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Model Predictive Control of an Inverted Pendulum

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***Abstract*- In this paper, model predictive control is applied to an inverted pendulum apparatus and the effect of input disturbance are studied. The optimization problem is solved on-line using quadratic programming approach on a PC hardware platform.**

I. INTRODUCTION

The Inverted Pendulum is a classical example of how the use of control may be employed to stabilize an inherently unstable system. The Inverted pendulum system represents also an accurate model for pitch and yaw behaviors of a flying rocket and can be used as a benchmark for many control methodologies. The Segway PT is a two wheeled (in parallel), self-balancing vehicle that transports a single person which uses the properties of the inverted pendulum [1].

Model predictive control (MPC), also referred to as moving horizon control or receding horizon control, has become an attractive feedback strategy, especially for linear or nonlinear systems subject to input and state constraints. In general, linear and nonlinear MPC are distinguished. Linear MPC refers to a family of MPC schemes in which linear models are used to predict the system dynamics, even though the dynamics of the closed loop system is nonlinear due to the presence of constraints. Linear MPC approaches have found successful applications, especially in the process industries. Important issues such as the online computations, the interplay between modeling, identification and control as well as system theoretic issues like stability are well addressed.

The success of MPC is due to the fact that it is perhaps the most general way of posing the control problem in the time domain. The use a finite-horizon strategy allows the explicit handling of process and operational constraints by the MPC [2].

In general, industrial processes are nonlinear, but many MPC applications are based on the use of linear models [3]. There are two main reasons for this: on one hand, the identification of a linear model based on process data is relatively easy and, on the other hand, linear models provide good results when the plant is operating in the neighborhood of the operating point. Besides, the use of a linear model together with a quadratic objective function gives rise to a convex problem whose solution is well studied with many commercial products available. Notice that MPC of a linear plant with linear constraints gives rise to a nonlinear controller, and that this combination of linear dynamics and linear constraints has influenced on the commercial success of MPC.

Standard predictive control involves recalculating at every sampling instant the input that minimizes a criterion defined over a horizon window in the future, taking into account the current state of the process. Only the first part of the computed optimal input is applied to the process.

A key element in predictive control is the extensive use of the dynamic process model that, unfortunately, may not represent the reality accurately. Thus, the predicted state evolution may differ from the future plant evolution.

When the difference between the predicted and the true plant evolutions is significant, robust predictive control is able to provide the desired performance. Robust predictive control computes an input that represents a compromise solution for the range of uncertainty considered [4-6].

Disturbance rejection is another topic in predictive control that requires special consideration in low-level control applications [7]. The open-loop design philosophy of the traditional MPC leads to efficient feedforward performance, but the methods are less effective in rejection of unmeasured disturbances.

This paper presents an application of a predictive controller on an inverted pendulum, a process that is unstable and non-minimum phase.

The outline of the paper is as follows: The following section describes the inverted pendulum process. Section 3 presents the predictive controller and the tuning procedure. Section 4 details the application of MPC to the inverted pendulum, and Section 5 provides experimental results.

II. PROCESS DESCRIPTION

The rotary pendulum module consists of a flat arm which is instrumented with a sensor at one end such that the sensor shaft is aligned with the longitudinal axis of the arm. A fixture is supplied to attach the pendulum to the sensor shaft. The opposite end of the arm is designed to be mounted on a rotary servo plant resulting in a horizontally rotating arm with a pendulum at the end. The inverted pendulum made by QUANSER Company is shown in Fig. 1.

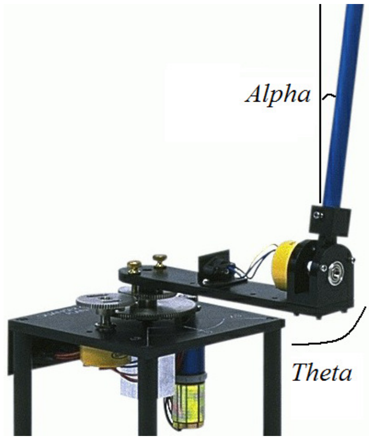

Fig. 1. Inverted pendulum, α and θ

The following equations describe the complete non-linear system;

$$K_1\ddot{\theta} - K_2\cos(\alpha)\ddot{\alpha} + K_2\sin(\alpha)\dot{\alpha}^2 + K_3\dot{\theta} = K_4 u \tag{1}$$

$$K_5\ddot{\alpha} - K_2\cos(\alpha)\ddot{\theta} - K_6\sin(\alpha) = 0 \tag{2}$$

where $K_{i=1,\ldots,5}$ are constants, u is the signal applied to the DC motor, and α and θ are shown in Fig 1.

The linearized model of the system is derived by linearizing equations (1) and (2) about $\alpha = 0$.

$$x(k+1) = Ax(k) + Bu(k) \tag{3}$$

$$y(k) = Cx(k) \tag{4}$$

where $x^T = [\theta \quad \alpha \quad \dot{\theta} \quad \dot{\alpha}]$, and

$$A = \begin{bmatrix} 1 & 0.001876 & 0.009308 & 6.3\times10^{-6} \\ 0 & 1 & -0.00066 & 0.01 \\ 0 & 0.3665 & 0.8647 & 0.001876 \\ 0 & 0.7927 & -0.1303 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.001218 \\ 0.001173 \\ 0.2379 \\ 0.2292 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

A sampling period, T_s, of 10ms is used. The open loop system is unstable.

III. Model Predictive Controller

Consider the problem of regulating to the origin the discrete-time linear time invariant system described by Eqs. (3) and (4) while fulfilling the constraints

$$y_{min} \le y(k) \le y_{max}, \tag{5}$$

$$u_{min} \le u(k) \le u_{max} \tag{6}$$

at all instants $k > 0$.

Model predictive control solves such a constrained regulation problem in the following way. Assume that a full measurement of the state $x(k)$ is available at the current time k. Then, the optimization problem

$$\min_U \{J(U, x(k))\} \tag{7}$$

s.t.

$y_{min} \le y_{k+i|k} \le y_{max}$, $i = 1, \cdots, H_p$,
$u_{min} \le u_{k+i} \le u_{max}$, $i = 0, 1, \cdots, H_p$,
$x_{k|k} = x(k)$,
$x_{k+i+1|k} = Ax_{k+i|k} + Bu_{k+i}$, $i \ge 0$,
$y_{k+i|k} = Cx_{k+i|k}$, $i \ge 0$,

is solved at each time k, where

$$U \triangleq \{u_k, \cdots, u_{k+H_u-1}\}, \tag{8}$$

$$J(U, x(k)) \triangleq \sum_{i=1}^{H_p} x_{k+i|k}^T Q x_{k+i|k} + \sum_{i=0}^{H_u-1} u_{k+i}^T R u_{k+i} \tag{9}$$

and $x_{k+i|k}$ denotes the predicted state vector at time $k+i$, obtained by applying the input sequence $u_k, \cdots, u_{k+i-1}$ to model described by Eqs (3) and (4) starting from the state $x(k)$. H_p is prediction horizon, and H_u is control horizon, and Q and R are square, symmetric and positive definite matrices.

This optimization problem is formulated that predicts over a finite horizon H_p, and the results of the optimization procedure is an optimal control input sequence U. Even though a whole sequence of inputs is at hand, only the first element u_k is applied to the system. In order to compensate for possible modeling errors or disturbances acting on the system, a new state measurement $x(k+1)$ is taken at the next sampling instance $k+1$ and the whole procedure is repeated.

Closed-loop performances will significantly decline unless model predictive controllers are operated with a set of outstanding parameters. Thus how to obtain the optimal parameters of predictive controllers is a challenging problem. A number of researchers have made great efforts to solve this problem and published many effective tuning methods. A model predictive control scheme is infinite-horizon optimal if the resulting control sequence minimizes a performance index over an infinite horizon. In finite horizon case, choosing the prediction horizon is very important.

IV. MPC Applied to an Inverted Pendulum

In the remainder of this paper we consider the application of MPC to an inverted pendulum apparatus shown in Fig. 2. There are physical limits on the control input and the arm position, which correspond to constraints on the supply current to the motor and the angle of the arm, respectively.

The input to the motor is constrained to lie between $-12 \leq u(k) \leq 12$ and the arm position must lie between $-120 \leq \theta \leq 120$.

We would like the state $x(k)$ to be at the origin, which corresponds to the arm position at the null position, the pendulum angle of 0 radians (i.e. upright), the arm not moving and the pendulum not rotating. Furthermore, we would like to be economical with control action and thus penalize input movements. This objective can be described in terms of the cost function in Eq. (9) via the choices

$$Q = \mathrm{diag}(q_1, q_2, q_3, q_4),\ R = r\ . \tag{10}$$

Since it is important that the pendulum angle is zero, q_2 receives a high value. It is less important, but not insignificant, that the arm position is zero, so q_1 has the next highest value. Arm velocity and pendulum angular velocity are not so important, so q_3 and q_4 receive zero value. For the experimental results shown in Section V this corresponds to

$$q_1 = 1,\ \ q_2 = 5,\ \ q_3 = 0,\ \ q_4 = 0,\ \ r = 0.1 \tag{11}$$

Both prediction horizon H_p and control horizon H_u have been established based on the assumptions that large values lead to increased computational effort and short values produce short-sighted control policy. The value of

$$H_p = 100 \tag{12}$$

was selected for the prediction horizon. The choice of a smaller H_p leads to short-sighted control associated with more aggressive control action. The closed loop system is unstable if $H_p < 10$. An additional consequence of reducing H_p is that the constraint violations are only checked over a short horizon, leading to a deadzone with inefficient control effect. The prediction horizon H_p is not established very long (relative to open loop settling time) in order to prevent sluggish control action (having in fact a stabilization effect) and raising the computational load. For the control horizon the value of

$$H_u = 4 \tag{13}$$

has been taken. The control horizon is established not too long, to prevent aggressive control action, but also not too short, to determine an inefficient control and to provide a sufficient number of degrees of freedom.

V. Results

The experimental results from applying the model predictive controller described above to the inverted pendulum shown in Fig. 2 are provided in this section. By way of summary, there are 4-states, 1-input, the sample time is $T_s = 10$ ms, the prediction horizon is $H_p = 100$ samples, the input is constrained via $|u(t)| \leq 12$v and the arm position is constrained via $|\theta(t)| \leq 120^{\circ}$. The quadratic programming is used to solve the optimization problem. All plots in this section show data recorded from the physical apparatus by the PC and data acquisition hardware.

In Fig. 3 and Fig. 4 the initial position of the arm is in the null position and the pendulum tip is down (in the stable position). First the swing up controller is switched on and changes the position of the pendulum to upright position [8, 9]. Then the model predictive controller is switched on and keeps the pendulum upward and rejects the disturbances. The input signal can be seen in the Fig. 3. The pendulum angle is depicted in Fig. 4 as the response of system.

To gauge the utility of the model predictive controller, a large disturbance was manually applied to the pendulum tip while it was in the upright position. Fig. 5 shows the response of pendulum angle to this disturbance. The response of arm angle is depicted in Fig. 6. The output signal of the model predictive controller which is applied to DC motor is depicted in Fig. 7. Note that the arm position and input obey their respective limits.

Friction is the most important nonlinear dynamic of the system which creates the limit cycle [10]. Fig. 5 and Fig. 6 show that there is stable oscillation around the equilibrium point.

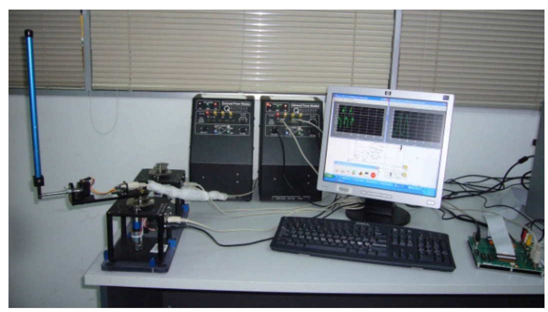

Fig. 2. Inverted pendulum and controller

VI. Conclusions

This paper presents the application of MPC to an inverted pendulum apparatus. While the good performance of MPC for this application may be of independent interest, the key point is that a reasonably challenging control problem can be dealt with via MPC in realtime on a modest hardware platform at a 100Hz sample rate. MPC can reject the manual disturbance of the pendulum angle and the arm position and input obey their respective limits. Friction is the most important nonlinear dynamic of the system which creates the limit cycle.

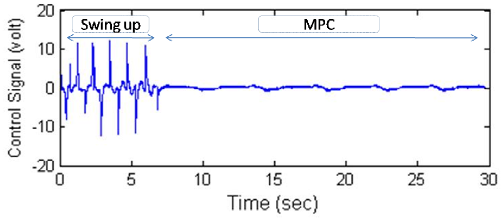

Figure 3. Signal generated by swing up controller and MPC

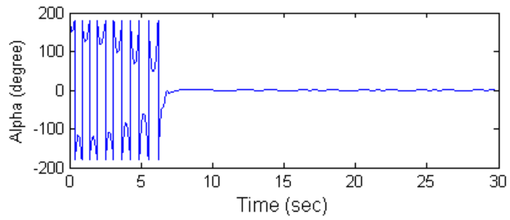

Figure 4. Pendulum angle

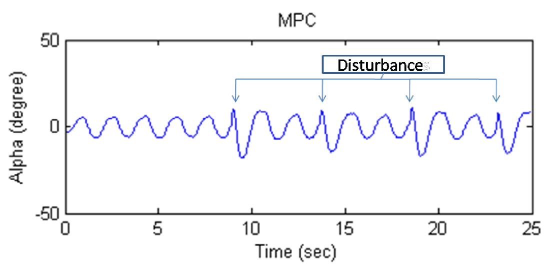

Figure 5. Pendulum angle while there is disturbance.

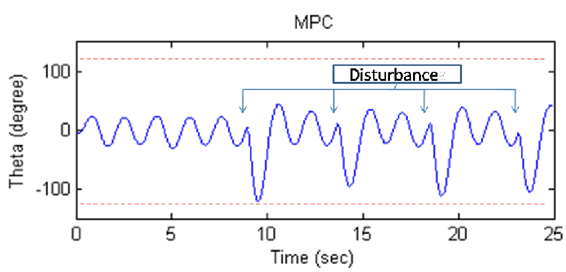

Figure 6. Arm angle

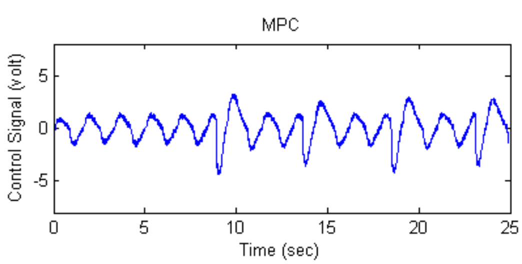

Figure 7. Control signal generated by MPC

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