VAC Degeneracy :

Let $|\psi_k\rangle$ denote state vector at solar surface with basis $|e\rangle . |\mu\rangle$

We detect

$$\sum_k \sum_{(S\to E)_i} \sum_{(\text{zenith})_j} |\langle e| \hat{H}_j \hat{H}_i |\psi_k\rangle|^2 .$$

$$= \sum_\gamma \langle \psi_\gamma | \sum_\beta H_\beta^\dagger (\sum_\alpha N(\alpha,\beta,\gamma) H_\alpha^\dagger |e\rangle\langle e| H_\alpha) H_\beta |\psi_\gamma\rangle$$

If $|1\rangle$ degenerates from $|2\rangle$, i.e.

$$\langle 1|\psi\rangle = A e^{i\varphi} \langle 2|\psi\rangle,$$

in which φ is uniform in $[0, 2\pi)$.

$$\text{then detect} = \sum_{\alpha,\beta} \Big[(\langle e|H_{\alpha\beta}|1\rangle\langle 1|\psi\rangle + \langle e|H_{\alpha\beta}|2\rangle\langle 2|\psi\rangle) \times c.c. \Big]$$

$$= \sum_{\alpha,\beta} \Big[\underbrace{(\langle e|H_{\alpha\beta}|1\rangle e^{i\varphi} \langle 2|\psi\rangle + \langle e|H_{\alpha\beta}|2\rangle\langle 2|\psi\rangle) \times c.c.}_{} \Big].$$

$\Rightarrow$ for one α, β.

$$\sim = |\langle e|H_{\alpha\beta}|1\rangle\langle 1|\psi\rangle|^2 + |\langle e|H_{\alpha\beta}|2\rangle\langle 2|\psi\rangle|^2 .$$

$$+ e^{i\varphi} \langle e|H_{\alpha\beta}|1\rangle\langle 2|H_{\alpha\beta}|e\rangle |\langle\varphi|2\rangle|^2$$

$$+ e^{-i\varphi} \langle e|H_{\alpha\beta}|2\rangle\langle 1|H_{\alpha\beta}|e\rangle |\langle\varphi|2\rangle|^2 .$$

Sum up for φ, since $\int_0^{2\pi} a e^{i\varphi} = 0$. inter~~fer~~ence.