Calculus Reference

Algorithms

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Suppose f is a function from the set of real numbers to the set of real numbers. The following notations mean the same thing:

1. $f'(x)$
2. $\frac{d}{dx}\, f(x)$
3. $\frac{df}{dx}$

Facts About Derivatives

(1) $\frac{d}{dx}\, a = 0$ for any real number a.

(2) $\frac{d}{dx}\, x^r = rx^{r-1}$, for any real number $r \neq 0$.

(3) If $f(x)$ has a derivative and a is any real number $\frac{d}{dx}\, af(x) = a\frac{d}{dx}\, f(x)$.

(4) $\frac{d}{dx}\, e^x = e^x$.

(5) $\frac{d}{dx}\, 2^x = 2^x\, \ln 2$

(6) $\frac{d}{dx}\, b^x = b^x\, \ln b$ whenever $b > 0$

(7) $\frac{d}{dx}\, \ln x = \frac{1}{x}$

(8) $\frac{d}{dx}\, \log x = \frac{1}{x} \cdot \log e$

(9) $\frac{d}{dx}\, \log_b x = \frac{1}{x} \cdot \log_b e$

(10) For any functions $f(x), g(x)$ (whose derivatives exist)

- (a) $\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}\, f(x) + \frac{d}{dx}\, g(x)$
- (b) $\frac{d}{dx}(f(x) \cdot g(x)) = f(x) \cdot \frac{d}{dx}\, g(x) + g(x) \cdot \frac{d}{dx}\, f(x)$
- (c) $\frac{d}{dx}(\frac{1}{f(x)}) = \frac{-f'(x)}{[f(x)]^2}$

(d) $\frac{d}{dx}(f(g(x)) = f'(g(x)) \cdot g'(x)$

(11) Important special cases (a denotes a real number)

(a) $\frac{d}{dx}\, ax = a$

(b) $\frac{d}{dx}\, ax^2 = 2ax$

(c) $\frac{d}{dx}\, ax^3 = 3ax^2$

(d) $\frac{d}{dx}\, \frac{1}{x} = \frac{-1}{x^2}$.

(e) $\frac{d}{dx}\, \sqrt{x} = \frac{d}{dx}\, x^{1/2} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$.

Facts About Limits

(1) $\lim_{n\to\infty} \frac{1}{n^r} = 0$ for any $r > 0$

(2) $\lim_{n\to\infty} n = \infty$

(3) $\lim_{n\to\infty} 2^n = \infty$

(4) Suppose $p(n)$ and $q(n)$ are polynomials If $\deg(p(n)) < \deg(q(n))$ then $\lim_{n\to\infty} \frac{p(n)}{q(n)} = 0$ and $\lim_{n\to\infty} \frac{q(n)}{p(n)} = \infty$

(5) [L'Hopital's Rule] Suppose f and g have derivatives (at least when x is large) and their limits as $x \to \infty$ are either both 0 or both infinite. Then

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)}$$

as long as these limits exist.