

Knapsack Problem $S = \{s_0, s_1, s_2, s_3\}$, $W = 7$

$v[] = \{1, 2, 3, 4\}$, $w[] = \{2, 3, 4, 5\}$

	0	1	2	3	4	5	6	7
0								
1								
2								
3								

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	0	1	2	3	4	5	6	7
0	{}	{}	{0}	{0}	{0}	{0}	{0}	{0}
1								
2								
3								

$A[i, j] = T \subseteq \{0, 1, \dots, i\}$ where $\sum_{t \in T} w_t \leq j$ and $\sum_{t \in T} v_t$ is maximal.

Calculate first row:

$$A[0, t] = \begin{cases} \emptyset & \text{if } t < w_0 \\ \{0\} & \text{if } t \geq w_0 \end{cases}$$

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	0	1	2	3	4	5	6	7
0	{}	{}	{0}	{0}	{0}	{0}	{0}	{0}
1	{}	{}	{0}	{1}	{1}	{0,1}		
2								
3								

Calculate value for cell A[1,5]

$$T_a = A[i-1, j], T_b = A[i-1, j-w_i] \cup \{i\}, B_a = \sum_{t \in T_a} v_t, B_b = \sum_{t \in T_b} v_t$$

$$A[i, j] = \begin{cases} T_a & \text{if } B_a \geq B_b \\ T_b & \text{otherwise.} \end{cases}$$

$$T_a = A[0, 5] = \{0\}, B_a = 1$$

$$T_b = A[0, 5-3] \cup \{1\} = A[0, 2] \cup \{1\} = \{0, 1\},$$

$$B_b = 1 + 2 = 3$$

$$A[1,5] = T_b = \{0,1\}$$

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	0	1	2	3	4	5	6	7
0	{}	{}	{0}	{0}	{0}	{0}	{0}	{0}
1	{}	{}	{0}	{1}	{1}	{0,1}	{0,1}	{0,1}
2	{}	{}	{0}	{1}	{2}	{0,1}	{0,2}	{1,2}
3								

Calculate value for cell A[2,7]

$$T_a = A[i-1, j], T_b = A[i-1, j-w_i] \cup \{i\}, B_a = \sum_{t \in T_a} v_t, B_b = \sum_{t \in T_b} v_t$$

$$A[i, j] = \begin{cases} T_a & \text{if } B_a \geq B_b \\ T_b & \text{otherwise.} \end{cases}$$

$$T_a = A[1, 7] = \{0, 1\}, B_a = 1 + 2 = 3$$

$$T_b = A[1, 7-4] \cup \{2\} = A[1, 3] \cup \{2\} = \{1, 2\},$$

$$B_b = 2 + 3 = 5$$

$$A[2, 7] = T_b = \{1, 2\}$$

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$v[] = \{1, 2, 3, 4\}$, $w[] = \{2, 3, 4, 5\}$

	0	1	2	3	4	5	6	7
0	{}	{}	{0}	{0}	{0}	{0}	{0}	{0}
1	{}	{}	{0}	{1}	{1}	{0,1}	{0,1}	{0,1}
2	{}	{}	{0}	{1}	{2}	{0,1}	{0,2}	{1,2}
3	{}	{}	{0}	{1}	{2}	{3}	{0,2}	{1,2}

Calculate value for cell A[3,7]

$$T_a = A[i-1, j], T_b = A[i-1, j-w_i] \cup \{i\}, B_a = \sum_{t \in T_a} v_t, B_b = \sum_{t \in T_b} v_t$$

$$A[i, j] = \begin{cases} T_a & \text{if } B_a \geq B_b \\ T_b & \text{otherwise.} \end{cases}$$

$$T_a = A[2, 7] = \{1, 2\}, B_a = 2 + 3 = 5$$

$$T_b = A[2, 7-5] \cup \{3\} = A[2, 2] \cup \{3\} = \{0, 3\},$$

$$B_b = 1 + 4 = 5$$

$$A[2, 7] = T_b = \{1, 2\}$$