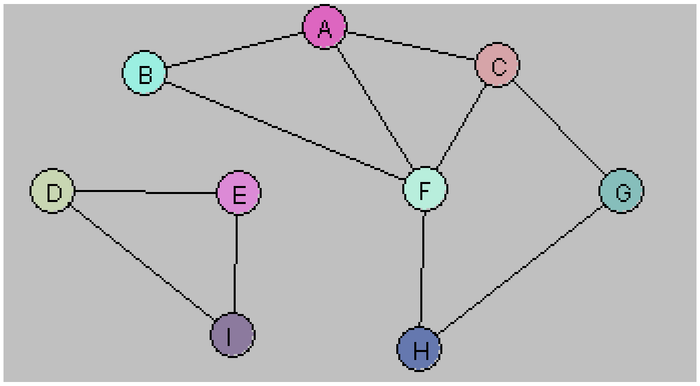
Lab 11

1. *Induced Graphs.* Answer questions about the graph G = (V,E) displayed below.

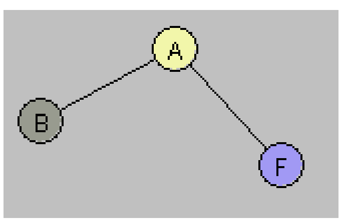


A. Let U = {A, B}. Draw G[U].

B. Let W = {A, C, G, F}. Draw G[W].

C. Let Y = {A, B, D, E}. Draw G[Y].

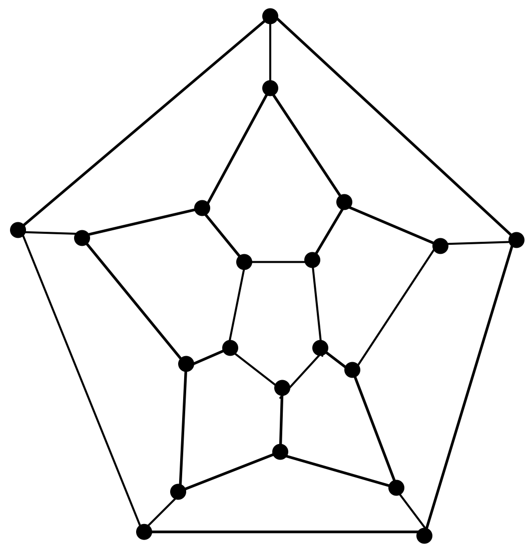
D. Consider the following subgraph H of G:



Is there a subset X of the vertex set V so that H = G[X]? Explain.

E. Find a way to partition the vertex set V into two subsets V_1, V_2 so that each of the induced graphs $G[V_1]$ and $G[V_2]$ is connected and $G = G[V_1] \cup G[V_2]$.

2. *Hamiltonian Graphs.* The following graph has a Hamiltonian cycle. Find it.



3. *Vertex Covers.* Create an algorithm for computing the smallest size of a vertex cover for a graph. The input of your algorithm is a set V of vertices along with a set E of edges. Assume you have the following functions available (no need to implement these):
   - computeEndpoints(edge) – returns the vertices that are at the endpoints of the input edge
   - belongsTo(vertex, set) – returns true if the input vertex is a member of the given set

   Hint: Loop through all subsets of V. For each subset W, check to see if W is a vertex cover. Do this by looping through all edges; for each edge e, check to see if at least one of its endpoints lies in W.

4. Show that if a graph G has |V| -1 edges and has no cycle, then G is connected. Hint: Assume G is disconnected, with connected components H_1, H_2, .. H_k. What can you say about each of these components? Do a computation to show that G must in this case have fewer than |V| - 1 edges (giving a contradiction).

5. The decision problem formulation of the Vertex Cover problem is this: Given a positive integer *k*, and a graph *G*, is there a vertex cover for *G* having size $\leq k$? Show that this decision problem belongs to *NP*.

6. (Extra Credit) (*Background for a minimum spanning tree algorithm*) Suppose $G = (V, E)$ is a connected graph.

   A. Suppose $V_1, V_2, \ldots, V_k$ are disjoint subsets of V and that $V_1 \cup V_2 \cup \ldots \cup V_k = V$. Show that there is an edge (x,y) in E such that for some $i \neq j$, $x \in V_i$ and $y \in V_j$.

   B. Suppose $S = (V_S, E_S)$ and $T = (V_T, E_T)$ are subtrees of G with no vertices in common (in other words, V_S and V_T are disjoint). Then for any edge (x,y) in E for which $x \in V_S$ and $y \in V_T$, the subgraph $U = (V_S \cup V_T, E_S \cup E_T \cup \{(x,y)\})$ is also a tree.

   C. Suppose W is a subset of the set E of edges of G and $|W| < n - 1$. Suppose also that if we consider the subgraph H formed from W (the edges of the graph are W, the vertices are the endpoints of edges in W), then H contains no cycle. Then there exists an edge (x,y) in G not in W so that the graph formed by $W \cup \{(x,y)\}$ also contains no cycle.

      Hint. Consider two cases: (1) H has less than n vertices. (2) H has n vertices.