Name: **Thyrso Munhoz Queiroz Gesta de Araújo**

AE-245 FINITE ELEMENT I

FINAL EXAM

Due: December 05th, 2015

São José dos Campos - SP

2015

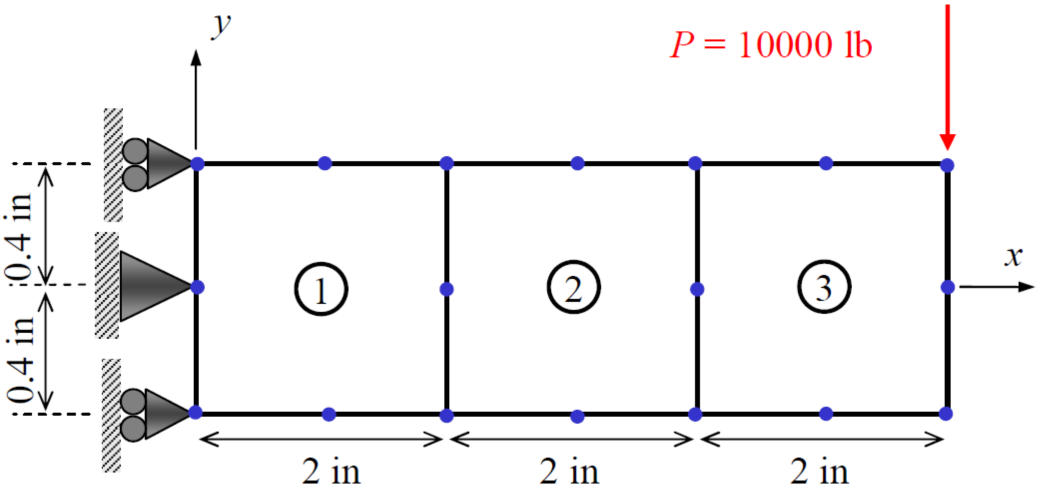
AE-245: Final Assignment
Cantilever beam

Analyze the cantilever beam shown below using three 8-noded isoperimetric elements. Compare your results for stress Sigma x and transverse displacements along $y = 0$ with:

a) Model with six constant strain triangle elements;

b) Classical beam theory in bending.

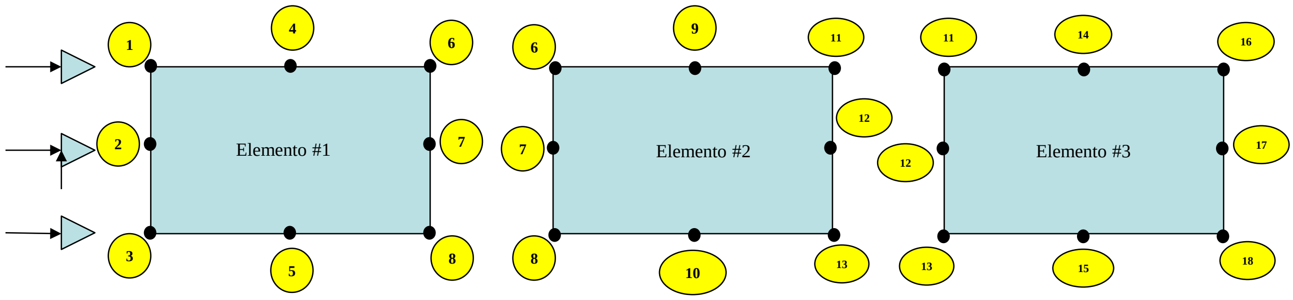
Show the computed stiffness matrices of the elements used in the analysis.
Consider: thickness $t = 1.0$ in, $E = 30*10^6$ psi e v= 0.3.



a) Model with six constant strain triangle elements;

Quadrilateral Elements: 11 steps

0# - Numerar os elementos:



1# - Coordenadas dos Nós dos Elementos:

$P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}, P_{12}, P_{13}, P_{14}, P_{15}, P_{16,} P_{17}, P_{18}$

```
%========================================================================
%Coordinate of Elements  - Nodes - DEPENDE DA INCIDENCIA DE NOS
syms qsi eta

syms N1(qsi,eta) N2(qsi,eta) N3(qsi,eta) N4(qsi,eta) N5(qsi,eta) N6(qsi,eta) N7(qsi,eta) N8(qsi,eta)
N9(qsi,eta)...
    N10(qsi,eta) N11(qsi,eta) N12(qsi,eta) N13(qsi,eta) N14(qsi,eta) N15(qsi,eta) N16(qsi,eta) N17(qsi,n)
N18(qsi,eta)

       % 1   2    3   4    5    6 7    8   9   10   11 12  13  14  15   16  17  18
vec_x = [0   0    0   1    1    2 2    2   3    3    4 4    4   5   5    6  6    6];
vec_y = [0.4 0 -0.4 0.4 -0.4  0.4 0 -0.4 0.4 -0.4 0.4 0 -0.4 0.4 -0.4 0.4  0 -0.4];
```

2# - Funções de Interpolação:

$$
\begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \\ N_8 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \xi & \xi & \xi & \xi & \xi & \xi & \xi & \xi \\ \eta & \eta & \eta & \eta & \eta & \eta & \eta & \eta \\ \xi^2 & \xi^2 & \xi^2 & \xi^2 & \xi^2 & \xi^2 & \xi^2 & \xi^2 \\ \xi\eta & \xi\eta & \xi\eta & \xi\eta & \xi\eta & \xi\eta & \xi\eta & \xi\eta \\ \xi^2 & \xi^2 & \xi^2 & \xi^2 & \xi^2 & \xi^2 & \xi^2 & \xi^2 \\ \xi^2\eta & \xi^2\eta & \xi^2\eta & \xi^2\eta & \xi^2\eta & \xi^2\eta & \xi^2\eta & \xi^2\eta \\ \xi\eta^2 & \xi\eta^2 & \xi\eta^2 & \xi\eta^2 & \xi\eta^2 & \xi\eta^2 & \xi\eta^2 & \xi\eta^2 \end{bmatrix} \begin{bmatrix} 1 \\ \xi \\ \eta \\ \xi^2 \\ \xi\eta \\ \xi^2 \\ \xi^2\eta \\ \xi\eta^2 \end{bmatrix} onde\, \xi = [-1{,}1], \eta = [-1{,}1]
$$

```
%========================================================================
%Interpolation Functions of 8-noded Serendipity element - DEPENDE DA INCIDENCIA DE NOS

N5(qsi,eta)  = 1/2*(1-qsi^2)*(1-eta)%N5
N7(qsi,eta)  = 1/2*(1+qsi)*(1-eta^2)%N6
N4(qsi,eta)  = 1/2*(1-qsi^2)*(1+eta)%N7
N2(qsi,eta)  = 1/2*(1-qsi)*(1-eta^2)%N8


N3(qsi,eta)  = 1/4*(1-qsi)*(1-eta)  - 1/2*(N2(qsi,eta)  + N5(qsi,eta))  %N1
N8(qsi,eta)  = 1/4*(1+qsi)*(1-eta)  - 1/2*(N5(qsi,eta)  + N7(qsi,eta))  %N2
N6(qsi,eta)  = 1/4*(1+qsi)*(1+eta)  - 1/2*(N7(qsi,eta)  + N4(qsi,eta))  %N3
N1(qsi,eta)  = 1/4*(1-qsi)*(1+eta)  - 1/2*(N4(qsi,eta)  + N2(qsi,eta))  %N4


%========================================================================
%Derivation Interpolation Functions of 8-noded Serendipity element
```

```
dN1_dE= diff(N1,qsi)
dN2_dE= diff(N2,qsi)
dN3_dE= diff(N3,qsi)
dN4_dE= diff(N4,qsi)
dN5_dE= diff(N5,qsi)
dN6_dE= diff(N6,qsi)
dN7_dE= diff(N7,qsi)
dN8_dE= diff(N8,qsi)

dN1_dn= diff(N1,eta)
dN2_dn= diff(N2,eta)
dN3_dn= diff(N3,eta)
dN4_dn= diff(N4,eta)
dN5_dn= diff(N5,eta)
dN6_dn= diff(N6,eta)
dN7_dn= diff(N7,eta)
dN8_dn= diff(N8,eta)
```

dN1_dE(qsi, eta) =(ξ*(η+ 1))/2 - η/4 - η^2/4

dN2_dE(qsi, eta) = η^2/2 - 1/2

dN3_dE(qsi, eta) = η/4 - (ξ*(η- 1))/2 - η^2/4

dN4_dE(qsi, eta) = -ξ*(η+ 1)

N5_dE(qsi, eta) = ξ*(η- 1)

dN6_dE(qsi, eta) = η/4 + (ξ*(η+ 1))/2 + η^2/4

dN7_dE(qsi, eta) = 1/2 - η^2/2

dN8_dE(qsi, eta) = η^2/4 - (ξ*(η- 1))/2 - η/4

dN1_dn(qsi, eta) = ξ^2/4 - ξ/4 - η*(ξ/2 - 1/2)

dN2_dn(qsi, eta) = 2*η*(ξ/2 - 1/2)

dN3_dn(qsi, eta) = ξ/4 - ξ^2/4 - η*(ξ/2 - 1/2)

dN4_dn(qsi, eta) = 1/2 - ξ^2/2

dN5_dn(qsi, eta) = ξ^2/2 - ½

dN6_dn(qsi, eta) = ξ/4 + ξ^2/4 + η*(ξ/2 + 1/2)

dN7_dn(qsi, eta) = -2*η*(ξ/2 + 1/2)

dN8_dn(qsi, eta) = η*(ξ/2 + 1/2) - ξ^2/4 - ξ/4

3# - Calcular: $x(\xi,\eta)=\sum_{i=1}^{m} Ni(\xi,\eta)*x_i$ e $y(\xi,\eta)=\sum_{i=1}^{m} Ni(\xi,\eta)*x_i$

```
%=========================================================================================
%Isoparametric Formulation - DEPENDE DA INCIDENCIA DE NOS

syms X1(qsi,eta) Y1(qsi,eta) X2(qsi,eta) Y2(qsi,eta) X3(qsi,eta) Y3(qsi,eta)

%Element 1#

X1(qsi,eta) = vec_x(1)*N1(qsi,eta) + vec_x(2)*N2(qsi,eta) + vec_x(3)*N3(qsi,eta) + vec_x(4)*N4(qsi,eta) +
vec_x(5)*N5(qsi,eta) + vec_x(6)*N6(qsi,eta) + vec_x(7)*N7(qsi,eta) + vec_x(8)*N8(qsi,eta)

Y1(qsi,eta) = vec_y(1)*N1(qsi,eta) + vec_y(2)*N2(qsi,eta) + vec_y(3)*N3(qsi,eta) + vec_y(4)*N4(qsi,eta) +
vec_y(5)*N5(qsi,eta) + vec_y(6)*N6(qsi,eta) + vec_y(7)*N7(qsi,eta) + vec_y(8)*N8(qsi,eta)

%Element 2#

X2(qsi,eta) = vec_x(6)*N6(qsi,eta) + vec_x(7)*N7(qsi,eta) + vec_x(8)*N8(qsi,eta) + vec_x(9)*N9(qsi,eta) +
vec_x(10)*N10(qsi,eta) + vec_x(11)*N11(qsi,eta) + vec_x(12)*N12(qsi,eta) + vec_x(13)*N13(qsi,eta)

Y2(qsi,eta) = vec_y(6)*N6(qsi,eta) + vec_y(7)*N7(qsi,eta) + vec_y(8)*N8(qsi,eta) + vec_y(9)*N9(qsi,eta) +
vec_y(10)*N10(qsi,eta) + vec_y(11)*N11(qsi,eta) + vec_y(12)*N12(qsi,eta) + vec_y(13)*N13(qsi,eta)

%Element 3#

X3(qsi,eta) = vec_x(11)*N11(qsi,eta) + vec_x(12)*N12(qsi,eta) + vec_x(13)*N13(qsi,eta) + vec_x(14)*N14(qsi,eta) +
vec_x(15)*N15(qsi,eta) + vec_x(16)*N16(qsi,eta) + vec_x(17)*N17(qsi,eta) + vec_x(18)*N18(qsi,eta)
```

```
Y3(qsi,eta) = vec_y(11)*N11(qsi,eta) + vec_y(12)*N12(qsi,eta) + vec_y(13)*N13(qsi,eta) + vec_y(14)*N14(qsi,eta) +
vec_y(15)*N15(qsi,eta) + vec_y(16)*N16(qsi,eta) + vec_y(17)*N17(qsi,eta) + vec_y(18)*N18(qsi,eta)
```

4# - Montar a matriz Jacobiana: $J = \begin{bmatrix} X,_{\xi} & Y,_{\xi} \\ X,_{\xi} & Y,_{\xi} \end{bmatrix}$

```
%==========================================================================
%Jacobian Matrix
%[m,n] = size(X)
%Element 1#
J1 = zeros(2,2)

J1=[diff(X1,qsi), diff(Y1,qsi);
    diff(X1,eta), diff(Y1,eta)]
J1 = formula(J1)

J2 = J1;
J3 = J1;
%==========================================================================
%Inverse -> Jacobian Matrix

J1_inv = inv(J1)
J2_inv = inv(J2)
J3_inv = inv(J3)
```

```
J1 =
[ 1,   0]
[ 0, 2/5]

J2 =
[ 1,   0]
[ 0, 2/5]

J3 =
[ 1,   0]
[ 0, 2/5]
```

5# - Calcular: $\begin{bmatrix} N,_{x} \\ N,_{y} \end{bmatrix} = J^{-1} * \begin{bmatrix} N_i,_{\xi} \\ N_i,_{\eta} \end{bmatrix}$

```
%==================================================================================================
%DEPENDE DA INCIDENCIA DE NOS
%Element 1# - 1, 2, 3, 4, 5, 6, 7, 8

N1qsi_N1eta = [dN1_dE;
               dN1_dn]
N1qsi_N1eta = formula(N1qsi_N1eta)
AUX1 = J1_inv*N1qsi_N1eta
AUX1 = formula(AUX1)
dN1_dX = AUX1(1,1)
dN1_dY = AUX1(2,1)

N2qsi_N2eta = [dN2_dE;
               dN2_dn]
N2qsi_N2eta = formula(N2qsi_N2eta)
AUX2 = J1_inv*N2qsi_N2eta
dN2_dX = AUX2(1,1)
dN2_dY = AUX2(2,1)

N3qsi_N3eta = [dN3_dE;
               dN3_dn]
N3qsi_N3eta = formula(N3qsi_N3eta)
AUX3 = J1_inv*N3qsi_N3eta
dN3_dX = AUX3(1,1)
dN3_dY = AUX3(2,1)

N4qsi_N4eta = [dN4_dE;
               dN4_dn]
N4qsi_N4eta = formula(N4qsi_N4eta)
AUX4 = J1_inv*N4qsi_N4eta
dN4_dX = AUX4(1,1)
dN4_dY = AUX4(2,1)

N5qsi_N5eta = [dN5_dE;
               dN5_dn]
N5qsi_N5eta = formula(N5qsi_N5eta)
AUX5 = J1_inv*N5qsi_N5eta
```

```
dN5_dX = AUX5(1,1)
dN5_dY = AUX5(2,1)

N6qsi_N6eta = [dN6_dE;
               dN6_dn]
N6qsi_N6eta = formula(N6qsi_N6eta)
AUX6 = J1_inv*N6qsi_N6eta
dN6_dX = AUX6(1,1)
dN6_dY = AUX6(2,1)

N7qsi_N7eta = [dN7_dE;
               dN7_dn]
N7qsi_N7eta = formula(N7qsi_N7eta)
AUX7 = J1_inv*N7qsi_N7eta
dN7_dX = AUX7(1,1)
dN7_dY = AUX7(2,1)

N8qsi_N8eta = [dN8_dE;
               dN8_dn]
N8qsi_N8eta = formula(N8qsi_N8eta)
AUX8 = J1_inv*N8qsi_N8eta
dN8_dX = AUX8(1,1)
dN8_dY = AUX8(2,1)
```

```
dN1_dX =(ξ*(η + 1))/2 - η/4 - η^2/4
dN1_dY =(5*ξ^2)/8 - (5*ξ)/8 - (5*η*(ξ/2 - 1/2))/2
dN2_dX =η^2/2 - 1/2
dN2_dY =5*η*(qsi/2 - 1/2)
dN3_dX =η/4 - (ξ*(eta - 1))/2 - η^2/4
dN3_dY =(5*ξ)/8 - (5*ξ^2)/8 - (5*η*(ξ/2 - 1/2))/2
dN4_dX =-ξ*(η + 1)
dN4_dY =5/4 - (5*ξ^2)/4
dN5_dX =ξ*(η - 1)
dN5_dY =(5*ξ^2)/4 - 5/4
dN6_dX =eta/4 + (ξ*(η + 1))/2 + η^2/4
dN6_dY =(5*ξ)/8 + (5*ξ^2)/8 + (5*η*(ξ/2 + 1/2))/2
dN7_dX =1/2 - η^2/2
dN7_dY =-5*η*(ξ/2 + 1/2)
dN8_dX =η^2/4 - (ξ*(η - 1))/2 - η/4
dN8_dY =(5*η*(ξ/2 + 1/2))/2 - (5*ξ^2)/8 - (5*ξ)/8
```

6# - Calcular Matriz: $B = \begin{bmatrix} N,_x & 0 \\ 0 & N,_y \\ N,_y & N,_x \end{bmatrix}$

```
B1e = [dN1_dX 0 dN2_dX 0 dN3_dX 0 dN4_dX 0 dN5_dX 0 dN6_dX 0 dN7_dX 0 dN8_dX 0;
       0 dN1_dY 0 dN2_dY 0 dN3_dY 0 dN4_dY 0 dN5_dY 0 dN6_dY 0 dN7_dY 0 dN8_dY;
       dN1_dY dN1_dX dN2_dY dN2_dX dN3_dY dN3_dX dN4_dY dN4_dX dN5_dY dN5_dX dN6_dY dN6_dX dN7_dY
dN7_dX dN8_dY dN8_dX]
B2e = B1e
B3e = B1e
```

7# - Calcular a Matriz de Rigidez por elemento: $K_e = \iint_{\Omega} \left[B^T{}_{3x16} D_{3x3} B_{3x16} \right] \det(J) * t * d\xi d\eta$

```
%==========================================================================================================
%Stiffness Natrix of elements ke - %DEPENDE DA INCIDENCIA DE NOS
np = 4;
%---------------------------------------------------------------------------------------------------------
%Element 1# - 1, 2, 3, 4, 5, 6, 7, 8
```

```
Ke_1 = (B1e')*D*B1e*t*det(J1)
%Gauss Quadrature
solint1 = QuadraturaGauss(Ke_1,np)
Ke_element1 = double(solint1
%---------------------------------------------------------------------------------------------------------
%Element 2# - 5, 6, 7, 9, 10, 11, 12, 13
Ke_2 = (B2e')*D*B2e*t*det(J2)
%Gauss Quadrature
solint2 = QuadraturaGauss(Ke_2,np)
Ke_element2 = double(solint2)
% Ke_element2 = vpa(solint2,2)
%---------------------------------------------------------------------------------------------------------
%Element 3# - 10, 11, 12, 14, 15, 16, 17, 18
Ke_3 = (B3e')*D*B3e*t*det(J3)
%Gauss Quadrature
solint3 = QuadraturaGauss(Ke_3,np)
Ke_element3 = double(solint3)
%---------------------------------------------------------------------------------------------------------
function solint = QuadraturaGauss(f,n)
% QuadraturaGauss Soluciona a quadratura de Gauss para até 6 pontos de integração.
%
% $$ \int_{-1}^{1} \int_{-1}^{1} f(qsi,eta)*dqsi*deta $$
%
% SolNum = QuadraturaGauss(f,n)
% f - função simbólica nas variáveis "qsi" e "eta"
% n - inteiro de 1-6
syms qsi eta
Gaussian = {[0 2];...
    [0.5773502691896262 1;-0.5773502691896262 1];...
    [0.7745966692414833 0.555555555555556;-0.7745966692414833 0.555555555555556;0 0.888888888888889;0
0.888888888888889];...
    [0.8611363115940534 0.347854845137454;-0.8611363115940534 0.347854845137454;0.339981043584856
0.652145154862546;-0.339981043584856 0.652145154862546];...
    [0.9061798459386645 0.236926885056189;-0.9061798459386645 0.236926885056189;0.538469310105683
0.478628670499366;-0.538469310105683 0.478628670499366;0 0.568888888888889];...
    [0.9324695142031526 0.171324492379170;-0.9324695142031526 0.171324492379170;0.661209386466265
0.360761573048139;-0.661209386466265 0.360761573048139;0.238619186083197 0.467913934572691;-0.238619186083197
0.467913934572691]};
Coef = Gaussian{n};
solint = 0;
for i=1:n
    for j=1:n
        solint = solint + Coef(i,2)*Coef(j,2)*subs(f,{qsi,eta},{Coef(i,1) Coef(j,1)});
    end
end
```

K1e = 10^6*

24,07137	-10,5721	-24,1649	4,919014	11,3156	0,014028	-10,0905	5,031235	-7,86481	2,487562	9,515393	-0,01403	-13,4291	2,487562	10,64695	-4,35323
-10,5721	51,23555	5,031235	-74,5131	-0,01403	27,037	4,919014	1,601017	2,487562	-7,61606	0,014028	17,30146	2,487562	-37,7077	-4,35323	22,66188
-24,1649	5,031235	57,30745	0	-24,1649	-5,03123	0	-9,95025	0	9,950249	-13,4291	-2,48756	17,88052	0	-13,4291	2,487562
4,919014	-74,5131	0	152,0338	-4,91901	-74,5131	-9,95025	0	9,950249	0	-2,48756	-37,7077	0	72,40789	2,487562	-37,7077
11,3156	-0,01403	-24,1649	-4,91901	24,07137	10,57214	-7,86481	-2,48756	-10,0905	-5,03123	10,64695	4,353234	-13,4291	-2,48756	9,515393	0,014028
0,014028	27,037	-5,03123	-74,5131	10,57214	51,23555	-2,48756	-7,61606	-4,91901	1,601017	4,353234	22,66188	-2,48756	-37,7077	-0,01403	17,30146
-10,0905	4,919014	0	-9,95025	-7,86481	-2,48756	38,97804	0	-3,06737	0	-10,0905	-4,91901	0	9,950249	-7,86481	2,487562
5,031235	1,601017	-9,95025	0	-2,48756	-7,61606	0	52,90839	0	-40,8783	-5,03123	1,601017	9,950249	0	2,487562	-7,61606
-7,86481	2,487562	0	9,950249	-10,0905	-4,91901	-3,06737	0	38,97804	0	-7,86481	-2,48756	0	-9,95025	-10,0905	4,919014
2,487562	-7,61606	9,950249	0	-5,03123	1,601017	0	-40,8783	0	52,90839	-2,48756	-7,61606	-9,95025	0	5,031235	1,601017
9,515393	0,014028	-13,4291	-2,48756	10,64695	4,353234	-10,0905	-5,03123	-7,86481	-2,48756	24,07137	10,57214	-24,1649	-4,91901	11,3156	-0,01403
-0,01403	17,30146	-2,48756	-37,7077	4,353234	22,66188	-4,91901	1,601017	-2,48756	-7,61606	10,57214	51,23555	-5,03123	-74,5131	0,014028	27,037
-13,4291	2,487562	17,88052	0	-13,4291	-2,48756	0	9,950249	0	-9,95025	-24,1649	-5,03123	57,30745	0	-24,1649	5,031235
2,487562	-37,7077	0	72,40789	-2,48756	-37,7077	9,950249	0	-9,95025	0	-4,91901	-74,5131	0	152,0338	4,919014	-74,5131
10,64695	-4,35323	-13,4291	2,487562	9,515393	-0,01403	-7,86481	2,487562	-10,0905	5,031235	11,3156	0,014028	-24,1649	4,919014	24,07137	-10,5721
-4,35323	22,66188	2,487562	-37,7077	0,014028	17,30146	2,487562	-7,61606	4,919014	1,601017	-0,01403	27,037	5,031235	-74,5131	-10,5721	51,23555

```
K2e = 10^6*
```

24,07137	-10,5721	-24,1649	4,919014	11,3156	0,014028	-10,0905	5,031235	-7,86481	2,487562	9,515393	-0,01403	-13,4291	2,487562	10,64695	-4,35323
-10,5721	51,23555	5,031235	-74,5131	-0,01403	27,037	4,919014	1,601017	2,487562	-7,61606	0,014028	17,30146	2,487562	-37,7077	-4,35323	22,66188
-24,1649	5,031235	57,30745	0	-24,1649	-5,03123	0	-9,95025	0	9,950249	-13,4291	-2,48756	17,88052	0	-13,4291	2,487562
4,919014	-74,5131	0	152,0338	-4,91901	-74,5131	-9,95025	0	9,950249	0	-2,48756	-37,7077	0	72,40789	2,487562	-37,7077
11,3156	-0,01403	-24,1649	-4,91901	24,07137	10,57214	-7,86481	-2,48756	-10,0905	-5,03123	10,64695	4,353234	-13,4291	-2,48756	9,515393	0,014028
0,014028	27,037	-5,03123	-74,5131	10,57214	51,23555	-2,48756	-7,61606	-4,91901	1,601017	4,353234	22,66188	-2,48756	-37,7077	-0,01403	17,30146
-10,0905	4,919014	0	-9,95025	-7,86481	-2,48756	38,97804	0	-3,06737	0	-10,0905	-4,91901	0	9,950249	-7,86481	2,487562
5,031235	1,601017	-9,95025	0	-2,48756	-7,61606	0	52,90839	0	-40,8783	-5,03123	1,601017	9,950249	0	2,487562	-7,61606
-7,86481	2,487562	0	9,950249	-10,0905	-4,91901	-3,06737	0	38,97804	0	-7,86481	-2,48756	0	-9,95025	-10,0905	4,919014
2,487562	-7,61606	9,950249	0	-5,03123	1,601017	0	-40,8783	0	52,90839	-2,48756	-7,61606	-9,95025	0	5,031235	1,601017
9,515393	0,014028	-13,4291	-2,48756	10,64695	4,353234	-10,0905	-5,03123	-7,86481	-2,48756	24,07137	10,57214	-24,1649	-4,91901	11,3156	-0,01403
-0,01403	17,30146	-2,48756	-37,7077	4,353234	22,66188	-4,91901	1,601017	-2,48756	-7,61606	10,57214	51,23555	-5,03123	-74,5131	0,014028	27,037
-13,4291	2,487562	17,88052	0	-13,4291	-2,48756	0	9,950249	0	-9,95025	-24,1649	-5,03123	57,30745	0	-24,1649	5,031235
2,487562	-37,7077	0	72,40789	-2,48756	-37,7077	9,950249	0	-9,95025	0	-4,91901	-74,5131	0	152,0338	4,919014	-74,5131
10,64695	-4,35323	-13,4291	2,487562	9,515393	-0,01403	-7,86481	2,487562	-10,0905	5,031235	11,3156	0,014028	-24,1649	4,919014	24,07137	-10,5721
-4,35323	22,66188	2,487562	-37,7077	0,014028	17,30146	2,487562	-7,61606	4,919014	1,601017	-0,01403	27,037	5,031235	-74,5131	-10,5721	51,23555

```
K3e = 10^6*
```

24,07137	-10,5721	-24,1649	4,919014	11,3156	0,014028	-10,0905	5,031235	-7,86481	2,487562	9,515393	-0,01403	-13,4291	2,487562	10,64695	-4,35323
-10,5721	51,23555	5,031235	-74,5131	-0,01403	27,037	4,919014	1,601017	2,487562	-7,61606	0,014028	17,30146	2,487562	-37,7077	-4,35323	22,66188
-24,1649	5,031235	57,30745	0	-24,1649	-5,03123	0	-9,95025	0	9,950249	-13,4291	-2,48756	17,88052	0	-13,4291	2,487562
4,919014	-74,5131	0	152,0338	-4,91901	-74,5131	-9,95025	0	9,950249	0	-2,48756	-37,7077	0	72,40789	2,487562	-37,7077
11,3156	-0,01403	-24,1649	-4,91901	24,07137	10,57214	-7,86481	-2,48756	-10,0905	-5,03123	10,64695	4,353234	-13,4291	-2,48756	9,515393	0,014028
0,014028	27,037	-5,03123	-74,5131	10,57214	51,23555	-2,48756	-7,61606	-4,91901	1,601017	4,353234	22,66188	-2,48756	-37,7077	-0,01403	17,30146
-10,0905	4,919014	0	-9,95025	-7,86481	-2,48756	38,97804	0	-3,06737	0	-10,0905	-4,91901	0	9,950249	-7,86481	2,487562
5,031235	1,601017	-9,95025	0	-2,48756	-7,61606	0	52,90839	0	-40,8783	-5,03123	1,601017	9,950249	0	2,487562	-7,61606
-7,86481	2,487562	0	9,950249	-10,0905	-4,91901	-3,06737	0	38,97804	0	-7,86481	-2,48756	0	-9,95025	-10,0905	4,919014
2,487562	-7,61606	9,950249	0	-5,03123	1,601017	0	-40,8783	0	52,90839	-2,48756	-7,61606	-9,95025	0	5,031235	1,601017
9,515393	0,014028	-13,4291	-2,48756	10,64695	4,353234	-10,0905	-5,03123	-7,86481	-2,48756	24,07137	10,57214	-24,1649	-4,91901	11,3156	-0,01403
-0,01403	17,30146	-2,48756	-37,7077	4,353234	22,66188	-4,91901	1,601017	-2,48756	-7,61606	10,57214	51,23555	-5,03123	-74,5131	0,014028	27,037
-13,4291	2,487562	17,88052	0	-13,4291	-2,48756	0	9,950249	0	-9,95025	-24,1649	-5,03123	57,30745	0	-24,1649	5,031235
2,487562	-37,7077	0	72,40789	-2,48756	-37,7077	9,950249	0	-9,95025	0	-4,91901	-74,5131	0	152,0338	4,919014	-74,5131
10,64695	-4,35323	-13,4291	2,487562	9,515393	-0,01403	-7,86481	2,487562	-10,0905	5,031235	11,3156	0,014028	-24,1649	4,919014	24,07137	-10,5721
-4,35323	22,66188	2,487562	-37,7077	0,014028	17,30146	2,487562	-7,61606	4,919014	1,601017	-0,01403	27,037	5,031235	-74,5131	-10,5721	51,23555

8# - Montar a Matriz de Rigidez Global: $K_{36x36} = \begin{bmatrix} K_{ele1} & \ldots & 0 \\ 0 & K_{ele2} & 0 \\ 0 & \ldots & K_{ele3} \end{bmatrix}$

```
Ke1=[Ke_element1 sym(zeros(16,20));
     sym(zeros(20,36))];

Ke2=[sym(zeros(10,36));
     sym(zeros(16,10)) Ke_element2 sym(zeros(16,10));
     sym(zeros(10,36))];

Ke3=[sym(zeros(20,36));
     sym(zeros(16,20)) Ke_element3];

K =Ke1+Ke2+Ke3;
```

```
K_global = double(K)*10^6
```

Special program of Doctorate in New Aircraft Concepts and Intelligent Structures (PE-AERO)

AE 245 – Finite Element I

Prof. Alfredo - 2º. Semester of 2015

24,07137	-10,5721	-24,1649	4,919014	11,3156	0,014028	-10,0905	5,031235	-7,86481	2,487562	9,515393	-0,01403	-13,4291	2,487562	10,64695	-4,35323	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-10,5721	51,23555	5,031235	-74,5131	-0,01403	27,037	4,919014	1,601017	2,487562	-7,61606	0,014028	17,30146	2,487562	-37,7077	-4,35323	22,66188	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-24,1649	5,031235	57,30745	0	-24,1649	-5,03123	0	-9,95025	0	9,950249	-13,4291	-2,48756	17,88052	0	-13,4291	2,487562	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4,919014	-74,5131	0	152,0338	-4,91901	-74,5131	-9,95025	0	9,950249	0	-2,48756	-37,7077	0	72,40789	2,487562	-37,7077	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11,3156	-0,01403	-24,1649	-4,91901	24,07137	10,57214	-7,86481	-2,48756	-10,0905	-5,03123	10,64695	4,353234	-13,4291	-2,48756	9,515393	0,014028	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,014028	27,037	-5,03123	-74,5131	10,57214	51,23555	-2,48756	-7,61606	-4,91901	1,601017	4,353234	22,66188	-2,48756	-37,7077	-0,01403	17,30146	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-10,0905	4,919014	0	-9,95025	-7,86481	-2,48756	38,97804	0	-3,06737	0	-10,0905	-4,91901	0	9,950249	-7,86481	2,487562	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5,031235	1,601017	-9,95025	0	-2,48756	-7,61606	0	52,90839	0	-40,8783	-5,03123	1,601017	9,950249	0	2,487562	-7,61606	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-7,86481	2,487562	0	9,950249	-10,0905	-4,91901	-3,06737	0	38,97804	0	-7,86481	-2,48756	0	-9,95025	-10,0905	4,919014	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2,487562	-7,61606	9,950249	0	-5,03123	1,601017	0	-40,8783	0	52,90839	-2,48756	-7,61606	-9,95025	0	5,031235	1,601017	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9,515393	0,014028	-13,4291	-2,48756	10,64695	4,353234	-10,0905	-5,03123	-7,86481	-2,48756	48,14274	0	-48,3298	0	22,6312	0	-10,0905	5,031235	-7,86481	2,487562	9,515393	-0,01403	-13,4291	2,487562	10,64695	-4,35323	0	0	0	0	0	0	0	0	0	0
-0,01403	17,30146	-2,48756	-37,7077	4,353234	22,66188	-4,91901	1,601017	-2,48756	-7,61606	0	102,4711	0	-149,026	0	54,07399	4,919014	1,601017	2,487562	-7,61606	0,014028	17,30146	2,487562	-37,7077	-4,35323	22,66188	0	0	0	0	0	0	0	0	0	0
-13,4291	2,487562	17,88052	0	-13,4291	-2,48756	0	9,950249	0	-9,95025	-48,3298	0	114,6149	0	-48,3298	0	0	-9,95025	0	9,950249	-13,4291	-2,48756	17,88052	0	-13,4291	2,487562	0	0	0	0	0	0	0	0	0	0
2,487562	-37,7077	0	72,40789	-2,48756	-37,7077	9,950249	0	-9,95025	0	0	-149,026	0	304,0676	0	-149,026	-9,95025	0	9,950249	0	-2,48756	-37,7077	0	72,40789	2,487562	-37,7077	0	0	0	0	0	0	0	0	0	0
10,64695	-4,35323	-13,4291	2,487562	9,515393	-0,01403	-7,86481	2,487562	-10,0905	5,031235	22,6312	0	-48,3298	0	48,14274	0	-7,86481	-2,48756	-10,0905	-5,03123	10,64695	4,353234	-13,4291	-2,48756	9,515393	0,014028	0	0	0	0	0	0	0	0	0	0
-4,35323	22,66188	2,487562	-37,7077	0,014028	17,30146	2,487562	-7,61606	4,919014	1,601017	0	54,07399	0	-149,026	0	102,4711	-2,48756	-7,61606	-4,91901	1,601017	4,353234	22,66188	-2,48756	-37,7077	-0,01403	17,30146	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-10,0905	4,919014	0	-9,95025	-7,86481	-2,48756	38,97804	0	-3,06737	0	-10,0905	-4,91901	0	9,950249	-7,86481	2,487562	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	5,031235	1,601017	-9,95025	0	-2,48756	-7,61606	0	52,90839	0	-40,8783	-5,03123	1,601017	9,950249	0	2,487562	-7,61606	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-7,86481	2,487562	0	9,950249	-10,0905	-4,91901	-3,06737	0	38,97804	0	-7,86481	-2,48756	0	-9,95025	-10,0905	4,919014	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	2,487562	-7,61606	9,950249	0	-5,03123	1,601017	0	-40,8783	0	52,90839	-2,48756	-7,61606	-9,95025	0	5,031235	1,601017	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	9,515393	0,014028	-13,4291	-2,48756	10,64695	4,353234	-10,0905	-5,03123	-7,86481	-2,48756	48,14274	0	-48,3298	0	22,6312	0	-10,0905	5,031235	-7,86481	2,487562	9,515393	-0,01403	-13,4291	2,487562	10,64695	-4,35323
0	0	0	0	0	0	0	0	0	0	-0,01403	17,30146	-2,48756	-37,7077	4,353234	22,66188	-4,91901	1,601017	-2,48756	-7,61606	0	102,4711	0	-149,026	0	54,07399	4,919014	1,601017	2,487562	-7,61606	0,014028	17,30146	2,487562	-37,7077	-4,35323	22,66188
0	0	0	0	0	0	0	0	0	0	-13,4291	2,487562	17,88052	0	-13,4291	-2,48756	0	9,950249	0	-9,95025	-48,3298	0	114,6149	0	-48,3298	0	0	-9,95025	0	9,950249	-13,4291	-2,48756	17,88052	0	-13,4291	2,487562
0	0	0	0	0	0	0	0	0	0	2,487562	-37,7077	0	72,40789	-2,48756	-37,7077	9,950249	0	-9,95025	0	0	-149,026	0	304,0676	0	-149,026	-9,95025	0	9,950249	0	-2,48756	-37,7077	0	72,40789	2,487562	-37,7077
0	0	0	0	0	0	0	0	0	0	10,64695	-4,35323	-13,4291	2,487562	9,515393	-0,01403	-7,86481	2,487562	-10,0905	5,031235	22,6312	0	-48,3298	0	48,14274	0	-7,86481	-2,48756	-10,0905	-5,03123	10,64695	4,353234	-13,4291	-2,48756	9,515393	0,014028
0	0	0	0	0	0	0	0	0	0	-4,35323	22,66188	2,487562	-37,7077	0,014028	17,30146	2,487562	-7,61606	4,919014	1,601017	0	54,07399	0	-149,026	0	102,4711	-2,48756	-7,61606	-4,91901	1,601017	4,353234	22,66188	-2,48756	-37,7077	-0,01403	17,30146
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-10,0905	4,919014	0	-9,95025	-7,86481	-2,48756	38,97804	0	-3,06737	0	-10,0905	-4,91901	0	9,950249	-7,86481	2,487562
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5,031235	1,601017	-9,95025	0	-2,48756	-7,61606	0	52,90839	0	-40,8783	-5,03123	1,601017	9,950249	0	2,487562	-7,61606
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-7,86481	2,487562	0	9,950249	-10,0905	-4,91901	-3,06737	0	38,97804	0	-7,86481	-2,48756	0	-9,95025	-10,0905	4,919014
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2,487562	-7,61606	9,950249	0	-5,03123	1,601017	0	-40,8783	0	52,90839	-2,48756	-7,61606	-9,95025	0	5,031235	1,601017
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9,515393	0,014028	-13,4291	-2,48756	10,64695	4,353234	-10,0905	-5,03123	-7,86481	-2,48756	24,07137	10,57214	-24,1649	-4,91901	11,3156	-0,01403
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0,01403	17,30146	-2,48756	-37,7077	4,353234	22,66188	-4,91901	1,601017	-2,48756	-7,61606	10,57214	51,23555	-5,03123	-74,5131	0,014028	27,037
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-13,4291	2,487562	17,88052	0	-13,4291	-2,48756	0	9,950249	0	-9,95025	-24,1649	-5,03123	57,30745	0	-24,1649	5,031235
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2,487562	-37,7077	0	72,40789	-2,48756	-37,7077	9,950249	0	-9,95025	0	-4,91901	-74,5131	0	152,0338	4,919014	-74,5131
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10,64695	-4,35323	-13,4291	2,487562	9,515393	-0,01403	-7,86481	2,487562	-10,0905	5,031235	11,3156	0,014028	-24,1649	4,919014	24,07137	-10,5721
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-4,35323	22,66188	2,487562	-37,7077	0,014028	17,30146	2,487562	-7,61606	4,919014	1,601017	-0,01403	27,037	5,031235	-74,5131	-10,5721	51,23555

9# - Zerar a linhas e colunas que possuem os constraints: 1, 3, 4 e 5 da Matriz Global de Rigidez

```
%==================================================================================================
%Cancelando as linhas que tem reacoes
nr = 5;% Numero de Reacoes
nn = 18; %Numero de Nós
F = zeros(36,1);
F(16*2,1) = (1/6)*10000
F(17*2,1) = (4/6)*10000
F(18*2,1) = (1/6)*10000

K_global(1,:)=0;
K_global(:,1)=0;
K_global(3,:)=0;
K_global(:,3)=0;
K_global(4,:)=0;
K_global(:,4)=0;
K_global(5,:)=0;
K_global(:,5)=0;
K_global(1,1)=1;
K_global(3,3)=1;
K_global(4,4)=1;
K_global(5,5)=1;
```

10# - Determinar os deslocamento, strass e strains: $u = Nd, \sigma = DBd$ utilizando o vetor de carregamento igual a $F = \left[0...0...F/6^{nó=32}...F*4/6^{nó=34}...F/6^{nó=36}\right]$

```
K_global
Disp = inv(K_global)*F

%Element 1:
sigma_e1 = D*B1e*Disp(1:16,1)
sigma_e2 = D*B2e*Disp(11:26,1)
sigma_e3 = D*B3e*Disp(21:36,1)
```

Deslocamento (in)							
0	**1**	-0,03124	**11**	-0,04998	**21**	-0,05628	**31**
0,001224	**2**	0,084927	**12**	0,293588	**22**	0,564827	**32**
0	**3**	-1,27E-17	**13**	-5,99E-17	**23**	-8,64E-17	**33**
0	**4**	0,084096	**14**	0,293186	**24**	0,564787	**34**
0	**5**	0,031238	**15**	0,049979	**25**	0,056278	**35**
0,001224	**6**	0,084927	**16**	0,293588	**26**	0,564827	**36**
-0,01671	**7**	-0,04171	**17**	-0,05421	**27**		
0,023552	**8**	0,177544	**18**	0,425271	**28**		
0,016715	**9**	0,041715	**19**	0,054215	**29**		
0,023552	**10**	0,177544	**20**	0,425271	**30**		

sigma_element1(:,:,1) =

Nó	1	2	3	4	5	6	7	8
Coord.	ξ=-1, η=1 1.0e+05 *	ξ=-1, η=0 1.0e+04 *	ξ=-1, η=-1 1.0e+05 *	ξ=0, η=-1 1.0e+05 *	ξ=-1, η=1 1.0e+05 *	ξ=-1, η=1 1.0e+05 *	ξ=-1, η=1 1.0e+04 *	ξ=-1, η=1 1.0e+05 *
σ_x	53.126	-0.0000	-53.126	46.875	-46.875	40.624	-0.0000	-40.624
σ_y	-0.0791	-0.0000	0.0791	0.0037	-0.0037	0.0865	-0.0000	-0.0865
τ_{xy}	-0.3194	-34.023	-0.3194	-0.0070	-0.0070	-0.3194	-34.023	-0.3194

sigma_element2(:,:,1) =

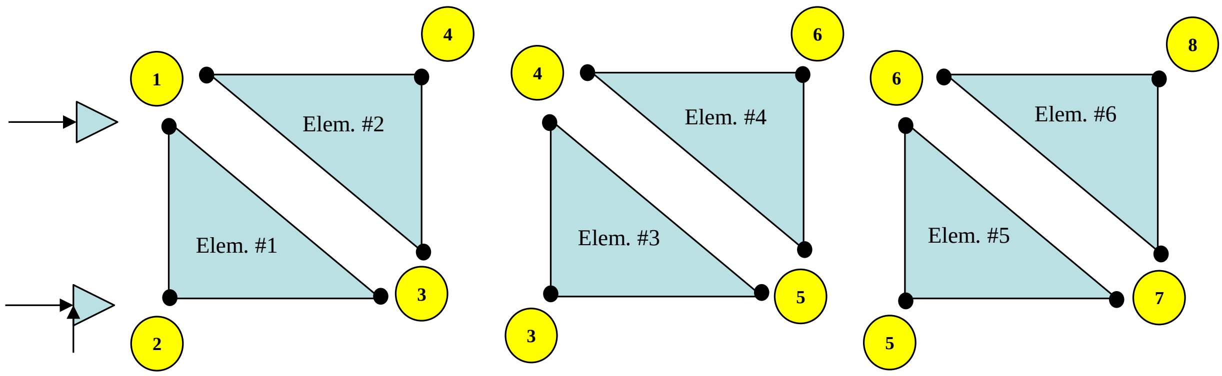
Nó	1	2	3	4	5	6	7	8
Coord.	ξ=-1, η=1 1.0e+05 *	ξ=-1, η=0 1.0e+04 *	ξ=-1, η=-1 1.0e+05 *	ξ=0, η=-1 1.0e+05 *	ξ=-1, η=1 1.0e+05 *	ξ=-1, η=1 1.0e+05 *	ξ=-1, η=1 1.0e+04 *	ξ=-1, η=1 1.0e+05 *
σ_x	34.353	-0.0000	-34.353	28.125	-28.125	21.897	-0.0000	-21.897
σ_y	-0.1016	-0.0000	0.1016	0.0040	-0.0040	0.1096	-0.0000	-0.1096
τ_{xy}	-0.3198	-34.228	-0.3198	-0.0051	-0.0051	-0.3198	-34.228	-0.3198

sigma_element3(:,:,1) =

Nó	1	2	3	4	5	6	7	8
Coord.	ξ=-1, η=1 1.0e+05 *	ξ=-1, η=0 1.0e+05 *	ξ=-1, η=-1 1.0e+05 *	ξ=0, η=-1 1.0e+04 *	ξ=-1, η=1 1.0e+04 *	ξ=-1, η=1 1.0e+04 *	ξ=-1, η=1 1.0e+04 *	ξ=-1, η=1 1.0e+04 *
σ_x	-15.650	-15.650	-15.650	-93.750	-93.750	-30.999	-30.999	-30.999
σ_y	0.0778	0.0778	0.0778	0.2020	0.2020	-0.3738	-0.3738	-0.3738
τ_{xy}	-0.3191	-0.3191	-0.3191	-0.0906	-0.0906	-31.905	-31.905	-31.905

Triangular Elements: 10 steps

0# - Numerar os elementos:



1# - Coordenadas dos Nós dos Elementos: $P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8$

```
%=======================================================================
%Coordinate of Elements  - Nodes - DEPENDE DA INCIDENCIA DE NOS
syms qsi eta

syms N(qsi,eta) N1(qsi,eta) N2(qsi,eta) N3(qsi,eta) N4(qsi,eta) N5(qsi,eta) N6(qsi,eta) N7(qsi,eta) N8(qsi,eta)
N9(qsi,eta)...
    N10(qsi,eta) N11(qsi,eta) N12(qsi,eta) N13(qsi,eta) N14(qsi,eta) N15(qsi,eta) N16(qsi,eta) N17(qsi,n)
N18(qsi,eta)

syms X1(qsi,eta) Y1(qsi,eta) X2(qsi,eta) Y2(qsi,eta) X3(qsi,eta) Y3(qsi,eta)

      %   1    2    3    4    5    6   7    8
vec_x=[0,0,2,2,4,4,6,6];
vec_y=[0.4, -0.4, 0.4, -0.4, 0.4, -0.4, 0.4, -0.4];
```

2# - Funções de Interpolação: $\begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} = \begin{bmatrix} 1\,1\,1 \\ \xi\,\xi\,\xi \\ \eta\,\eta\,\eta \end{bmatrix} \begin{bmatrix} 1 \\ \xi \\ \eta \end{bmatrix} onde\ \xi = [-1,1], \eta = [-1,1]$

```
%=======================================================================
%Interpolation Functions of 8-noded Serendipity element - DEPENDE DA INCIDENCIA DE NOS

        %1 %2  %3
N_c =  [ 1  1  1;
        -1  1  0;
        -1 -1  1;]

N = [1; qsi; eta;]

N_int = N_c*N

N1(qsi,eta) = N_int(1,1)
N2(qsi,eta) = N_int(2,1)
N3(qsi,eta) = N_int(3,1)

%=======================================================================
%Derivation Interpolation Functions of 8-noded Serendipity element

dN1_dE= diff(N1,qsi)
dN2_dE= diff(N2,qsi)
dN3_dE= diff(N3,qsi)
```

3# - Calcular: $x(\xi,\eta)=\sum_{i=1}^{m} Ni(\xi,\eta)*x_i$ e $y(\xi,\eta)=\sum_{i=1}^{m} Ni(\xi,\eta)*x_i$

```
%=========================================================================
%Isoparametric Formulation - DEPENDE DA INCIDENCIA DE NOS

syms X1(qsi,eta) Y1(qsi,eta) X2(qsi,eta) Y2(qsi,eta) X3(qsi,eta) Y3(qsi,eta)...
     X4(qsi,eta) Y4(qsi,eta) X5(qsi,eta) Y5(qsi,eta) X6(qsi,eta) Y6(qsi,eta)
%Element 1#
X1(qsi,eta) = vec_x(1)*N3(qsi,eta) + vec_x(3)*N2(qsi,eta) + vec_x(2)*N1(qsi,eta)
Y1(qsi,eta) = vec_y(1)*N3(qsi,eta) + vec_y(3)*N2(qsi,eta) + vec_y(2)*N1(qsi,eta)
%Element 2#
X2(qsi,eta) = vec_x(2)*N1(qsi,eta) + vec_x(4)*N2(qsi,eta) + vec_x(3)*N3(qsi,eta)
Y2(qsi,eta) = vec_y(2)*N1(qsi,eta) + vec_y(4)*N2(qsi,eta) + vec_y(3)*N3(qsi,eta)
%Element 3#
X3(qsi,eta) = vec_x(4)*N1(qsi,eta) + vec_x(3)*N3(qsi,eta) + vec_x(5)*N2(qsi,eta)
Y3(qsi,eta) = vec_y(4)*N1(qsi,eta) + vec_y(3)*N3(qsi,eta) + vec_y(5)*N2(qsi,eta)
%Element 4#
X4(qsi,eta) = vec_x(4)*N1(qsi,eta) + vec_x(6)*N2(qsi,eta) + vec_x(5)*N3(qsi,eta)
Y4(qsi,eta) = vec_y(4)*N1(qsi,eta) + vec_y(6)*N2(qsi,eta) + vec_y(5)*N3(qsi,eta)
%Element 5#
X5(qsi,eta) = vec_x(6)*N1(qsi,eta) + vec_x(5)*N3(qsi,eta) + vec_x(7)*N2(qsi,eta)
Y5(qsi,eta) = vec_y(6)*N1(qsi,eta) + vec_y(5)*N3(qsi,eta) + vec_y(7)*N2(qsi,eta)
%Element 6#
X6(qsi,eta) = vec_x(6)*N1(qsi,eta) + vec_x(8)*N2(qsi,eta) + vec_x(7)*N3(qsi,eta)
Y6(qsi,eta) = vec_y(6)*N1(qsi,eta) + vec_y(8)*N2(qsi,eta) + vec_y(7)*N3(qsi,eta)
%=========================================================================
%=========================================================================
%Derivation Interpolation Functions of 8-noded Serendipity element

dN1_dE= diff(N1,qsi)
dN2_dE= diff(N2,qsi)
dN3_dE= diff(N3,qsi)

dN1_dn= diff(N1,eta)
dN2_dn= diff(N2,eta)
dN3_dn= diff(N3,eta)
```

4# - Determinante da matriz Jacobiana: $J=\begin{bmatrix} X,_{\xi} & Y,_{\xi} \\ X,_{\xi} & Y,_{\xi} \end{bmatrix}$

```
%%Determinant. Jacobian:

det_J1 = (x1(1)-x1(3))*(y1(2)-y1(3))-(x1(2)-x1(3))*(y1(1)-y1(3));
det_J2 = (x2(1)-x2(3))*(y2(2)-y2(3))-(x2(2)-x2(3))*(y2(1)-y2(3));
det_J3 = (x3(1)-x3(3))*(y3(2)-y3(3))-(x3(2)-x3(3))*(y3(1)-y3(3));
det_J4 = (x4(1)-x4(3))*(y4(2)-y4(3))-(x4(2)-x4(3))*(y4(1)-y4(3));
det_J5 = (x5(1)-x5(3))*(y5(2)-y5(3))-(x5(2)-x5(3))*(y5(1)-y5(3));
det_J6 = (x6(1)-x6(3))*(y6(2)-y6(3))-(x6(2)-x6(3))*(y6(1)-y6(3));
```

det_J1 =-1.6000
det_J2 = 1.6000
det_J3 =-1.6000
det_J4 =1.6000
det_J5 =-1.6000
det_J6 =1.6000

5# - Calcular Matriz: $B=\sum_{i=1}^{3}\begin{bmatrix} N,_x & 0 \\ 0 & N,_y \\ N,_y & N,_x \end{bmatrix}$

B1e =

21,01648	10,71429	-14,4231	-4,94505	-6,59341	-5,76923
10,71429	43,51648	-5,76923	-41,2088	-4,94505	-2,30769
-14,4231	-5,76923	21,01648	0	0	10,71429

B2e =

-0,5	0	0	0	0,5	0
0	0	0	-1,25	0	1,25
0	-0,5	-1,25	0	1,25	0,5

B3e =

-0,5	0	0	0	0,5	0
0	-1,25	0	1,25	0	0
-1,25	-0,5	1,25	0	0	0,5

B4e =

-0,5	0	0	0	0,5	0
0	0	0	-1,25	0	1,25
0	-0,5	-1,25	0	1,25	0,5

B5e =

-0,5	0	0	0	0,5	0
0	-1,25	0	1,25	0	0
-1,25	-0,5	1,25	0	0	0,5

B6e =

-0,5	0	0	0	0,5	0
0	0	0	-1,25	0	1,25
0	-0,5	-1,25	0	1,25	0,5

6# - Calcular a Matriz de Rigidez por elemento:

K1 = 10^6*

21,01648	10,71429	-14,4231	-4,94505	-6,59341	-5,76923
10,71429	43,51648	-5,76923	-41,2088	-4,94505	-2,30769
-14,4231	-5,76923	14,42308	0	0	5,769231
-4,94505	-41,2088	0	41,20879	4,945055	0
-6,59341	-4,94505	0	4,945055	6,593407	0
-5,76923	-2,30769	5,769231	0	0	2,307692

K2 = 10^6*

6,593407	0	0	4,945055	-6,59341	-4,94505
0	2,307692	5,769231	0	-5,76923	-2,30769
0	5,769231	14,42308	0	-14,4231	-5,76923
4,945055	0	0	41,20879	-4,94505	-41,2088
-6,59341	-5,76923	-14,4231	-4,94505	21,01648	10,71429
-4,94505	-2,30769	-5,76923	-41,2088	10,71429	43,51648

K3 = 10^6*

21,01648	10,71429	-14,4231	-4,94505	-6,59341	-5,76923
10,71429	43,51648	-5,76923	-41,2088	-4,94505	-2,30769
-14,4231	-5,76923	14,42308	0	0	5,769231
-4,94505	-41,2088	0	41,20879	4,945055	0
-6,59341	-4,94505	0	4,945055	6,593407	0
-5,76923	-2,30769	5,769231	0	0	2,307692

K4 = 10^6*

6,593407	0	0	4,945055	-6,59341	-4,94505
0	2,307692	5,769231	0	-5,76923	-2,30769
0	5,769231	14,42308	0	-14,4231	-5,76923
4,945055	0	0	41,20879	-4,94505	-41,2088
-6,59341	-5,76923	-14,4231	-4,94505	21,01648	10,71429
-4,94505	-2,30769	-5,76923	-41,2088	10,71429	43,51648

K5 = 10^6*

21,01648	10,71429	-14,4231	-4,94505	-6,59341	-5,76923
10,71429	43,51648	-5,76923	-41,2088	-4,94505	-2,30769
-14,4231	-5,76923	14,42308	0	0	5,769231
-4,94505	-41,2088	0	41,20879	4,945055	0
-6,59341	-4,94505	0	4,945055	6,593407	0
-5,76923	-2,30769	5,769231	0	0	2,307692

K6 = 10^6*

6,593407	0	0	4,945055	-6,59341	-4,94505
0	2,307692	5,769231	0	-5,76923	-2,30769
0	5,769231	14,42308	0	-14,4231	-5,76923
4,945055	0	0	41,20879	-4,94505	-41,2088
-6,59341	-5,76923	-14,4231	-4,94505	21,01648	10,71429
-4,94505	-2,30769	-5,76923	-41,2088	10,71429	43,51648

7# - Montar a Matriz de Rigidez Global: $K_{16x16} = \begin{bmatrix} K_{ele1} & 0 & ... & 0 & 0 & 0 \\ 0 & K_{ele2} & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{ele3} & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{ele4} & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{ele5} & 0 \\ 0 & 0 & ... & 0 & 0 & K_{ele6} \end{bmatrix}$

K_Global = 10^6*

21,01648	10,71429	-14,4231	-4,94505	-6,59341	-5,76923	0	0	0	0	0	0	0	0	0	0
10,71429	43,51648	-5,76923	-41,2088	-4,94505	-2,30769	0	0	0	0	0	0	0	0	0	0
-14,4231	-5,76923	21,01648	0	0	10,71429	-6,59341	-4,94505	0	0	0	0	0	0	0	0
-4,94505	-41,2088	0	43,51648	10,71429	0	-5,76923	-2,30769	0	0	0	0	0	0	0	0
-6,59341	-4,94505	0	10,71429	42,03297	10,71429	-28,8462	-10,7143	-6,59341	-5,76923	0	0	0	0	0	0
-5,76923	-2,30769	10,71429	0	10,71429	87,03297	-10,7143	-82,4176	-4,94505	-2,30769	0	0	0	0	0	0
0	0	-6,59341	-5,76923	-28,8462	-10,7143	42,03297	10,71429	0	10,71429	-6,59341	-4,94505	0	0	0	0
0	0	-4,94505	-2,30769	-10,7143	-82,4176	10,71429	87,03297	10,71429	0	-5,76923	-2,30769	0	0	0	0
0	0	0	0	-6,59341	-4,94505	0	10,71429	42,03297	10,71429	-28,8462	-10,7143	-6,59341	-5,76923	0	0
0	0	0	0	-5,76923	-2,30769	10,71429	0	10,71429	87,03297	-10,7143	-82,4176	-4,94505	-2,30769	0	0
0	0	0	0	0	0	-6,59341	-5,76923	-28,8462	-10,7143	42,03297	10,71429	0	10,71429	-6,59341	-4,94505
0	0	0	0	0	0	-4,94505	-2,30769	-10,7143	-82,4176	10,71429	87,03297	10,71429	0	-5,76923	-2,30769
0	0	0	0	0	0	0	0	-6,59341	-4,94505	0	10,71429	21,01648	0	-14,4231	-5,76923
0	0	0	0	0	0	0	0	-5,76923	-2,30769	10,71429	0	0	43,51648	-4,94505	-41,2088
0	0	0	0	0	0	0	0	0	0	-6,59341	-5,76923	-14,4231	-4,94505	21,01648	10,71429
0	0	0	0	0	0	0	0	0	0	-4,94505	-2,30769	-5,76923	-41,2088	10,71429	43,51648

8# - Zerar a linhas e colunas que possuem os constraints: 1, 3 e 4 da Matriz Global de Rigidez

K_Global = 10^6*

1,00E-06	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	43,51648	0	0	-4,94505	-2,30769	0	0	0	0	0	0	0	0	0	0
0	0	1,00E-06	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1,00E-06	0	0	0	0	0	0	0	0	0	0	0	0
0	-4,94505	0	0	42,03297	10,71429	-28,8462	-10,7143	-6,59341	-5,76923	0	0	0	0	0	0
0	-2,30769	0	0	10,71429	87,03297	-10,7143	-82,4176	-4,94505	-2,30769	0	0	0	0	0	0
0	0	0	0	-28,8462	-10,7143	42,03297	10,71429	0	10,71429	-6,59341	-4,94505	0	0	0	0
0	0	0	0	-10,7143	-82,4176	10,71429	87,03297	10,71429	0	-5,76923	-2,30769	0	0	0	0
0	0	0	0	-6,59341	-4,94505	0	10,71429	42,03297	10,71429	-28,8462	-10,7143	-6,59341	-5,76923	0	0
0	0	0	0	-5,76923	-2,30769	10,71429	0	10,71429	87,03297	-10,7143	-82,4176	-4,94505	-2,30769	0	0
0	0	0	0	0	0	-6,59341	-5,76923	-28,8462	-10,7143	42,03297	10,71429	0	10,71429	-6,59341	-4,94505
0	0	0	0	0	0	-4,94505	-2,30769	-10,7143	-82,4176	10,71429	87,03297	10,71429	0	-5,76923	-2,30769
0	0	0	0	0	0	0	0	-6,59341	-4,94505	0	10,71429	21,01648	0	-14,4231	-5,76923
0	0	0	0	0	0	0	0	-5,76923	-2,30769	10,71429	0	0	43,51648	-4,94505	-41,2088
0	0	0	0	0	0	0	0	0	0	-6,59341	-5,76923	-14,4231	-4,94505	21,01648	10,71429
0	0	0	0	0	0	0	0	0	0	-4,94505	-2,30769	-5,76923	-41,2088	10,71429	43,51648

9# - Determinar os deslocamento, strass e strains: $u = Nd, \sigma = DBd$ utilizando o vetor de carregamento igual a $F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & F/2 & 0 & F/2 \end{bmatrix}$

```
%Displacement
d = K_Global\F
%Stress:
sigma_e1 = vpa(D*B_e1*d(1:6,1),2)
sigma_e2 = vpa(D*B_e2*d(3:8,1),2)
sigma_e3 = vpa(D*B_e3*d(5:10,1),2)
sigma_e4 = vpa(D*B_e4*d(7:12,1),2)
sigma_e5 = vpa(D*B_e5*d(9:14,1),2)
sigma_e6 = vpa(D*B_e6*d(11:16,1),2)
```

Deslocamento (in)			
0	1	-0,03368	10
-0,00097	2	0,004872	11
0	3	-0,03385	12
0	4	-0,00569	13
-0,00357	5	-0,06259	14
-0,01073	6	0,005595	15
0,002961	7	-0,06276	16
-0,0109	8		
-0,00523	9		

Sigmas	Sigma 1	Sigma 2	Sigma 3	Sigma 4	Sigma 5	Sigma 6
σ_x	-4.7e4	4.7e4	-2.9e4	2.9e4	-9.8e3	9.8e3
σ_y	2.3e4	7.8e3	-1.5e4	2.4e3	-9.3e3	-3.4e3
τ_{xy}	-5.6e4	3.1e4	-3.8e4	1.3e4	-2.1e4	-3.9e3

b) Classical beam theory in bending.

$$u = \left(\frac{P}{EI}\right)\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right) \quad onde \quad I = \frac{b*h^3}{12} = \frac{8^3*1}{12}$$

```
E = 30e6; % Unit: [PSI]
v = 0.3;
t = 1; % Unit: [in]
P=10000
L=6
h = 0.8
I = t*(h^3)/12
j=1
for i=1:6
    x=i;
    u(j) = (P/E*I)*(L*(x^2)/2 - (x^3)/6)
    j = j + 1;
end
```

Results:

Deslocamento (in)											
1		2		3		4		5		6	
x	y	x	y	x	y	x	y	x	y	x	y
0	4,03E-05	0	-0,0001517	0	-0,00032	0	-0,00053096	0	-0,00077037	0	-0,001024