Big Ideas in AI - Module 3
Traveling Salesman Problem

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a

The Traveling Salesman Problem (TSP), has been proven to belong to the NP-hard complexity class [1].

b

For any possible route in TSP, assuming it is undirected, the direction and the initial city does not impact its cost. Additionally, we can not revisit any city on the route. There are $n - 1$ choices for the second city to visit, $n - 2$ choices for the third and so on. This gives a total of $(n - 1)!$ possible tours. As the direction does not matter, half of these will be duplicates resulting in a total of $\frac{(n-1)!}{2}$ unique tours [2].

d

Given a tour T of length $n + 1$, consisting of the cities in the tour in the order they are visited given as their indices in M, with the first and last element being the same, the cost can be computed as:

$$\sum_{i=0}^{n} M(T_i, T_{i+1})$$

e

With the knowledge of M being bi-directional and symmetric and with M essentially being a weighted adjacency matrix, testing whether any valid tour exists at all is equivalent to testing the graph G as described by M for connectivity. If G is disconnected, no valid tour exists. If a Depth First Search on G, executed from an initial vertex v_i, is not able to visit all nodes in G it is said to be disconnected. It must be noted that this solution only works on symmetric matrices.

References

[1] Michael Jünger, Gerhard Reinelt, and Giovanni Rinaldi. "Chapter 4 The traveling salesman problem." In: *Handbooks in Operations Research and Management Science*. Elsevier, 1995, pp. 225–330. DOI: `10.1016/s0927-0507(05)80121-5`. URL: `https://doi.org/10.1016/s0927-0507(05)80121-5`.

[2] URL: `https://www.math.uwaterloo.ca/tsp/problem/pcb3cnt.html`.