

SubsetSum Problem $S = \{4, 2, 5, 6\}$, $k = 7$

$$s_0 = 4, s_1 = 2, s_2 = 5, s_3 = 6$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|-----------|
| 0 | | | | | | | | |
| 1 | | | | | | | | |
| 2 | | | | | | | | |
| 3 | | | | | | | | $T_{3,7}$ |

Seeking a subset $T_{3,7}$ of $S_3 = \{4, 2, 5, 6\}$ whose sum is 7.

Such a subset $T_{3,7}$ can be found if and only if either a subset of $S_2 = \{4, 2, 5\}$ sums to 7, or a subset of $S_2 = \{4, 2, 5\}$ sums to $7-6 = 1$.

SubsetSum Problem $S = \{4, 2, 5, 6\}$, $k = 7$

$$s_0 = 4, s_1 = 2, s_2 = 5, s_3 = 6$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|-----------|---|---|---|---|---|-----------|
| 0 | | | | | | | | |
| 1 | | | | | | | | |
| 2 | | $T_{2,1}$ | | | | | | $T_{2,7}$ |
| 3 | | | | | | | | $T_{3,7}$ |

Seeking a subset $T_{2,7}$ of $S_2 = \{4, 2, 5\}$ whose sum is 7 OR

a subset $T_{2,1}$ of $S_2 = \{4, 2, 5\}$ whose sum is $7-6=1$.

Can find $T_{2,7}$ iff a subset of $S_1 = \{4, 2\}$ has sum 7 or a subset of $S_1 = \{4, 2\}$ has sum $7-5=2$.

Can find $T_{2,1}$ iff a subset of $S_1 = \{4, 2\}$ has sum 1

(don't consider the possibility of sum $1-5$)

SubsetSum Problem $S = \{4, 2, 5, 6\}$, $k = 7$

$$s_0 = 4, s_1 = 2, s_2 = 5, s_3 = 6$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|-----------|-----------|---|---|---|---|-----------|
| 0 | | | | | | | | |
| 1 | | | $T_{1,2}$ | | | | | $T_{1,7}$ |
| 2 | | $T_{2,1}$ | | | | | | $T_{2,7}$ |
| 3 | | | | | | | | $T_{3,7}$ |

Starting from $T_{2,7} \subseteq S_2 = \{4, 2, 5\}$ with $k = 7$, we seek

- a subset $T_{1,7}$ of $S_1 = \{4, 2\}$ whose sum is 7 OR
- a subset $T_{1,2}$ of $S_1 = \{4, 2\}$ whose sum is $7-5=2$.

Can find $T_{1,7}$ iff a subset of $S_0 = \{4\}$ has sum 7 or a subset of $S_0 = \{4\}$ has sum $7-2=5$.

Can find $T_{1,2}$ iff a subset of $S_0 = \{4\}$ has sum 2 or a subset of $S_0 = \{4\}$ has sum $2-2 = 0$.

SubsetSum Problem $S = \{4, 2, 5, 6\}$, $k = 7$

$$s_0 = 4, s_1 = 2, s_2 = 5, s_3 = 6$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|-----------|-----------|---|---|---|---|-----------|
| 0 | | | | | | | | |
| 1 | | $T_{1,1}$ | $T_{1,2}$ | | | | | $T_{1,7}$ |
| 2 | | $T_{2,1}$ | | | | | | $T_{2,7}$ |
| 3 | | | | | | | | $T_{3,7}$ |

Starting from $T_{2,1} \subseteq S_2 = \{4, 2, 5\}$ with $k = 1$, we seek

- a subset $T_{1,1}$ of $S_1 = \{4, 2\}$ whose sum is 1

Can find $T_{1,1}$ iff a subset of $S_0 = \{4\}$ has sum 1

(don't consider the possibility of sum = 1 - 2)

SubsetSum Problem $S = \{4, 2, 5, 6\}$, $k = 7$

$$s_0 = 4, s_1 = 2, s_2 = 5, s_3 = 6$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|-----------|-----------|---|---|-----------|---|-----------|
| 0 | | | | | | $T_{0,5}$ | | $T_{0,7}$ |
| 1 | | $T_{1,1}$ | $T_{1,2}$ | | | | | $T_{1,7}$ |
| 2 | | $T_{2,1}$ | | | | | | $T_{2,7}$ |
| 3 | | | | | | | | $T_{3,7}$ |

Starting from $T_{1,7} \subseteq S_1 = \{4, 2\}$ with $k = 7$, we seek

- a subset $T_{0,7}$ of $S_0 = \{4\}$ whose sum is 7 OR
- a subset $T_{0,5}$ of $S_0 = \{4\}$ whose sum is $7-2=5$.

SubsetSum Problem $S = \{4, 2, 5, 6\}$, $k = 7$

$$s_0 = 4, s_1 = 2, s_2 = 5, s_3 = 6$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|-----------|-----------|-----------|---|---|-----------|---|-----------|
| 0 | $T_{0,0}$ | $T_{0,1}$ | $T_{0,2}$ | | | $T_{0,5}$ | | $T_{0,7}$ |
| 1 | | $T_{1,1}$ | $T_{1,2}$ | | | | | $T_{1,7}$ |
| 2 | | $T_{2,1}$ | | | | | | $T_{2,7}$ |
| 3 | | | | | | | | $T_{3,7}$ |

Starting from $T_{1,2} \subseteq S_1 = \{4, 2\}$ with $k=2$, we seek

- a subset $T_{0,2}$ of $S_0 = \{4\}$ whose sum is 2 OR
- a subset $T_{0,0}$ of $S_0 = \{4\}$ whose sum is $2-2=0$.

Starting from $T_{1,1} \subseteq S_1 = \{4, 2\}$ with $k=1$,

we seek a subset $T_{0,1}$ of $S_0 = \{4\}$ whose sum is 1

SubsetSum Problem $S = \{4, 2, 5, 6\}$, $k = 7$

$$s_0 = 4, s_1 = 2, s_2 = 5, s_3 = 6$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|------------------|-------------------------|-------------------------|---|---|-------------------------|---|-------------------------|
| 0 | $T_{0,0} = \{\}$ | $T_{0,1} = \text{NULL}$ | $T_{0,2} = \text{NULL}$ | | | $T_{0,5} = \text{NULL}$ | | $T_{0,7} = \text{NULL}$ |
| 1 | | $T_{1,1}$ | $T_{1,2}$ | | | | | $T_{1,7}$ |
| 2 | | $T_{2,1}$ | | | | | | $T_{2,7}$ |
| 3 | | | | | | | | $T_{3,7}$ |

$T_{0,0} = \{\}$ is a subset of $S_0 = \{4\}$ whose sum is 0.

SubsetSum Problem $S = \{4, 2, 5, 6\}$, $k = 7$

$$s_0 = 4, s_1 = 2, s_2 = 5, s_3 = 6$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|------------------|-------------------------|-----------------------------|---|---|-------------------------|---|-------------------------|
| 0 | $T_{0,0} = \{\}$ | $T_{0,1} = \text{NULL}$ | $T_{0,2} = \text{NULL}$ | | | $T_{0,5} = \text{NULL}$ | | $T_{0,7} = \text{NULL}$ |
| 1 | | $T_{1,1} = \text{NULL}$ | $T_{1,2} = \{\} \cup \{2\}$ | | | | | $T_{1,7} = \text{NULL}$ |
| 2 | | $T_{2,1}$ | | | | | | $T_{2,7}$ |
| 3 | | | | | | | | $T_{3,7}$ |

$T_{1,2} = \{\} \cup \{2\} = \{2\}$ is a subset of $S_1 = \{4, 2\}$ whose sum is 2

SubsetSum Problem $S = \{4, 2, 5, 6\}$, $k = 7$

$$s_0 = 4, s_1 = 2, s_2 = 5, s_3 = 6$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|------------------|-------------------------|-------------------------|---|---|-------------------------|---|-------------------------|
| 0 | $T_{0,0} = \{\}$ | $T_{0,1} = \text{NULL}$ | $T_{0,2} = \text{NULL}$ | | | $T_{0,5} = \text{NULL}$ | | $T_{0,7} = \text{NULL}$ |
| 1 | | $T_{1,1} = \text{NULL}$ | $T_{1,2} = \{u\{2\}$ | | | | | $T_{1,7} = \text{NULL}$ |
| 2 | | $T_{2,1} = \text{NULL}$ | | | | | | $T_{2,7} = \{2\}u\{5\}$ |
| 3 | | | | | | | | $T_{3,7}$ |

$T_{2,7} = \{2\}u\{5\} = \{2,5\}$ is a subset of $S_2 = \{4,2,5\}$ whose sum is 7.

SubsetSum Problem $S = \{4, 2, 5, 6\}$, $k = 7$

$$s_0 = 4, s_1 = 2, s_2 = 5, s_3 = 6$$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|------------------|-------------------------|-----------------------------|---|---|-------------------------|---|------------------------------|
| 0 | $T_{0,0} = \{\}$ | $T_{0,1} = \text{NULL}$ | $T_{0,2} = \text{NULL}$ | | | $T_{0,5} = \text{NULL}$ | | $T_{0,7} = \text{NULL}$ |
| 1 | | $T_{1,1} = \text{NULL}$ | $T_{1,2} = \{\} \cup \{2\}$ | | | | | $T_{1,7} = \text{NULL}$ |
| 2 | | $T_{2,1} = \text{NULL}$ | | | | | | $T_{2,7} = \{2\} \cup \{5\}$ |
| 3 | | | | | | | | $T_{3,7} = \{2, 5\}$ |

$T_{3,7} = T_{2,7} = \{2, 5\}$ is a subset of $S_3 = \{4, 2, 5, 6\}$ whose sum is 7.