Univariate Time Series Analysis | **Summer 2020**
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Foundations & Explorative Analyses

Exercise 1 (Basics and Motivation)

Let us first clarify some general concepts before we start with examining concrete processes and models. Consider the discrete stochastic process $\{y_t\}_{t\in\mathbb{Z}}$, of which we observe a finite number of realizations $\{1,\dots,T\}$.

(a) What is the idea behind time series analysis and its purpose? What is the main assumption?

(b) Give the definition of the *mean*, *(auto-)covariance* and *(auto-)correlation function* in general. How can these functions be interpreted?

(c) One central concept in time series analysis is *stationarity*. Why? How is *weak* and *strict* stationarity defined? Of what form are the autocovariance and autocorrelation in context of stationary time series?

(d) One major building block of discrete time series models are *white noise processes*. How are these processes defined? What is the purpose of this building block?

Exercise 2 (Linear Processes)

A linear process is defined as a linear combination of white noise variates ε_t with $t\in\mathbb{Z}$, a constant $\mu\in\mathbb{Z}$ and weights $\psi_j\in\mathbb{R}$. The process is of the form

$$y_t = \mu + \sum_{j=-\infty}^{\infty} \psi_j \varepsilon_{t-j}. \tag{1}$$

Furthermore, the sum sum over the coefficients/weights is well-defined, i.e.

$$\sum_{j=-\infty}^{\infty} |\psi_j| < \infty. \tag{2}$$

Show that the autocovariance function is give by

$$\gamma(h) = \sigma_\varepsilon^2 \sum_{j=-\infty}^{\infty} \psi_{j+h}\psi_j \quad \text{with} \quad h\in\mathbb{Z}. \tag{3}$$

Hint: Which pairs of the white noise variates do not vanish?

Exercise 3 (Components of Time Series)

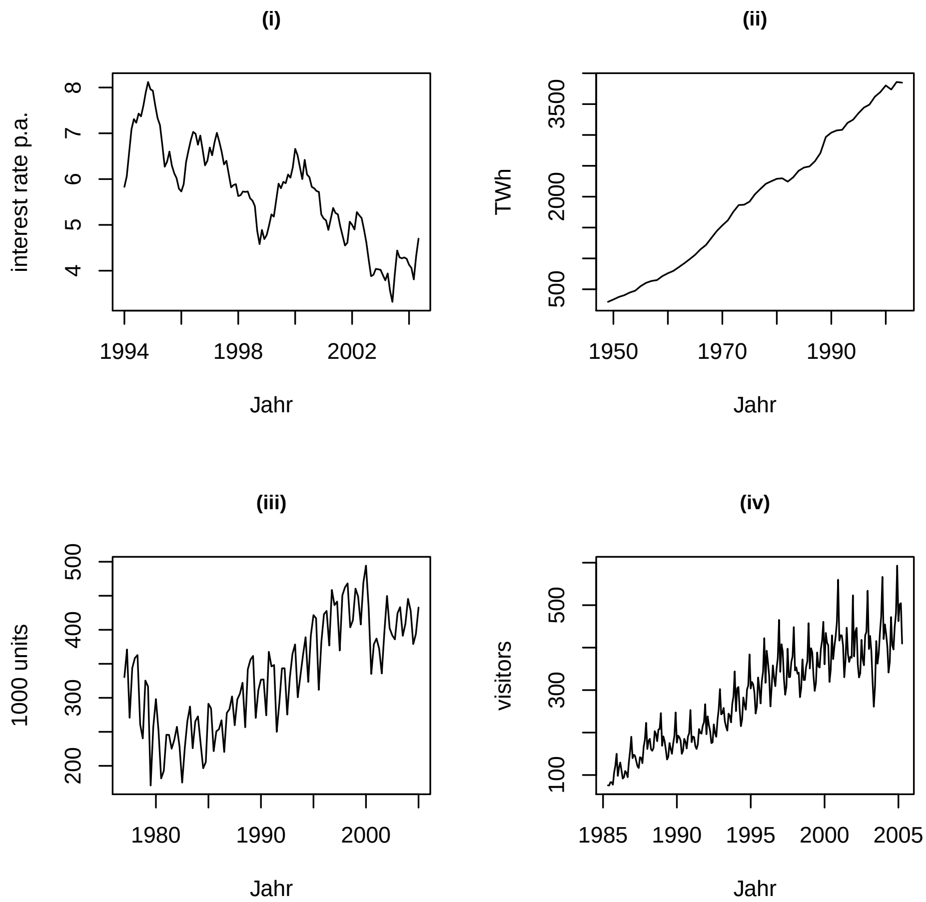
The following figures depict time series[1] of

(i) monthly interest rates of US treasury bonds with time to maturity of 10 years (January 1994 to May 2004),

(ii) yearly electricity production in the US (1949 to 2003),

(iii) quarterly UK car production (1977/1 to 2005/1),

(iv) monthly number of tourists in Australia (May 1985 to April 2005)



(a) Explain the three components of time series by means of these figures.

(b) One can distinguish between additive and multiplicative conjunction of the components. Which kind of conjunction do you think is appropriate for time series (i) to (iv)? What are the consequences for further analyses?

[1]The datasets are taken from the `R` package `expsmooth`: `bonds`, `usnetelec`, `ukcars` and `visitors`.

Exercise 4 (Deterministic Trend)

Consider the process

$$y_t = \beta_1 + \beta_2 t + \varepsilon_t \tag{4}$$

where $\beta_1,\ \beta_2 \in \mathbb{R}$ are known and non-zero weights. Let $\{\varepsilon_t\}_{t\in\mathbb{Z}}$ be a white noise process with variance σ^2.

(a) Determine whether the process y_t is stationary.

(b) Show that the process $x_t = y_t - y_{t-1}$ is stationary.

Exercise 5 (Smoothing: Linear Filter)

A general linear filter M_t of the t-th value of a time series is defined by

$$M_t = \sum_{i=-\infty}^{\infty} \lambda_i y_{t+i}. \tag{5}$$

Let $b \in \mathbb{Z}$. One specific linear filter, the running mean, can be defined as

$$D_t = \sum_{i=-b}^{b} \frac{1}{2b+1} y_{t-i}. \tag{6}$$

(a) Calculate the mean of the filter D_t with x_t as defined in exercise 4, i.e. $y_t = \beta_1 + \beta_2 t + \varepsilon_t$.

(b) Calculate the corresponding autocovariance function $\gamma(h)$ of (**??**) and derive a simplified expression.

Exercise 6 (Random Walk)

Consider the random walk with drift model

$$y_t = \delta + y_{t-1} + \varepsilon_t \tag{7}$$

for $t \in \mathbb{N}$, with $y_0 = 0$, where $\{\varepsilon_t\}_{t\in\mathbb{N}}$ is white noise with variance σ^2.

(a) Show that the model can be written as

$$y_t = \delta t + \sum_{k=1}^{t} \varepsilon_k. \tag{8}$$

(b) Find a representation of the mean function and the autocovariance function of y_t.

(c) Show

$$\rho(t-1,t) = \sqrt{\frac{t-1}{t}} \to 1 \text{ as } t \to \infty. \tag{9}$$

What is the implication of this result?