λ_Q : A Simple Quantum Programming Language

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1 Motivation

Quantum computing is getting more and more popular these days. twh: Add some background of quantum computing here @wxz. blah
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There are three main models of quantum computing: *Quantum Turing Machine*, *Quantum* λ*-Calculus* and *Quantum Circuit*, among which the third one is the most practical. Most quantum programming languages are *quantum-circuit description languages*, which means they are used to describe the architecture of quantum circuits. The current quantum programming languages can be categorized into two categories according to their styles: functional quantum programming languages and imperative quantum programming languages.

Functional:	Imperative:
• Qwire	• QASM
• QML	• QCL
• Quipper	• ScaffCC
• QuaFL	• Qiskit
• Silq	• Quil

Advanced quantum programming languages can use more powerful abstract constructs and type systems to make it easier for programmers to write correct quantum programs. In particular, we design a simple quantum programming language named λ_Q, which means **λ-calculus with quantum circuit**. Its syntax consists of a traditional part, which is a simple λ-calculus, and a quantum part, whose syntax is based on Qwire, a functional quantum programming language with linear type system. What's more, we implement a compiler from λ_Q to QASM (Quantum Assembly Language), an imperative quantum programming language with low-level instruction sets. The output QASM program can be run on the IBM cloud quantum machine.

The main feature of λ_Q is that the syntax for traditional computation and quantum computation are separated. They communicate with each other via some specific operations : quantum circuit can be *abstracted* or *lifted (measured)* into traditional term, and traditional term for quantum circuit can be applied to quantum bits. Thus, the syntax of quantum circuit can use *linear type system* to guarantee that the Quantum Non-cloning Theorem is not violated, meanwhile the λ-calculus of the traditional part makes it easier to write quantum programs.

2 Specification of λ_Q

In this section we give the language specification of λ_Q, including the syntax and typing rules. The syntax and typing rules of the quantum circuit are similar to Qwire [**?**]. The operational semantics are not given in this section. Instead, we give a transformation from λ_Q to $QASM$ in the section of frontend, which gives semantics to each syntax of λ_Q. Interested readers can refer to the operational semantics of simply-typed λ-calculus and Qwire.

2.1 Syntax

The syntax of the traditional part (terms, values, types) is an extended version of simply-typed λ-calcus with `run` C and $\kappa\ p : W.C$, which are used to interact with the quantum part. The syntax of the quantum part (circuits, wire types, wire patterns, gates, gate types) is similar to the syntax of Qwire.

t	::=		**terms**:
		x	variable
		`unit`	constant unit
		`true`	constant true
		`false`	constant false
		$\lambda\, x : T.t$	function abstraction
		$t\ t$	function application
		(t, t)	pair
		$t.1$	first projection
		$t.2$	second projection
		`if` t `then` t `else` t	conditional
		`run` C	static lifting
		$\kappa\, p : W.C$	circuit abstraction
v	::=		**values**:
		$\lambda\, x : T.t$	abstraction value
		(v, v)	pair value
		`unit`	unit value
		`true`	true value
		`false`	false value
		$\kappa\, p : W.C$	circuit value
T	::=		**types**:
		`Unit`	unit type
		`Bool`	boolean type
		$T \times T$	product type
		$T \to T$	function type
		$T \rightsquigarrow T$	circuit type
Γ	::=		**contexts**:
		$\varnothing$	empty context
		$\Gamma, x : T$	term variable binding
W	::=		**wire types**:
		`1`	wire unit type
		`Bit`	bit type
		`Qubit`	qubit type
		$W \otimes W$	wire product type
p	::=		**wire patterns**:
		$()$	empty
		w	wire variable
		(p, p)	wire pair
C	::=		**circuits**:
		`output` p	output a pattern

		$p_2 \leftarrow \texttt{gate}\ g\ p_1; C$	gate application
		$p \leftarrow C; C$	circuit composition
		$x \leftarrow\!\shortmid \texttt{lift}\ p; C$	dynamic lifting
		$\texttt{capp}\ t\ \texttt{to}\ p$	circuit application
g	::=		**gates**:
		$\texttt{new}_0$	generate a bit 0
		$\texttt{new}_1$	generate a bit 1
		$\texttt{init}_0$	generate a qubit 0
		$\texttt{init}_1$	generate a qubit 1
		$\texttt{meas}$	measurement gate
		$\texttt{discard}$	disgard gate
		$\texttt{H}$	Hadamard gate
		$\texttt{X}$	Pauli-X gate
		$\texttt{Z}$	Pauli-Z gate
		$\texttt{CNOT}$	CNOT gate
G	::=		**gate types**:
		$\mathcal{G}(W, W)$	simple gate type
Ω	::=		**wire contexts**:
		$\varnothing$	empty context
		$\Omega, w : W$	wire variable binding

2.2 Typing Rules

The main feature of the language design of λ_Q is to use linear type system to guarantee no quantum bit is used twice or not used at all. To state the type inference rules of λ_Q, we first need to define what is a **well-formed wire context**, which is used to maintain variables used by quantum circuit in the calculus. This context is actually corresponding to the context of linear variables in the traditional linear type system.

Definition 1 (Well-formed Wire Contexts)**.** *A wire context Ω is well-formed, if there are no duplicate wire variables in it. For simplicity, we always assume the wire contexts are well-formed in the following contexts. And when we write Ω_1, Ω_2, we require Ω_1 and Ω_2 to be disjoint to preserve the well-formedness.*

Since there are some different kinds of terms: (λ-)terms, wire patterns, gates and circuits, we have defined several different typing relations for each of them.

- $\Omega \vdash p : W$ is the typing relation for patterns.
- $\Gamma; \Omega \vdash C : W$ is the typing relation for circuits;
- $\Gamma \vdash t : T$ is the typing relation for (λ-)terms;
- $g : G$ is the typing relation for gates.

2.2.1 Typing rules for gates

Note that since we only support built-in gates, the typing rules for gates are extremely simple, just assigning a type to each built-in gate.
Typing rules for gates: $\boxed{g : G}$

$$\frac{}{\texttt{new}_0 : \mathcal{G}(\texttt{1}, \texttt{Bit})}$$

$$\frac{}{\texttt{new}_1 : \mathcal{G}(\texttt{1}, \texttt{Bit})}$$

$$\frac{}{\texttt{init}_0 : \mathcal{G}(\texttt{1}, \texttt{Qubit})}$$

$$\frac{}{\texttt{init}_1 : \mathcal{G}(\texttt{1}, \texttt{Qubit})}$$

$$\frac{}{\texttt{meas} : \mathcal{G}(\texttt{Qubit}, \texttt{Bit})}$$

$$\frac{}{\texttt{discard} : \mathcal{G}(\texttt{Bit}, \texttt{1})}$$

2.2.2 Typing rules for patterns

The patterns are used to construct complex gates. Patterns are destructed when doing pattern matching in gate application, circuit composition and dynamic lifting.
Typing rules for patterns: $\boxed{\Omega \vdash p : W}$

$$\frac{}{\varnothing \vdash () : One}$$

$$\frac{}{w : W \vdash w : W}$$

$$\frac{\Omega_1 \vdash p_1 : W_1 \quad \Omega_2 \vdash p_2 : W_2}{\Omega_1, \Omega_2 \vdash (p_1, p_2) : W_1 \otimes W_2}$$

2.2.3 Typing rules for circuits

The typing rules for circuit uses linear type. The context Γ is for normal (traditional) variables and the context Ω is for linear (quantum) variables.
Typing rules for circuits: $\boxed{\Gamma; \Omega \vdash C : W}$

$$\frac{\Omega \vdash p : W}{\Gamma; \Omega \vdash \texttt{output}\ p : W} \qquad \text{(C-OUTPUT)}$$

$$\frac{g : \mathcal{G}(W_1, W_2) \quad \Omega_1 \vdash p_1 : W_1 \quad \Omega_2 \vdash p_2 : W_2 \quad \Gamma; \Omega_2, \Omega \vdash C : W}{\Gamma; \Omega_1, \Omega \vdash p_2 \leftarrow \texttt{gate}\ g\ p_1; C : W} \qquad \text{(C-GATE)}$$

$$\frac{\Gamma;\Omega_1 \vdash C : W \quad \Omega \vdash p : W \quad \Gamma;\Omega,\Omega_2 \vdash C' : W'}{\Gamma;\Omega_1,\Omega_2 \vdash p \leftarrow C;C' : W'} \text{ (C-COMPOSE)}$$

$$\frac{\Omega \vdash p : W \quad \Gamma, x : |W|;\Omega' \vdash C : W'}{\Gamma;\Omega,\Omega' \vdash x \leftarrow \texttt{lift}\ p;C : W'} \text{ (C-LIFT)}$$

$$\frac{\Gamma \vdash t : W_1 \rightsquigarrow W_2 \quad \Omega \vdash p : W_1}{\Gamma;\Omega \vdash \texttt{capp}\ t\ \texttt{to}\ p : W_2} \text{ (C-CAPP)}$$

2.2.4 Typing rules for terms

Typing rules for (λ-)terms: $\boxed{\Gamma \vdash t : T}$

$$\frac{\Gamma;\varnothing \vdash C : W}{\Gamma \vdash \texttt{run}\ C : |W|} \text{ (T-RUN)}$$

$$\frac{\Omega \vdash p : W_1 \quad \Gamma;\Omega \vdash C : W_2}{\Gamma \vdash \kappa\ p : W_1.C : W_1 \rightsquigarrow W_2} \text{ (T-CABS)}$$

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T} \text{ (T-VAR)}$$

$$\frac{\Gamma, x : T_1 \vdash t_2 : T_2}{\Gamma \vdash \lambda\ x : T_1.t_2 : T_1 \to T_2} \text{ (T-ABS)}$$

$$\frac{\Gamma \vdash t_1 : T_{11} \to T_{12} \quad \Gamma \vdash t_2 : T_{11}}{\Gamma \vdash t_1\ t_2 : T_{12}} \text{ (T-APP)}$$

$$\frac{}{\Gamma \vdash \texttt{true} : Bool} \text{ (T-TRUE)}$$

$$\frac{}{\Gamma \vdash \texttt{false} : Bool} \text{ (T-FALSE)}$$

$$\frac{\Gamma \vdash t_1 : \texttt{Bool} \quad \Gamma \vdash t_2 : T \quad \Gamma \vdash t_3 : T}{\texttt{if}\ t_1\ \texttt{then}\ t_2\ \texttt{else}\ t_3 : T} \text{ (T-IF)}$$

$$\frac{}{\Gamma \vdash \texttt{unit} : Unit} \text{ (T-UNIT)}$$

$$\frac{\Gamma \vdash t_1 : T_1 \quad \Gamma \vdash t_2 : T_2}{\Gamma \vdash (t_1, t_2) : T_1 \times T_2} \text{ (T-PAIR)}$$

$$\frac{\Gamma \vdash t : T_1 \times T_2}{\Gamma \vdash t.1 : T_1} \text{ (T-FST)}$$

$$\frac{\Gamma \vdash t : T_1 \times T_2}{\Gamma \vdash t.2 : T_2} \qquad \text{(T-SEC)}$$

3 Frontend

In this section we illustrate the design and implementation of the frontend of compiler of λ_Q.

3.1 Lexer & Parser

3.2 Desugar

3.3 Type Inference

3.4

4 Backend