## **Lab 9, Problems 2,3,4,5**

**Problem 2.** Carry out the array-based version of HeapSort on the input array [1, 4, 3, 9, 12, 2, 4]

Show steps and outputs along the way. Make sure to distinguish between Phase I and Phase II of the algorithm.

## Solution.

```
Phase I
  [1 | 4 3 9 12 2 4]
-----
  [14|391224]
  upheap
  [41|391224]
  [4 1 3 | 9 12 2 4]
  upheap (nothing to do)
  [4 1 3 9 | 12 2 4]
  upheap
  [4 9 3 1 | 12 2 4] \Rightarrow [9 4 3 1 | 12 2 4]
_____
  [9 4 3 1 12 | 2 4]
  upheap
  [9 \ 12 \ 3 \ 1 \ 4 \ | \ 2 \ 4] \implies [12 \ 9 \ 3 \ 1 \ 4 \ | \ 2 \ 4]
 _____
  [12 9 3 1 4 2 | 4]
```

```
upheap (nothing to do)
-----
  [12 9 3 1 4 2 4]
  upheap
  [12 9 4 1 4 2 3]
Phase II
  [12 9 4 1 4 2 3]
  [3 9 4 1 4 2 | 12]
  downheap
  [9\ 3\ 4\ 1\ 4\ 2\ |\ 12] \implies [9\ 4\ 4\ 1\ 3\ 2\ |\ 12]
-----
  [2 4 4 1 3 | 9 12]
  downheap
   [4 \ 2 \ 4 \ 1 \ 3 \ | \ 9 \ 12] \Rightarrow [4 \ 3 \ 4 \ 1 \ 2 \ | \ 9 \ 12]
  [2 3 4 1 | 4 9 12]
  downheap
   [4 3 2 1 | 4 9 12]
  [1 3 2 | 4 4 9 12]
  downheap
  [3 1 2 | 4 4 9 12]
```

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downheap (nothing to do)

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**Problem 3.** Carry out the steps of the recursive algorithm BottomUpHeap for the input sequence 11, 5, 2, 3, 17, 24, 1

## Solution.

downheap

$$[24, 5, 11, 2, 3, 17, 1] \Rightarrow [24, 5, 17, 2, 3, 11, 1]$$

4. Draw an example of a MaxHeap whose keys are all the odd numbers lie in [1, 21] (with no repeats), such that the insertion of an item with key 14 would cause upheap to proceed all the way up to a child of the root (replacing that child's key with 14).

