2024.11.29

DANIELE IANNOTTI   ALIOSCIA HAMMA
MICHELE VISCARDI   LORENZO CAMPOS VENUTI
GIANLUCA ESPOSITO

INTERPLAY BETWEEN MAGIC AND ENTANGLEMET

Suppose $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ $[\dim(\mathcal{H}) = d = d_A d_B]$

$|\psi\rangle \in \mathcal{H}$ t.c. $\psi \equiv |\psi\rangle\langle\psi|$ $\mathrm{Tr}(\psi) = 1$

MAGIC

- $M_{lin}(\psi) = 1 - SP(\psi) = 1 - d\,\mathrm{Tr}[Q\,\psi^{\otimes 4}]$

$$Q = \frac{1}{d^2} \sum_{P \in \mathcal{P}_n} P^{\otimes 4}$$

Haar measure

$$\sigma^2_{M_{lin}}(\psi) = \mathbb{E}[M_{lin}^2(\psi)] - \mathbb{E}[M_{lin}(\psi)]^2$$

$$= \frac{96(d-1)}{(d+3)^2(d+5)(d+6)(d+7)} \xrightarrow[d\to\infty]{} O\left(\frac{1}{d^4}\right)$$

$\hookrightarrow$ CONCENTRATED

ENTANGLEMENT

- $E_{lin}(\psi) = 1 - Pur(\psi_A) = 1 - tr(\psi_A^2)$

$\psi_A = tr_B |\psi\rangle\langle\psi|$

Haar measure

$$\sigma^2_{E_{lin}}(\psi) = \mathbb{E}[E_{lin}^2(\psi)] - \mathbb{E}[E_{lin}(\psi)]^2$$

$$= \frac{2(d_A^2-1)(d_B^2-1)}{(d+1)^2(d+2)(d+3)} \xrightarrow[d\to\infty]{} O\left(\frac{1}{d^2}\right)$$

CORRELATION

- $Cov(E_{lin}(\psi), M_{lin}(\psi)) = \mathbb{E}[E_{lin}(\psi) M_{lin}(\psi)] - \mathbb{E}[E_{lin}(\psi)]\mathbb{E}[M_{lin}(\psi)] = 0$

$\forall d$

$\boxed{\text{Different measures}}$ $E \equiv S_2(\psi_A) = -\log[tr(\psi_A^2)]$ & $M \equiv M_2(\psi)$

$\Rightarrow$ 2 qubit $Cov(E, M) = -0{,}039\ldots$

$\Rightarrow$ Dependent but not correlated

AVERAGES AT FIXED ENTANGLEMENT

$$\mathbb{E}_{U_A \otimes U_B}[M_{lin}(\psi) \mid E_{lin}(\psi) = e] \equiv M_{lin}(e)$$

$$= 1 - d \sum_{\substack{i_1 i_2 \\ i_3 i_4}} \sum_{\substack{j_1 j_2 \\ j_3 j_4}} \lambda_{i_1} \bar{\lambda}_{j_1} \lambda_{i_2} \bar{\lambda}_{j_2} \lambda_{i_3} \bar{\lambda}_{j_3} \lambda_{i_4} \bar{\lambda}_{j_4}$$

$$\mathrm{tr}[\, \mathbb{E}_{U_A}[Q_A]\, |\phi_{i_1}\rangle^A\langle\phi_{j_1}| \otimes |\phi_{i_2}\rangle^A\langle\phi_{j_2}| \otimes |\phi_{i_3}\rangle^A\langle\phi_{j_3}| \otimes |\phi_{i_4}\rangle^A\langle\phi_{j_4}| \,]$$

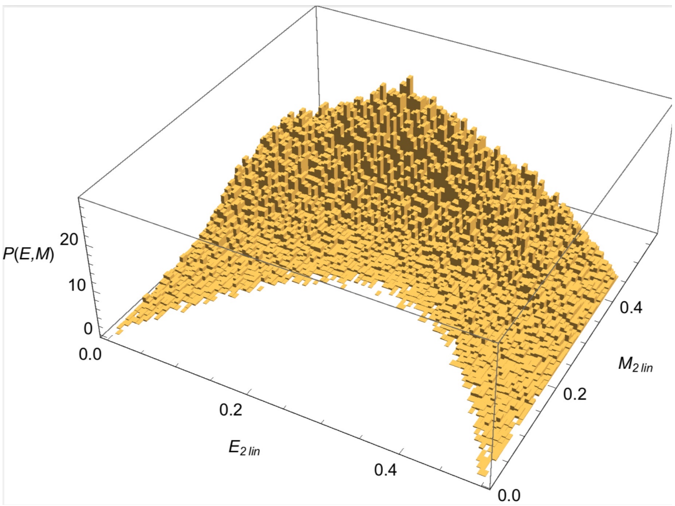
$$\mathrm{tr}[\, \mathbb{E}_{U_B}[Q_B]\, |\xi_{i_1}\rangle^B\langle\xi_{j_1}| \otimes |\xi_{i_2}\rangle^B\langle\xi_{j_2}| \otimes |\xi_{i_3}\rangle^B\langle\xi_{j_3}| \otimes |\xi_{i_4}\rangle^B\langle\xi_{j_4}| \,]$$

$$\psi_e = \sum_i \sum_j \lambda_i \bar{\lambda}_j \; |\phi_i\rangle^A\langle\phi_j| \otimes |\xi_i\rangle^B\langle\xi_j|$$

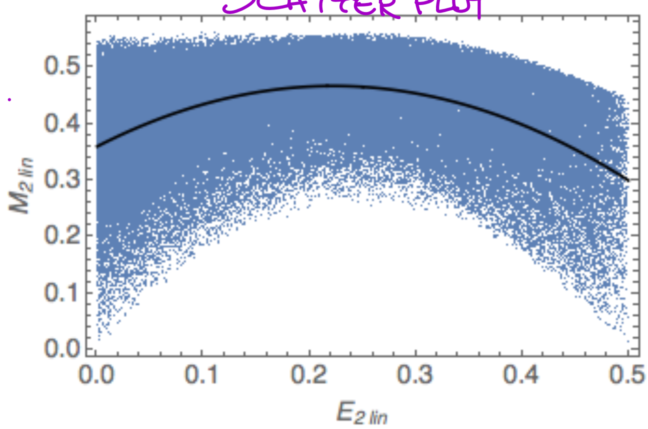
NUMERICS (2 qubits)

$$M_{lin}(e) = \frac{9 + 6e(4 - 9e)}{25}$$

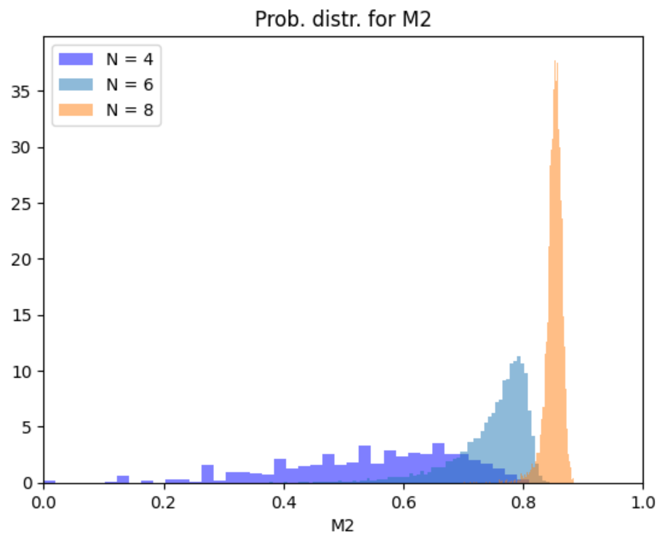
PROBABILITY DENSITY [HISTOGRAM]



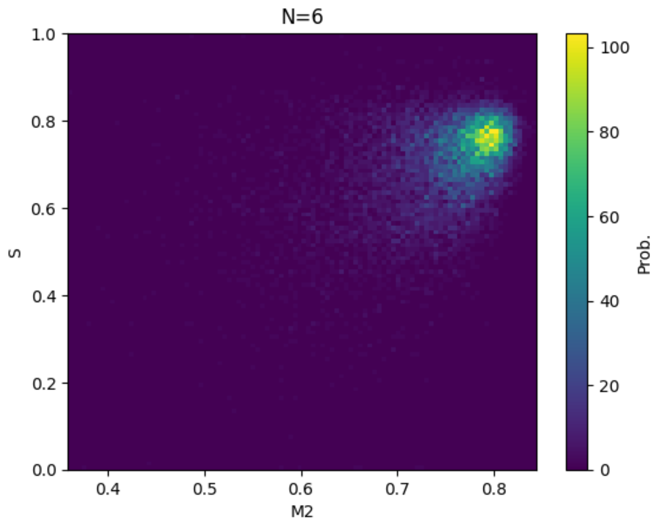
SCATTER PLOT

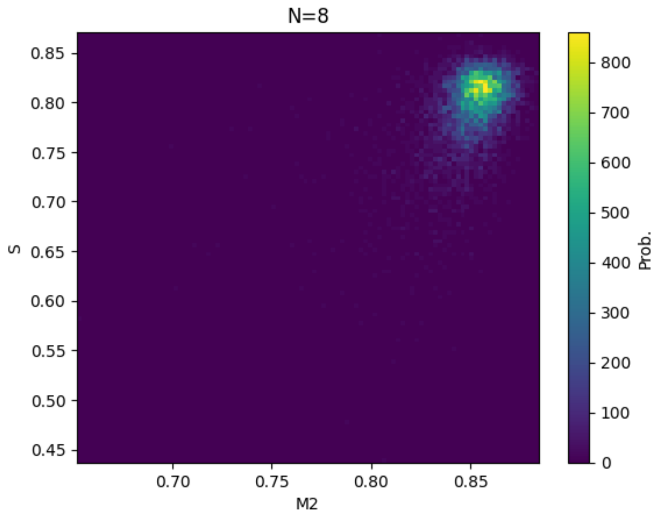


Numerics (N qubits)



Joint probability distribution





$$\log \Lambda(\rho_A) = 2\left(S_2(\rho_A) - S_3(\rho_A)\right)$$