dynamic ride-share applications simply let each potential participant specify a desired departure time. The provider then attempts to find an assignment with a departure time that is as close as possible to this desired departure time. This approach minimizes the information that participants must supply, but, at the same time, provides only limited information regarding a participant's time preferences and flexibility. Therefore, most studies capture a participant's time preferences by a time window representation. In Agatz et al. (2011), for example, we let a participant specify an earliest possible departure time and latest possible arrival time (see Fig. 1). Baldacci et al. (2004) and Amey (2011) also allow limits on the actual time that users spend traveling on a given trip. That is, they allow participants to specify the maximum excess travel time (over the direct travel time for their origin to destination) they are willing to accept.

In addition to time, there are other important feasibility considerations that determine whether a particular ride-share match is one that the participants would accept. For example, female participants may not feel safe sharing a ride alone with a male stranger (Levin et al., 1977), while smoking may be another critical issue (Ghoseiri et al., 2011). The user may only feel comfortable sharing a ride with certain groups of people, where the group preferences may be motivated by personal safety or social considerations. For example, one may not be willing to share a ride with a complete stranger and may only want to share rides with friends and colleagues. Of course, the more restrictions a potential user places on his pool of potential ride-share partners, the more difficult it will be to find successful matches for that user (Dailey et al., 1999).

Lastly, ride-share users may choose to participate primarily to reduce their travel costs. Therefore, it is probably also necessary to include constraints that restrict feasible matches to those that reduce the travel costs of each ride-share participant. Of course, determining whether or not a user reduces his costs via a ride-share depends on how costs are shared in such systems, which is the subject of the next section.

2.5. Cost considerations

Some people may choose to participate in ride-sharing primarily for potential cost-savings; trip-related expenses, such as fuel and tolls, are shared. For others the social and environmental benefits of ride-sharing may be reason enough to participate (they may even be willing to accept a small increase in their trip costs). For riders without a car at their disposal, ride-sharing may provide a (convenient) transit option that would otherwise not be available to them.

The literature on ride-sharing typically considers variable travel costs that are roughly proportional to vehicle-miles. If the cost of ride-share trip is less than the sum of the costs of individual trips of its participants, it is always feasible to allocate the cost savings among the participants such that each individual receives cost savings. Note that a complete ride-share trip should be defined as all travel required to move each participant from his origin to his destination. For example, in the case where a driver shares a ride with a single rider this would include travel from the driver's origin to the rider's origin, then onto to the rider's destination, and finally onto the driver's destination.

Each driver can reduce his trip cost by receiving compensation that is greater than the marginal cost required to accommodate the

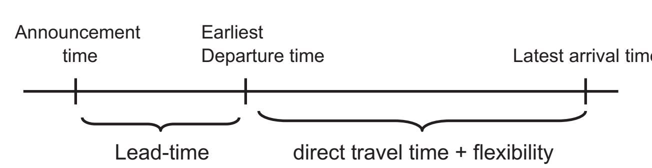

Fig. 1. Time schedule information in ride-sharing.

rider(s), *i.e.*, the marginal travel cost required by detours necessary to serve the riders. For the ride-share to be beneficial for a rider, the compensation he pays to the driver(s) should be lower than the cost of alternative means of transit, *e.g.*, driving themselves with their own car, using public transport or taking a taxi.

There are various ways to divide the trip costs between the ride-share partners. Geisberger et al. (2010) suggest to divide the cost of the shared part of the trip evenly between the ride-share partners. In Agatz et al. (2011) we propose a way to allocate the costs of the joint trip that is proportional to the distances of the separate trips. Kleiner et al. (2011) propose an auction-based mechanism to determine the driver's compensation. This approach takes into account the different ride valuations of individual riders. For each rider, they simulate an individual willingness to pay per mile that lies between the cost of driving alone in a private car and the cost of taking a taxi. In this environment, higher driver compensations correspond with more ride-share matches because drivers accept longer detours.

The participants of a traditional carpool for recurring trips to work typically do not share costs on a per trip basis but take turns driving. If the composition of the group of participants differs per trip, it is not trivial to establish a *fair* driver schedule (see *e.g.*, Fagin and Williams (1983); Ajtai et al. (1998) and Naor, 2005).

2.6. System vs. user benefits

It is important to recognize that a system-wide optimal solution aimed at minimizing the external societal costs may not necessarily optimize the cost-savings of all individual ride-share participants. Consider a system with two drivers d_1 and d_2, two riders r_1 and r_2, and locations and distances given in Fig. 2. Each driver can accommodate a single rider. When minimizing system-wide vehicle-miles, the optimal solution has value 20 and matches d_1 with r_1 and d_2 with r_2; represented by bold paths in Fig. 2. When costs are allocated proportionally, each driver and rider pays the costs of 5 miles, *i.e.*, $\frac{6}{12}$ of the joint trip length of 10 miles. However, driver d_1 and rider r_2 could reduce their trip costs even more by establishing a ride-share on their own, since the joint trip length would be 9 miles of which each would pay for 4.5 miles. In this case, driver d_2 and rider r_1 would be without a ride-share since their joint trip length would be 17 miles. In the terminology of cooperative game theory, the system-wide solution is not stable. This is an example of the *price of anarchy*, the degradation of system-wide performance if participants act selfishly (Koutsoupias and Papadimitriou, 1999). While the literature on mitigating the price of anarchy is vast, see for example (Roughgarden, 2005), we are not aware of any papers that specifically address the issue in the context of ride-sharing.

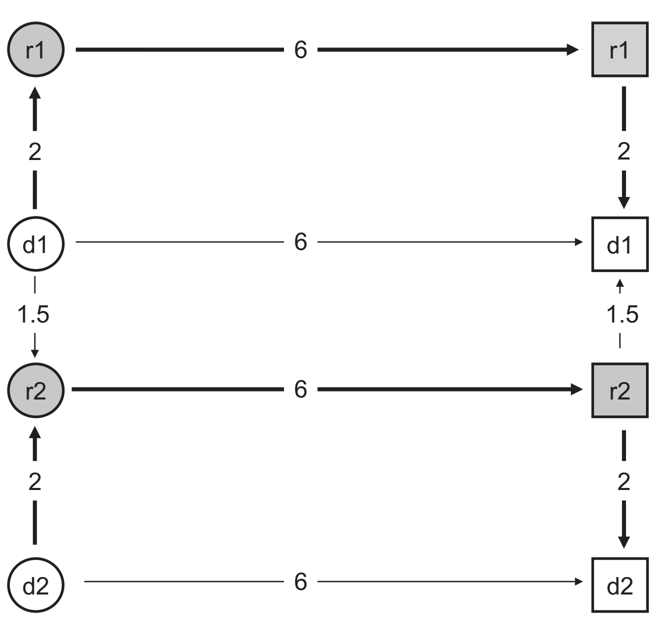

Fig. 2. Riders (grey) and drivers (white) traveling from origin (circle) to destination (square).

Table 1
Problem characteristics considered in the optimization literature.

Reference	Objective	Constraints	Dynamics	Riders/trip
Baldacci et al. (2004)	M, P	Time	–	Multiple
Calvo et al. (2004)	M, P	Time	–	Multiple
Winter and Nittel (2006)	P, T	Time	Yes	One
Xing et al. (2009)	P	Time, personal	Yes	One
Agatz et al. (2011)	M	Time	Yes	One
Amey (2011)	M	Time	–	One
Ghoseiri et al. (2011)	P	Time, personal	Yes	Multiple
Kleiner et al. (2011)	M, P	Time, personal	Yes	One

M = minimizing vehicle miles, P = maximizing the number of participants T = minimizing system travel time.

To conclude this section, we present in Table 1 a brief overview of the problem characteristics considered in the papers that deal with optimization aspects of ride-share problems.

3. Basic ride-sharing problems

In this section, we describe optimization models that can be used to address these matching problems. We limit our attention here to what we will denote as static ride-sharing variants, where it is assumed that all driver and rider requests are known in advance prior to the execution of a matching process. In Section 4, this restriction will be relaxed as we examine the more relevant problems of dynamic matching.

Drivers offering a ride may want to take a *single* rider or may be willing to take *multiple* riders. Similarly, riders requesting a ride may want a ride with a *single* driver or may be willing ride with *multiple* drivers and transfer from one to another en route to their destinations. Thus, we can distinguish four basic ride-sharing system variants as shown in Table 2.

As will be shown shortly, optimally matching drivers offering a ride and riders requesting a ride is easy for the static variant in which a single driver takes along a single rider. This variant is easy because there are a polynomial number of potential matches, and determining the optimal route sequence for a given potential match is simple. In all other variants, determining the best route sequence for a given match, which may involve multiple drivers and riders, can be more complicated.

To be able to properly discuss the ride-share problems, we start by introducing some notation.

We are given a set of locations P and travel time t_{ij} and travel distance d_{ij} between each pair of locations $i, j \in P$. Furthermore, we are given a set of drivers D and a set of riders R. Each driver $d \in D$ (rider $r \in R$) wants to travel from his origin $v(d) \in P$ ($v(r) \in P$) to his destination $w(d) \in P$ ($w(r) \in P$). Each driver $d \in D$ (rider $r \in R$) has an earliest time $e(d)$ ($e(r)$) at which he can depart from his origin $v(d)$ ($v(r)$) and a latest time $l(d)$ ($l(r)$) at which he can arrive at his destination $w(d)$ ($w(r)$). Each driver d has $q(d)$ spare seats available.

Table 2
Ride-Share Variants

	Single Rider	Multiple Riders
Single Driver	Matching of pairs of drivers and riders	Routing of drivers to pickup and deliver riders
Multiple Drivers	Routing of riders to transfer between drivers	Routing of riders and drivers

3.1. Single rider, single driver arrangements

If a driver would like to share a ride with at most a single rider, then at most one pickup and delivery take place during his trip. Thus, if driver d and rider r are matched, then their joint trip length is $d_{v(d),v(r)} + d_{v(r),w(r)} + d_{w(r),w(d)}$. By comparing the vehicle-miles of the joint trip with the two separate trips, we can easily calculate the potential savings for each driver-rider match (see Fig. 3).

If we want to match drivers and riders in the system in a way that minimizes the total system-wide vehicle-miles, the driver-rider match optimization problem can be represented as a maximum-weight bipartite matching problem (also known as the assignment problem). The bipartite graph consists of two disjoint sets of vertices, a set representing drivers D and a set representing riders R. An edge between a driver and a rider exists if the match is feasible, with a weight that represents the positive savings in distance when traveling together compared to when each of them drives separately. More formally, a constraint on positive cost savings implies a necessary condition for the feasibility of a match between driver d and rider r: only if $d_{v(d),w(d)} + d_{v(r),w(r)} - (d_{v(d),v(r)} + d_{v(r),w(r)} + d_{w(r),w(d)}) > 0$. Moreover, matches must also be time feasible, where both the rider's and the driver's travel windows are respected. The assignment problem has been studied extensively in the literature with algorithmic approaches abound (see Pentico (2007) for a review).

Amey (2011) studies the ride-share potential at the MIT campus in Cambridge, Massachusetts. Given the home locations and time schedules of the faculty and staff, the author identifies potential ride-share arrangements in which two commuters share a trip together. The ride-share match optimization in this case must not only decide on the assignment of riders to drivers but also assign a role to each of the participants. It is therefore not possible to model this problem using bipartite matching. The author formulates the problem as a general network flow problem with side constraints to ensure that a commuter was not matched up as both a driver and a rider in separate ride-share arrangements. The study indicates a potential reduction of system vehicle miles of between 9% and 27%, depending on the maximum acceptable driver detour. Note that when a single driver travels with at most a single rider and riders do not transfer, the total system-wide vehicle-miles traveled by participants can be reduced by no more than 50%. The reason for this is that the length of the joint trip cannot be smaller than the larger of the individual trips of the ride-share partners. Again when transfers are not allowed and a driver can ride with at most $q(d)$ passengers, we can save at most $1/q(d)$ of the system-wide vehicle-miles by ride-sharing.

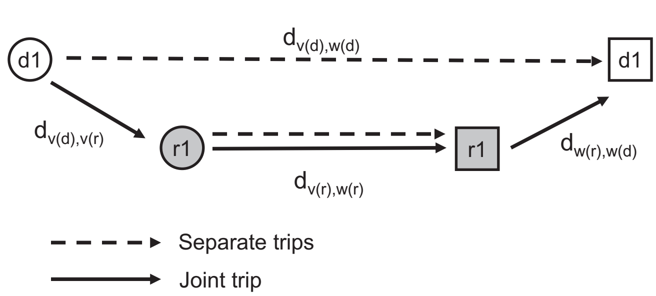

Fig. 3. Example of single rider, single driver ride-share arrangement.

As an additional note, we point out that it is likely that the vast majority of ride-share participants will need to plan round trips. In a dynamic ride-share system with a sufficient amount of capacity, the rider should be able to arrange the trips separately shortly before departure. However, some riders may not feel comfortable going to certain destinations without a guarantee that they will be able to find a ride back (for example, because the alternatives may be very costly). The need for round trip planning may necessitate that systems allow riders to place two transportation requests at the same time. These two request are directly linked if the rider's alternative means of transportation is his own car. In this case, a rider may only want to commit to a ride-share if both legs are covered. The return trip does not necessarily have to be conducted by the same driver that provided the departing trip.

3.2. Single driver, multiple rider arrangements

If drivers have sufficient time flexibility, they may be willing to provide rides to several riders on a trip, either one after the other or simultaneously for portions of the time. The pickup and drop-off of multiple riders in a single trip gives rise to more complex routing decisions.

The carpool problem is a special case of this ride-sharing variant. In the carpool problem workers, partitioned into riders and drivers, want to go to their common work location from their homes. The objective is to assign riders to drivers and construct feasible routes for drivers to minimize the travel costs plus a penalty associated with unserved riders. Each worker has an earliest time he can leave home and a latest time he can arrive at work. Furthermore, each driver has a maximum time he is willing to spend driving from home to work.

Baldacci et al. (2004) address the *to-work* variant of the carpool problem separately from the *return-from-work* variant. They propose both an exact and heuristic method to solve the problem based on two integer programming formulations. They solve several instances, some based on real-world data, with the number of workers ranging from 50 to 250. Calvo et al. (2004) study the problem using a model that allows different network travel times at different times of the day. They develop a heuristic approach to solve the problem based on construction and local search. In a computational study using real-life carpool data, the authors investigate the impact of varying the ratio of drivers to riders and show that the total system-wide travel time increases with the ratio between driver and riders.

The carpool problem is a special case of the so-called pickup and delivery problem, which has been studied extensively in the operations research literature; see *e.g.*, Savelsbergh and Sol (1995). These problems involve the construction of vehicle routes and schedules to satisfy transportation requests between origins and destinations. A fleet of vehicles with a given capacity is available to operate the routes, typically based at one or more depot locations. It is also usually assumed that the pickup and delivery of each individual request is made by one vehicle.

The dial-a-ride problem is a special case of the pickup and delivery problem that focuses on the transportation of passengers (Cordeau and Laporte, 2007; Berbeglia et al., 2007). Consequently, passenger convenience considerations become important. Passenger service quality may be measured, for example, in terms of the ratio of actual drive time and direct drive time, the waiting time, the number of stops while on board, and the difference between actual and desired delivery times (Paquette et al., 2009). These criteria may be treated as constraints or may be incorporated into the objective function.

Dynamic ride-sharing differs from conventional on-demand transportation primarily with regards to the supply of drivers and vehicles. Instead of being employed by a company, drivers in a ride-sharing system are private independent entities. Like riders, they arise dynamically over time at various locations in a process that may be difficult to predict with certainty. Since they are independent, they are not obligated to accept ride-share arrangements that they do not like. Therefore, driver preferences need to be accounted for when matching drivers and riders in a ride-sharing system. Driver preferences may include a maximum deviation from the direct trip duration, a maximum number of simultaneous riders, and a maximum number of stops.

Another important difference between a dial-a-ride system and a ride-share system is that in a dial-a-ride system all vehicles typically operate out of one or more depot locations, whereas in a ride-share system each driver may have a unique origin and destination. This implies that in a ride-share system, routing decisions are represented and evaluated as deviations from a driver's direct path from origin to destination. Deviations from a given path are also at the heart of a Mobility Allowance Shuttle Transport (MAST) service, in which a vehicle has a predefined route but is allowed to deviate from this route to pick-up and drop-off passengers at preferred locations within a certain service area (Quadrifoglio et al., 2008; Zhao and Dessouky, 2008). In addition, customers who board the vehicle at a scheduled stop can request a drop off location that is within half a mile from the predefined route (Zhao and Dessouky, 2008). The MAST concept aims at combining the flexibility and convenience of on-demand transportation with the cost-efficiency of fixed route transit. Los Angeles County operates a MAST during the night hours. Passengers located within half a mile off the route may call-in for pick-ups at off-route locations.

3.3. Single rider, multiple driver arrangements

If we allow riders to transfer between drivers, a rider may travel with more than one driver to reach his final destination. Gruebele (2008) describes such a multi-hop ride-share system in detail. Potential transfer points could include public transport stops, shopping malls, or park-and-ride lots. Herbawi and Weber (2011) consider a version of the multi-hop ride-share problem in which drivers do not deviate from their routes and time schedules. As such, the drivers' ride-share offers form the transportation network for the rider, who has to find a route that minimizes costs, time and number of transfers. The authors model this problem as a multi-objective shortest path problem on a time-expanded graph representing the drivers' offers. They present an evolutionary solution approach to solve the problem and show that this approach is able to provide good quality solutions in reasonable running times.

The multi-hop ride-sharing problem is a lot more difficult when also considering the routing of the drivers. We are not aware of any dedicated modeling efforts on this topic in ride-sharing. In a related dial-a-ride environment, Cortes et al. (2010) extend the standard pickup and delivery problem formulation to facilitate passenger transfers from one vehicle to another. The formulation allows the specification of one or more potential transfer points and a maximum passenger waiting times at these points. The authors present an exact solution method and shown its effectiveness on small instances with one transfer point, 2 vehicles and up to 6 requests. Furthermore, they show that allowing transfers between vehicles can be beneficial in some settings.

An interesting area of research related to multi-hop passenger transportation systems also concerns system design. Rather than focusing only on effectively routing passengers through a given network with transfer points, another important focus could be on where to locate the transfer locations. There is a huge area of research on this topic in freight transportation and airline passenger networks, where transhipment points are typically called hubs (see Alumur and Kara (2008) and Campbell (2005) for a comprehensive review).

4. Dynamic ride-sharing problems

In any practical dynamic ride-share implementation, new riders and drivers continuously enter and leave the system. A driver enters the system by announcing a planned trip and offering a ride, while a rider enters the system by announcing a planned trip and requesting a ride. Drivers and riders leave the system when a ride-share arrangement has been planned and accepted, or when their planned trips "expire," *i.e.*, when the latest possible departure time of a planned trip occurs before a successful arrangement can be found.

To avoid one potential worry for potential participants, it may be better to have each participant specify a trip expiration time in addition to (and which may be earlier than) his latest possible departure time. If the announcement time of a trip for driver d (rider r) is denoted $a(d)$ $(a(r))$ and the expiration time of the trip by $b(d)$ $(b(r))$, then the window for matching driver d (rider r) is $[a(d), b(d)]$ $([a(r), b(r)])$. Due to these matching windows, a match between driver d and rider r (assuming they have otherwise compatible trips) can be established only in the time interval $[\max\{a(d), a(r), \min\{b(d), b(r)]$.

4.1. Arrival of riders and drivers

Since new drivers and riders continuously arrive, not all relevant offers and requests may be known at the time the ride-share provider executes an algorithm for planning ride-sharing arrangements. In recent years, several authors have addressed this issue in dynamic ride-sharing.

In Agatz et al. (2011), we deal with this planning uncertainty by using a rolling horizon solution approach. In this approach, the optimization problem to be solved includes all of the offered rides (drivers) and requested rides (riders) that are known at the time of execution and that have not yet been matched. We evaluate different re-optimization frequencies and run the algorithm for finding ride-share arrangements each time a new request arrives or at fixed time intervals. Moreover, we experiment with different commitment strategies, *i.e.*, immediately notify drivers and riders of the matches identified by the optimization, or wait before notifying so as to find improved matches at the next execution time. We observe that systems that employ the latest commitment strategy for matches should be optimized more frequently. However, in a system that immediately commits matches, we observe that there are advantages of optimizing less frequently since it allows the accumulation of more trip announcements between optimization runs. Moreover, the results indicate that sophisticated optimization methods outperform simple greedy matching rules in terms of the number of established rides-share matches and vehicle miles savings.

Several papers consider an agent-based system where autonomous rider and driver agents *locally* establish ride-shares. Winter and Nittel (2006) and Xing et al. (2009) consider such agent-based ride-share systems with the objective of maximizing the number of served riders. Winter and Nittel (2006) consider a setting in which wireless communication devices are used that only enable short-range communications (*e.g.*, Bluetooth or WiFi). They show that limiting the information dissemination between agents does not significantly impact the solution quality. Xing et al. (2009) consider a highly dynamic ride-share system where drivers and riders are matched *en-route*. The participants announce their trips (offers and requests) at their departure time, *i.e.*, $a(d) = e(d)(a(r) = e(r))$. They incorporate gender and smoking preferences and specify a maximum acceptable service response time for the riders. Riders may walk to a pickup-point to facilitate easy pickup by the driver. Simulation experiments on a real-life urban map of the Bremen metropolitan area show that the probability of a successful ride-share arrangement increases with the number of available drivers. The experiments also suggest that with sufficient drivers, dynamic ride-sharing may be an attractive alternative to public transportation in terms of travel time.

In a similar setting, Kleiner et al. (2011) apply a rolling horizon solution approach where arrangements are committed as late as possible given the time considerations. They present an auction-based solution mechanism that takes into account the individual preferences of the participants for ride-share partners and rides. The simulation experiments show that the auction-based approach provide close-to-optimal solutions to the ride-sharing problem.

The dynamics of the arrival of new rider requests and driver offers is not unique to ride-sharing. Various other passenger transit settings, such as a taxi service, share similar features. For excellent recent reviews on *dynamic* pick-up-and-delivery problems see Berbeglia et al. (2010) and Cordeau et al. (2007). A transportation request for an urban taxi typically arrives only a short time before the desired departure (Lee et al., 2004) and vehicle routes and schedules are updated each time a new transportation request arrives. The dynamic ride-sharing environment resembles an urban taxi environment in terms of the arrival process of transportation requests, *i.e.*, rides, but also has the added complexity of an arrival process of transportation resources, *i.e.*, drivers. That is, a urban taxi system tends to have better information about where and when individual resources will become available.

Note that it is often passenger convenience rather than physical capacity that keeps taxis from serving multiple passengers simultaneously. Horn (2002) demonstrate that allowing multiple passenger parties together in a single taxi trip may decrease system-wide vehicle miles but increase the individual travel times of the passengers. They present a dispatching software to manage a fleet of demand-responsive taxis taking into account both passenger service quality considerations and fleet efficiency considerations. The system assigns new travel requests to vehicles based on minimum cost criteria and then periodically applies improvement procedures. The author conducts a number of simulation studies based on data from a real-life taxi operator in Australia. The tests show that the software tool operates effectively in a fairly dynamic environment and realistic problems sizes.

Dial (1995) proposes an autonomous dial-a-ride taxi service that shares many similarities with dynamic ride-sharing. The fully automated system lets passengers reserve trips by phone or computer on short-notice. For routing and dispatching, the author suggests the use of a dynamic programming approach of Psaraftis (1980). The dynamic algorithm reoptimizes the not yet executed part of the tentative optimal route each time a new request appears. Since the algorithm can only solve very small instances the system only includes passenger requests that want to be served in the near future and runs the algorithm for each vehicle individually.

In the area of freight transportation, full truckload carriers have to manage fleets of vehicles (e.g. containers, trailers, boxcars) that serve one load at a time, with orders continuously arriving over time (for a review see Powell et al. (2007)). The problem of sequentially assigning transportation requests to vehicles is typically referred to as the dynamic assignment problem (Spivey and Powell, 2004) or the dynamic stacker crane problem (Berbeglia et al., 2010). Yang et al. (2004) consider the real-time multi-vehicle truckload pickup and delivery problem. In this problem, requests for truck-load moves arrive over time. Each request has a time window during which a pickup must take place. The authors have modeled the static problem as an integer linear program. To handle the dynamics, the static problem is solved repeatedly in a rolling-horizon framework. The authors compare three rolling-horizon strategies, as well as two reoptimization policies.

4.2. Anticipation of future requests

When ride-share systems have been in operation for a while, it is likely that some information about future unknown ride offers and ride requests may become available. If participants make a round trip in which they arrange the trips separately, then a departing trip may provide information about a potential returning trip in the future. Instead of myopically optimizing for the offered trips and requested trips that are known, it may be possible to incorporate information that partially describes the stochastic future into a modeling and solution approach in order to improve system-wide cost savings. Several papers have shown the potential of incorporation stochastic information in a dynamic planning environment (see *e.g.*, Bent and Van Hentenryck, 2004; Powell, 1996). In a dial-a-ride setting, Schilde et al. (2011) show that incorporating stochastic information on future return requests may lead to substantial system improvements. Ride-share participants may typically want to confirm matches as soon as possible, so "waiting" for yet-to-be-revealed requests may be undesirable. Therefore, based on the forecast, the system may want to communicate a likely ride-share arrangement early and then confirm to a specific match later, once more information is available. In the context of ride-share it may be quite challenging to establish an accurate forecast because the forecast should incorporate both space and time, and the flexibility is likely to be small.

4.3. Deviations from planned trips

Even if the participants agree on a specific ride-share arrangement, the identified arrangement may not be executed as planned because of no-shows, last-minute cancelations or delays. In case there are many such deviations from the agreed plan, other, more robust, solution methods may have to be considered. For example, Powell et al. (2004) study the deviation of recommended plans in a truckload-trucking setting. They conclude that even in a situation with a relatively small number of deviations, simple greedy solutions can outperform optimal solutions.

Reputation systems, which are commonly used to support online sales transactions (the online marketplace eBay uses perhaps one of the most widely known reputation systems), could help to establish trust among participants and encourage reliable system behavior. A ride-share reputation system could provide drivers and riders the opportunity to rate each other. In addition, the ride-share provider could rate participants by monitoring cancelations, no-shows, and late arrivals. Such ratings could be converted into a reliability score. The reliability and feedback scores could then be used in the matching optimization to favor matches involving participants with high scores.

5. The multi-modal ride-sharing problem

Instead of providing door-to-door transportation, the ride-share concept could be integrated with other modes of transportation, such as public transit. Ride-sharing may provide a very effective means to increase the use of a scheduled public transportation system if it can be used as a feeder service. In such a setting, a driver would first take a rider from the rider's origin to a public transport stop, then he would use public transit to get close to his destination, and finally he would walk or use another ride-share driver to travel from the transit stop to his destination. Aktalita, a project currently in development in Guadalajara, Mexico (www.aktalita.com) aims at developing such an integrated ride-share system.

To increase public transit usage, Liaw et al. (1996) and Lee et al. (2005) have proposed on-demand taxis to serve as a feeder for scheduled transit. Liaw et al. (1996) consider the integration of paratransit dial-a-ride vehicles with fixed-route buses, in a system where transportation bookings are made in advance. They show that the combination of on-demand vehicles and scheduled transit allows for an increase of the number of accommodated requests while at the same time decreasing the number of required taxis. Lee et al. (2005) consider the integration of dial-a-ride taxis with a metropolitan rapid transit line. They study a highly dynamic environment where new transportation requests continuously arrive and are to be assigned to taxis en-route. They propose a dispatch strategy and determine an optimal required fleet-size taking into account passenger waiting and travel time, number of satisfied requests, and system costs. Li and Quadrifoglio (2010) study the service performance of an on-demand taxi feeder system known as the demand responsive connecter. They demonstrate that the on-demand system outperforms a scheduled feeder system if customer densities are relatively low.

An effective integration of a ride-share system and a scheduled public transit system will potentially increase, in a sense, the coverage area of the public transit system, which has many societal and environmental benefits. However, the transfer from one mode of transportation to another must be seamless and efficient, and without long waiting times, before large numbers of people will make use of the integrated system. Consistent seamless and efficient mode transfers will only be possible with effective optimization technology.

6. Conclusions

New dynamic ride-sharing systems have the potential to provide huge societal and environmental benefits. The development of algorithms for optimally matching drivers and riders in real-time is at the heart of the ride-sharing concept. We have formally defined dynamic ride-sharing, have highlighted many of the interesting optimization challenges that arise when developing technology to support dynamic ride-sharing and have reviewed the relevant operations research models in this area. We have seen that there is a growing interest from the research community to address the optimization issues in dynamic ride-sharing but that the number of specific contributions to date is still small. We see room for contributions in all areas of dynamic ride-sharing. In particular, we see the following broad areas for future research: (1) fast optimization approaches for real-life instance sizes (2) incentive schemes to build critical mass and (3) optimization approaches that allow choice.

Optimization. It may be the case that realistic-size instances of the model cannot be solved fast enough to be of use in a matching engine of an actual, sustainable ride-share system. In a major metropolitan area, thousands of riders and drivers travel between thousands of origins and destinations during the same time periods, which leads to very large optimization problems that have to be solved very quickly as often as once every few minutes. Thus there is a clear need for fast solution approaches producing high-quality results.

A related area of research that should be of interest to the transportation science and logistics community is the design of de-centralized ride-share matching technology. Centralized ride-share matching may not be practical, or computationally feasible, for many larger metropolitan areas. In such cases, effective decomposition approaches will be necessary. A simple decomposition based on a geographic partition is likely to be challenging since driver and rider trips involve both an origin and a destination location, and these locations may often be separated by significant distances. The existence of trip requests for which the origin location is in one subregion and the destination location is in another subregion is therefore quite likely. It is also not clear whether a static partition of the region suffices or whether the partition of the region should be adjusted dynamically based on the set of driver and rider trip requests that need to be matched.

Incentives. The financial benefits of sharing trip-related expenses may motivate people to participate in ride-sharing. Rising fuel costs, pay-per-mile auto insurance and congestion pricing may further increase the cost of private car use in the future, and thus strengthen the advantages of ride-sharing (Huang et al., 2000; Ben-Akiva and Atherton, 1977). However, without a sufficient number of drivers and riders, the chance of finding a ride, especially one close to the desired departure time, may be very small, and thus the inconvenience may outweigh the financial benefits. To achieve the required density, *i.e.*, the number of necessary drivers and riders for a sustainable ride-share system, local governments and businesses may need to subsidize ride-share initiatives (Brownstone and F Golob, 1992). These subsidies can be used to reward ride-share participants, either on a per-trip basis or a per-offer basis. Subsidizing commercial urban taxis to act as drivers may also be advantageous especially in a start-up phase.

Subsidized ride-share systems may provide a relatively inexpensive way to increase the capacity and efficiency of the transportation system, a potentially interesting alternative to the capital investments required to build or expand the road network or expand public transportation.

Choice. A good understanding of participant behavior and participant preferences will be essential when designing a dynamic ride-sharing system. If ride-share matches do not satisfy participant preferences, the match may not be accepted, or the participant may not make use of the ride-share system in the future. Unfortunately, providing comprehensive preferences may be difficult and time-consuming for participants, partly because preferences may be interdependent and may change from one day to the next. For example, a driver's time flexibility may depend on the day of the week, the financial benefits, and the specific rider. Moreover, some participants may be hesitant or unwilling to disclose certain preferences for privacy reasons.

Rather than being notified of a specific single ride-share match, participants may prefer to choose from a menu of available ride-share options. However, the selection process must not take too much time. Minimally, then, the ride-share provider should present only the best options and only the most relevant information regarding these options, which may include the pickup and drop-off times, the travel time, the financial benefits, but also person specific information, such as gender, age, professional profile, and feedback and reliability scores. Providing a menu of ride-share options introduces various system synchronization issues. If driver trips (rider trips) may appear as an option for several riders (drivers) simultaneously, there is a chance that preferred options clash, *i.e.*, the same driver trip or the same rider trip is chosen multiple times. Designing a selection-based matching process is non-trivial and would likely pose interesting new additional challenges for the underlying matching optimization engine.

We believe these challenges provide great research opportunities for the transportation science and logistics community and we hope that the introduction and review that we provide in this paper leads to many future contributions in this exciting, emerging area of public transportation.

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