

AE 245 - Finite Element I

Prof. Alfredo - 2º. Semester of 2015

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AE-245 FINITE ELEMENT I FINAL EXAM

Due: December 05th, 2015



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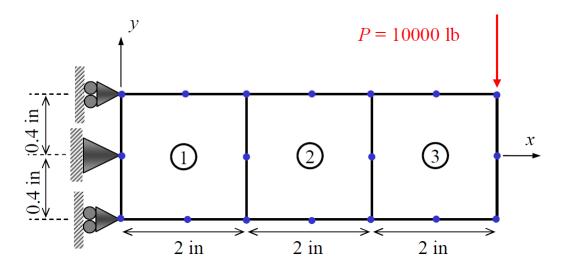
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AE-245: Final Assignment Cantilever beam

Analyze the cantilever beam shown below using three 8-noded isoperimetric elements. Compare your results for stress Sigma x and transverse displacements along y = 0 with:

- a) Model with six constant strain triangle elements;
- b) Classical beam theory in bending.

Show the computed stiffness matrices of the elements used in the analysis. Consider: thickness t = 1.0 in, $E = 30*10^6$ psi e v= 0.3.





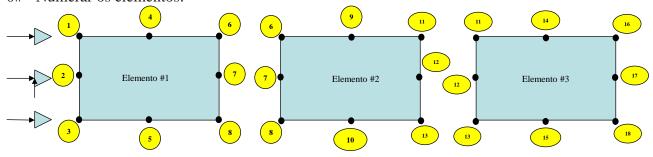
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a) Model with six constant strain triangle elements;

Quadrilateral Elements: 11 steps

0# - Numerar os elementos:



1# - Coordenadas dos Nós dos Elementos:

$$P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}$$

2# - Funções de Interpolação:

$$\begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \\ N_8 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \xi & \xi \\ \eta & \eta \\ \xi^2 & \xi^2 \\ \xi \eta & \xi \eta \\ \xi^2 & \xi^2 \\ \xi \eta & \xi \eta \\ \xi^2 & \xi^2 \\ \xi^2 \eta & \xi^2 \eta \\ \xi \eta^2 & \xi \eta^2 \end{bmatrix} \begin{bmatrix} 1 \\ \xi \\ \eta \\ \xi^2 \\ \xi^2 \\ \xi^2 \eta \\ \xi \eta^2 & \xi \eta^2 \\ \xi \eta^2 & \xi \eta^2 \end{bmatrix} \begin{bmatrix} 1 \\ \xi \\ \xi \eta \\ \xi^2 \\ \xi^2 \\ \xi^2 \eta \\ \xi \eta^2 & \xi \eta^2 \end{bmatrix} \begin{bmatrix} 1 \\ \xi \\ \xi \eta \\ \xi \eta \\ \xi \eta^2 \end{bmatrix}$$

%Interpolation Functions of 8-noded Serendipity element - DEPENDE DA INCIDENCIA DE NOS

```
N5(qsi,eta) = 1/2*(1-qsi^2)*(1-eta)%N5

N7(qsi,eta) = 1/2*(1+qsi)*(1-eta^2)%N6

N4(qsi,eta) = 1/2*(1-qsi^2)*(1+eta)%N7

N2(qsi,eta) = 1/2*(1-qsi)*(1-eta^2)%N8

N3(qsi,eta) = 1/4*(1-qsi)*(1-eta) - 1/2*(N2(qsi,eta) + N5(qsi,eta))%N1

N8(qsi,eta) = 1/4*(1+qsi)*(1-eta) - 1/2*(N5(qsi,eta) + N7(qsi,eta))%N2

N6(qsi,eta) = 1/4*(1+qsi)*(1+eta) - 1/2*(N7(qsi,eta) + N4(qsi,eta))%N3

N1(qsi,eta) = 1/4*(1-qsi)*(1+eta) - 1/2*(N4(qsi,eta) + N2(qsi,eta))%N4
```

 $Derivation \ Interpolation \ Functions of 8-noded \ Serendipity element$



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```
dN1_dE= diff(N1,qsi)
dN2_dE= diff(N2,qsi)
dN3_dE= diff(N3,qsi)
dN4_dE= diff(N4,qsi)
dN5 dE= diff(N5,qsi)
dN6_dE= diff(N6,qsi)
dN7_dE= diff(N7,qsi)
dN8_dE= diff(N8,qsi)
dN1_dn= diff(N1,eta)
dN2 dn= diff(N2,eta)
dN3_dn= diff(N3,eta)
dN4_dn= diff(N4,eta)
dN5_dn= diff(N5,eta)
dN6_dn= diff(N6,eta)
dN7_dn= diff(N7,eta)
dN8_dn= diff(N8,eta)
dN1_dE(qsi, eta) = (\xi * (\eta + 1))/2 - \eta/4 - \eta^2/4
dN2_dE(qsi, eta) = \eta^2/2 - 1/2
dN3 dE(qsi, eta) = \eta/4 - (\xi * (\eta - 1))/2 - \eta^2/4
dN4 dE(qsi, eta) = -\xi * (\eta + 1)
N5 dE(qsi, eta) = \xi * (\eta - 1)
dN6_dE(qsi, eta) = \eta/4 + (\xi * (\eta + 1))/2 + \eta^2/4
dN7_dE(qsi, eta) = 1/2 - \eta^2/2
dN8_dE(qsi, eta) = \eta^2/4 - (\xi * (\eta - 1))/2 - \eta/4
dN1_dn(qsi, eta) = \xi^2/4 - \xi/4 - \eta*(\xi/2 - 1/2)
dN2_dn(qsi, eta) = 2*\eta*(\xi/2 - 1/2)
dN3_dn(qsi, eta) = \xi/4 - \xi^2/4 - \eta*(\xi/2 - 1/2)
dN4 dn(qsi, eta) = 1/2 - \xi^2/2
dN5 dn(qsi, eta) = \xi^{2/2} - \frac{1}{2}
dN6_dn(qsi, eta) = \xi/4 + \xi^2/4 + \eta * (\xi/2 + 1/2)
dN7_dn(qsi, eta) = -2*\eta*(\xi/2 + 1/2)
dN8_dn(qsi, eta) = \eta * (\xi/2 + 1/2) - \xi^2/4 - \xi/4
```

```
3# - Calcular: x(\xi,\eta) = \sum_{i=1}^{m} Ni(\xi,\eta) * x_i e y(\xi,\eta) = \sum_{i=1}^{m} Ni(\xi,\eta) * x_i
```

```
%Isoparametric Formulation - DEPENDE DA INCIDENCIA DE NOS
```

```
syms X1(qsi,eta) Y1(qsi,eta) X2(qsi,eta) Y2(qsi,eta) X3(qsi,eta) Y3(qsi,eta)
 %Element 1#
 X1(qsi,eta) = vec x(1)*N1(qsi,eta) + vec x(2)*N2(qsi,eta) + vec x(3)*N3(qsi,eta) + vec x(4)*N4(qsi,eta) + vec x(
 \texttt{vec}_{\texttt{x}}(\texttt{5}) * \texttt{N5}(\texttt{qsi}, \texttt{eta}) + \texttt{vec}_{\texttt{x}}(\texttt{6}) * \texttt{N6}(\texttt{qsi}, \texttt{eta}) + \texttt{vec}_{\texttt{x}}(\texttt{7}) * \texttt{N7}(\texttt{qsi}, \texttt{eta}) + \texttt{vec}_{\texttt{x}}(\texttt{8}) * \texttt{N8}(\texttt{qsi}, \texttt{eta})
 Y1(qsi,eta) = vec_y(1)*N1(qsi,eta) + vec_y(2)*N2(qsi,eta) + vec_y(3)*N3(qsi,eta) + vec_y(4)*N4(qsi,eta) + vec_y(5)*N2(qsi,eta) + vec_y(5)*N3(qsi,eta) + vec_y(6)*N4(qsi,eta) + vec_y(
 vec_y(5)*N5(qsi,eta) + vec_y(6)*N6(qsi,eta) + vec_y(7)*N7(qsi,eta) + vec_y(8)*N8(qsi,eta)
vec_x(10)*N10(qsi,eta) + vec_x(11)*N11(qsi,eta) + vec_x(12)*N12(qsi,eta) + vec_x(13)*N13(qsi,eta)
 Y2(qsi,eta) = vec y(6)*N6(qsi,eta) + vec y(7)*N7(qsi,eta) + vec y(8)*N8(qsi,eta) + vec y(9)*N9(qsi,eta) + vec y(8)*N8(qsi,eta) + vec y(
 vec_y(10)*N10(qsi,eta) + vec_y(11)*N11(qsi,eta) + vec_y(12)*N12(qsi,eta) + vec_y(13)*N13(qsi,eta)
```

X3(qsi,eta) = vec x(11)*N11(qsi,eta) + vec x(12)*N12(qsi,eta) + vec x(13)*N13(qsi,eta) + vec x(14)*N14(qsi,eta) + vec xvec_x(15)*N15(qsi,eta) + vec_x(16)*N16(qsi,eta) + vec_x(17)*N17(qsi,eta) + vec_x(18)*N18(qsi,eta)



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 $Y3 (qsi,eta) = vec_y (11) *N11 (qsi,eta) + vec_y (12) *N12 (qsi,eta) + vec_y (13) *N13 (qsi,eta) + vec_y (14) *N14 (qsi,eta) + vec_y (15) *N15 (qsi,eta) + vec_y (16) *N16 (qsi,eta) + vec_y (17) *N17 (qsi,eta) + vec_y (18) *N18 (qsi,eta) \\$

```
5# - Calcular: \begin{bmatrix} N_{,x} \\ N_{,y} \end{bmatrix} = J^{-1} * \begin{bmatrix} N_{i}, \xi \\ N_{i}, \eta \end{bmatrix}
```

J3 =
[1, 0]
[0, 2/5]

```
%DEPENDE DA INCIDENCIA DE NOS
%Element 1# - 1, 2, 3, 4, 5, 6, 7, 8
N1qsi_N1eta = [dN1_dE;
dN1_dn]
N1qsi_N1eta = formula(N1qsi_N1eta)
AUX1 = J1_inv*N1qsi_N1eta
AUX1 = formula(AUX1)
dN1_dx = AUX1(1,1)
dN1 dY = AUX1(2,1)
N2qsi_N2eta = [dN2_dE;
dN2_dn]
N2qsi N2eta = formula(N2qsi N2eta)
AUX2 = J1 inv*N2qsi N2eta
dN2_dX = AUX2(1,1)

dN2_dY = AUX2(2,1)
N3qsi N3eta = [dN3 dE;
                 dN3_dn]
N3qsi_N3eta = formula(N3qsi_N3eta)
AUX3 = J1_inv*N3qsi_N3eta
dN3_dX = AUX3(1,1)
dN3_dY = AUX3(2,1)
N4qsi N4eta = [dN4 dE;
                  dN4_dn]
N4qsi_N4eta = formula(N4qsi_N4eta)
AUX4 = J1_inv*N4qsi_N4eta
dN4_dX = AUX4(1,1)
dN4_dY = AUX4(2,1)
N5qsi_N5eta = [dN5_dE;
dN5_dn]
N5qsi_N5eta = formula(N5qsi_N5eta)
AUX5 = J1_inv*N5qsi_N5eta
```



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```
dN5_dX = AUX5(1,1)

dN5_dY = AUX5(2,1)
N6qsi N6eta = [dN6 dE;
                  dN6 dn]
N6qsi_N6eta = formula(N6qsi_N6eta)
AUX6 = J1_inv*N6qsi_N6eta
dN6_dX = AUX6(1,1)
dN6_dY = AUX6(2,1)
N7qsi_N7eta = [dN7_dE;
dN7_dn]
N7qsi_N7eta = formula(N7qsi_N7eta)
AUX7 = J1_inv*N7qsi_N7eta
dN7_dX = AUX7(1,1)
dN7_dY = AUX7(2,1)
N8qsi_N8eta = [dN8_dE;
                  dN8_dn]
N8qsi_N8eta = formula(N8qsi_N8eta)
AUX8 = J1_inv*N8qsi_N8eta
dN8_dX = AUX8(1,1)
dN8 dY = AUX8(2,1)
dN1_dx = (\xi * (\eta + 1))/2 - \eta/4 - \eta^2/4
dN1_dY = (5*\xi^2)/8 - (5*\xi^2)/8 - (5*\eta^*(\xi^2-1/2))/2
dN2_dx = \eta^2 - 1/2
dN2_dY = 5 * \eta * (qsi/2 - 1/2)
dN3_dx = \eta / 4 - (\xi * (eta - 1)) / 2 - \eta ^2 / 4
dN3_dY = (5*\xi)/8 - (5*\xi^2)/8 - (5*\eta*(\xi/2 - 1/2))/2
dN4_dx = -\xi * (\eta + 1)
dN4_dy = 5/4 - (5*\xi^2)/4
dN5_dx = \xi * (\eta - 1)
dN5_dY = (5*\xi^2)/4 - 5/4
dN6_dX = eta/4 + (\xi * (\eta + 1))/2 + \eta^2/4
dN6_{dY} = (5*\xi)/8 + (5*\xi^2)/8 + (5*\eta*(\xi/2 + 1/2))/2
dN7_dx = 1/2 - \eta^2/2
dN7_dY = -5 * \eta * (\xi /2 + 1/2)
dN8_dx = \eta^2/4 - (\xi * (\eta - 1))/2 - \eta/4
dN8_dY = (5*\eta*(\xi/2 + 1/2))/2 - (5*\xi^2)/8 - (5*\xi^2)/8
```

6# - Calcular Matriz:
$$B = \begin{bmatrix} N_{x} & 0 \\ 0 & N_{y} \\ N_{y} & N_{x} \end{bmatrix}$$

7# - Calcular a Matriz de Rigidez por elemento: $K_e = \iint_{\Omega} \left[B^T_{3x16} D_{3x3} B_{3x16} \right] \det(J) * t * d\xi d\eta$

%Element 1# - 1, 2, 3, 4, 5, 6, 7, 8



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```
Ke_1 = (Ble')*D*Ble*t*det(J1)
Gauss Quadrature
solint1 = QuadraturaGauss (Ke 1, np)
Ke element1 = double(solint1
%Element 2# - 5, 6, 7, 9, 10, 11, 12, 13
Ke 2 = (B2e')*D*B2e*t*det(J2)
%Gauss Quadrature
solint2 = OuadraturaGauss(Ke 2.np)
Ke element2 = double(solint2)
 % Ke element2 = vpa(solint2,2)
%Element 3# - 10, 11, 12, 14, 15, 16, 17, 18
Ke 3 = (B3e')*D*B3e*t*det(J3)
%Gauss Quadrature
solint3 = QuadraturaGauss(Ke_3,np)
Ke_element3 = double(solint3)
function solint = QuadraturaGauss(f,n)
% QuadraturaGauss Soluciona a quadratura de Gauss para até 6 pontos de integração.
  $$ \int {-1}^{1} \int {-1}^{1} f(qsi,eta)*dqsi*deta $$
% SolNum = QuadraturaGauss(f,n)
% f - função simbólica nas variáveis "qsi" e "eta"
% n - inteiro de 1-6
syms qsi eta
Gaussian = \{[0 2];.
        [0.5773502691896262 1;-0.5773502691896262 1];.
        0 888888888888891:
         \begin{smallmatrix} 10.8611363115940534 & 0.347854845137454 \end{smallmatrix}; -0.8611363115940534 & 0.347854845137454 \end{smallmatrix}; 0.339981043584856 
0.652145154862546;-0.339981043584856 0.652145154862546];...
        [0.9061798459386645 \ \ 0.236926885056189; -0.9061798459386645 \ \ 0.236926885056189; 0.538469310105683]
0.478628670499366; -0.538469310105683 \ \ 0.478628670499366; 0 \ \ 0.5688888888888889]; \dots \\
        [0.9324695142031526 \ 0.171324492379170; -0.9324695142031526 \ 0.171324492379170; 0.661209386466265]
0.360761573048139;-0.661209386466265 0.360761573048139;0.238619186083197 0.467913934572691;-0.238619186083197
0.467913934572691]};
Coef = Gaussian{n};
solint = 0;
for i=1:n
        for j=1:n
               solint = solint + Coef(i, 2) *Coef(i, 2) *subs(f, {gsi, eta}, {Coef(i, 1) Coef(i, 1)});
       end
K1e = 10^6*
 24,07137 -10,5721 -24,1649 4,919014 11,3156 0,014028 -10,0905 5,031235 -7,86481 2,487562 9,515393 -0,01403 -13,4291 2,487562 10,64695 -4,35323
  -10,5721 51,23555 5,031235 -74,5131 -0,01403
                                                                          27,037 4,919014 1,601017 2,487562 -7,61606 0,014028 17,30146 2,487562 -37,7077 -4,35323 22,66188
  -24,1649 5,031235 57,30745
                                                     0 -24.1649 -5.03123
                                                                                                 0 -9,95025
                                                                                                                             0 9,950249 -13,4291 -2,48756 17,88052
                                                                                                                                                                                                     0 -13.4291 2.487562
                                        0 152,0338 -4,91901 -74,5131 -9,95025
                                                                                                                                             0 -2,48756 -37,7077
 4 919014 -74 5131
                                                                                                               0 9 950249
                                                                                                                                                                                        0 72.40789 2.487562 -37.7077
   11,3156 -0,01403 -24,1649 -4,91901 24,07137 10,57214 -7,86481 -2,48756 -10,0905 -5,03123 10,64695 4,353234 -13,4291 -2,48756 9,515393 0,014028
                 27,037 -5,03123 -74,5131 10,57214 51,23555 -2,48756 -7,61606 -4,91901 1,601017 4,353234 22,66188 -2,48756 -37,7077 -0,01403 17,30146
                                                                                                              0 -3,06737 0 -10,0905 -4,91501
839 0 -40,8783 -5,03123 1,601017 9,950249
7 96/81 -2,48756 0
  -10,0905 4,919014
                                        0 -9,95025 -7,86481 -2,48756 38,97804
                                                                                                                                                                                       0 9,950249 -7,86481 2,487562
 5.031235 1,601017 -9,95025
                                                                                                                                                                                                     0 2,487562 -7,61606
                                                     0 -2,48756 -7,61606
                                                                                                 0 52,90839
  -7,86481 2,487562
                                       0 9,950249 -10,0905 -4,91901 -3,06737
                                                                                                                                                                                       0 -9,95025 -10,0905 4,919014
                                                                                                                             0 52,90839 -2,48756 -7,61606 -9,95025
 2.487562 -7.61606 9.950249
                                                     0 -5.03123 1.601017
                                                                                                 0 -40.8783
                                                                                                                                                                                                      0 5.031235 1.601017
 9,515393 0,014028 -13,4291 -2,48756 10,64695 4,353234 -10,0905 -5,03123 -7,86481 -2,48756 24,07137 10,57214 -24,1649 -4,91901 11,3156 -0,01403
  -0,01403 \quad 17,30146 \quad -2,48756 \quad -37,7077 \quad 4,353234 \quad 22,66188 \quad -4,91901 \quad 1,601017 \quad -2,48756 \quad -7,61606 \quad 10,57214 \quad 51,23555 \quad -5,03123 \quad -74,5131 \quad 0,014028 \quad -1,014028 \quad -1,0140
                                                                                                                                                                                                                           27.037
  -13,4291 2,487562 17,88052
                                                      0 -13,4291 -2,48756
                                                                                                  0 9,950249
                                                                                                                             0 -9,95025 -24,1649 -5,03123 57,30745
                                                                                                                                                                                                     0 -24,1649 5,031235
                                        0 72,40789 -2,48756 -37,7077 9,950249
                                                                                                              0 -9,95025
                                                                                                                                             0 -4,91901 -74,5131
                                                                                                                                                                                       0 152,0338 4,919014 -74,5131
  2,487562 -37,7077
 10,64695 -4,35323 -13,4291 2,487562 9,515393 -0,01403 -7,86481 2,487562 -10,0905 5,031235 11,3156 0,014028 -24,1649 4,919014 24,07137 -10,5721 -4,35323 22,66188 2,487562 -37,7077 0,014028 17,30146 2,487562 -7,61606 4,919014 1,601017 -0,01403 27,037 5,031235 -74,5131 -10,5721 51,23555
```



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K2e = 10	0^6*														
24,07137	-10,5721	-24,1649	4,919014	11,3156	0,014028	-10,0905	5,031235	-7,86481	2,487562	9,515393	-0,01403	-13,4291	2,487562	10,64695	-4,35323
-10,5721	51,23555	5,031235	-74,5131	-0,01403	27,037	4,919014	1,601017	2,487562	-7,61606	0,014028	17,30146	2,487562	-37,7077	-4,35323	22,66188
-24,1649	5,031235	57,30745	0	-24,1649	-5,03123	0	-9,95025	0	9,950249	-13,4291	-2,48756	17,88052	0	-13,4291	2,487562
4,919014	-74,5131	0	152,0338	-4,91901	-74,5131	-9,95025	0	9,950249	0	-2,48756	-37,7077	0	72,40789	2,487562	-37,7077
11,3156	-0,01403	-24,1649	-4,91901	24,07137	10,57214	-7,86481	-2,48756	-10,0905	-5,03123	10,64695	4,353234	-13,4291	-2,48756	9,515393	0,014028
0,014028	27,037	-5,03123	-74,5131	10,57214	51,23555	-2,48756	-7,61606	-4,91901	1,601017	4,353234	22,66188	-2,48756	-37,7077	-0,01403	17,30146
-10,0905	4,919014	0	-9,95025	-7,86481	-2,48756	38,97804	0	-3,06737	0	-10,0905	-4,91901	0	9,950249	-7,86481	2,487562
5,031235	1,601017	-9,95025	0	-2,48756	-7,61606	0	52,90839	0	-40,8783	-5,03123	1,601017	9,950249	0	2,487562	-7,61606
-7,86481	2,487562	0	9,950249	-10,0905	-4,91901	-3,06737	0	38,97804	0	-7,86481	-2,48756	0	-9,95025	-10,0905	4,919014
2,487562	-7,61606	9,950249	0	-5,03123	1,601017	0	-40,8783	0	52,90839	-2,48756	-7,61606	-9,95025	0	5,031235	1,601017
9,515393	0,014028	-13,4291	-2,48756	10,64695	4,353234	-10,0905	-5,03123	-7,86481	-2,48756	24,07137	10,57214	-24,1649	-4,91901	11,3156	-0,01403
-0,01403	17,30146	-2,48756	-37,7077	4,353234	22,66188	-4,91901	1,601017	-2,48756	-7,61606	10,57214	51,23555	-5,03123	-74,5131	0,014028	27,037
-13,4291	2,487562	17,88052	0	-13,4291	-2,48756	0	9,950249	0	-9,95025	-24,1649	-5,03123	57,30745	0	-24,1649	5,031235
2,487562	-37,7077	0	72,40789	-2,48756	-37,7077	9,950249	0	-9,95025	0	-4,91901	-74,5131	0	152,0338	4,919014	-74,5131
10,64695	-4,35323	-13,4291	2,487562	9,515393	-0,01403	-7,86481	2,487562	-10,0905	5,031235	11,3156	0,014028	-24,1649	4,919014	24,07137	-10,5721
											27.027		74.5434	40 5704	F1 22FFF
-4,35323	22,66188	2,487562	-37,7077	0,014028	17,30146	2,487562	-7,61606	4,919014	1,601017	-0,01403	27,037	5,031235	-/4,5131	-10,5721	51,23555
K3e = 10	0^6*														
K3e = 10 24,07137	0 ^6* -10,5721	-24,1649	4,919014	11,3156	0,014028	-10,0905	5,031235	-7,86481	2,487562	9,515393	-0,01403	-13,4291	2,487562	10,64695	-4,35323
K3e = 10 24,07137 -10,5721	0 ^6* -10,5721 51,23555	-24,1649 5,031235	4,919014 -74,5131	11,3156 -0,01403	0,014028	-10,0905 4,919014	5,031235 1,601017	-7,86481 2,487562	2,487562 -7,61606	9,515393 0,014028	-0,01403 17,30146	-13,4291 2,487562	2,487562 -37,7077	10,64695 -4,35323	-4,35323 22,66188
K3e = 10 24,07137 -10,5721 -24,1649	0^6* -10,5721 51,23555 5,031235	-24,1649 5,031235 57,30745	4,919014 -74,5131 0	11,3156 -0,01403 -24,1649	0,014028 27,037 -5,03123	-10,0905 4,919014 0	5,031235 1,601017 -9,95025	-7,86481 2,487562 0	2,487562 -7,61606 9,950249	9,515393 0,014028 -13,4291	-0,01403 17,30146 -2,48756	-13,4291 2,487562 17,88052	2,487562 -37,7077 0	10,64695 -4,35323 -13,4291	-4,35323 22,66188 2,487562
K3e = 10 24,07137 -10,5721 -24,1649 4,919014	0^6* -10,5721 51,23555 5,031235 -74,5131	-24,1649 5,031235 57,30745 0	4,919014 -74,5131 0 152,0338	11,3156 -0,01403 -24,1649 -4,91901	0,014028 27,037 -5,03123 -74,5131	-10,0905 4,919014 0 -9,95025	5,031235 1,601017 -9,95025 0	-7,86481 2,487562 0 9,950249	2,487562 -7,61606 9,950249 0	9,515393 0,014028 -13,4291 -2,48756	-0,01403 17,30146 -2,48756 -37,7077	-13,4291 2,487562 17,88052 0	2,487562 -37,7077 0 72,40789	10,64695 -4,35323 -13,4291 2,487562	-4,35323 22,66188 2,487562 -37,7077
K3e = 10 24,07137 -10,5721 -24,1649 4,919014 11,3156	0^6* -10,5721 51,23555 5,031235 -74,5131 -0,01403	-24,1649 5,031235 57,30745 0 -24,1649	4,919014 -74,5131 0 152,0338 -4,91901	11,3156 -0,01403 -24,1649 -4,91901 24,07137	0,014028 27,037 -5,03123 -74,5131 10,57214	-10,0905 4,919014 0 -9,95025 -7,86481	5,031235 1,601017 -9,95025 0 -2,48756	-7,86481 2,487562 0 9,950249 -10,0905	2,487562 -7,61606 9,950249 0 -5,03123	9,515393 0,014028 -13,4291 -2,48756 10,64695	-0,01403 17,30146 -2,48756 -37,7077 4,353234	-13,4291 2,487562 17,88052 0 -13,4291	2,487562 -37,7077 0 72,40789 -2,48756	10,64695 -4,35323 -13,4291 2,487562 9,515393	-4,35323 22,66188 2,487562 -37,7077 0,014028
K3e = 10 24,07137 -10,5721 -24,1649 4,919014 11,3156 0,014028	0^6* -10,5721 51,23555 5,031235 -74,5131 -0,01403 27,037	-24,1649 5,031235 57,30745 0 -24,1649 -5,03123	4,919014 -74,5131 0 152,0338 -4,91901 -74,5131	11,3156 -0,01403 -24,1649 -4,91901 24,07137 10,57214	0,014028 27,037 -5,03123 -74,5131 10,57214 51,23555	-10,0905 4,919014 0 -9,95025 -7,86481 -2,48756	5,031235 1,601017 -9,95025 0 -2,48756 -7,61606	-7,86481 2,487562 0 9,950249 -10,0905 -4,91901	2,487562 -7,61606 9,950249 0 -5,03123 1,601017	9,515393 0,014028 -13,4291 -2,48756 10,64695 4,353234	-0,01403 17,30146 -2,48756 -37,7077 4,353234 22,66188	-13,4291 2,487562 17,88052 0 -13,4291 -2,48756	2,487562 -37,7077 0 72,40789 -2,48756 -37,7077	10,64695 -4,35323 -13,4291 2,487562 9,515393 -0,01403	-4,35323 22,66188 2,487562 -37,7077 0,014028 17,30146
24,07137 -10,5721 -24,1649 4,919014 11,3156 0,014028 -10,0905	0^6* -10,5721 51,23555 5,031235 -74,5131 -0,01403 27,037 4,919014	-24,1649 5,031235 57,30745 0 -24,1649 -5,03123 0	4,919014 -74,5131 0 152,0338 -4,91901 -74,5131 -9,95025	11,3156 -0,01403 -24,1649 -4,91901 24,07137 10,57214 -7,86481	0,014028 27,037 -5,03123 -74,5131 10,57214 51,23555 -2,48756	-10,0905 4,919014 0 -9,95025 -7,86481 -2,48756 38,97804	5,031235 1,601017 -9,95025 0 -2,48756 -7,61606	-7,86481 2,487562 0 9,950249 -10,0905 -4,91901 -3,06737	2,487562 -7,61606 9,950249 0 -5,03123 1,601017	9,515393 0,014028 -13,4291 -2,48756 10,64695 4,353234 -10,0905	-0,01403 17,30146 -2,48756 -37,7077 4,353234 22,66188 -4,91901	-13,4291 2,487562 17,88052 0 -13,4291 -2,48756 0	2,487562 -37,7077 0 72,40789 -2,48756 -37,7077 9,950249	10,64695 -4,35323 -13,4291 2,487562 9,515393 -0,01403 -7,86481	-4,35323 22,66188 2,487562 -37,7077 0,014028 17,30146 2,487562
24,07137 -10,5721 -24,1649 4,919014 11,3156 0,014028 -10,0905 5,031235	0^6* -10,5721 51,23555 5,031235 -74,5131 -0,01403 27,037 4,919014 1,601017	-24,1649 5,031235 57,30745 0 -24,1649 -5,03123 0 -9,95025	4,919014 -74,5131 0 152,0338 -4,91901 -74,5131 -9,95025 0	11,3156 -0,01403 -24,1649 -4,91901 24,07137 10,57214 -7,86481 -2,48756	0,014028 27,037 -5,03123 -74,5131 10,57214 51,23555 -2,48756 -7,61606	-10,0905 4,919014 0 -9,95025 -7,86481 -2,48756 38,97804 0	5,031235 1,601017 -9,95025 0 -2,48756 -7,61606 0 52,90839	-7,86481 2,487562 0 9,950249 -10,0905 -4,91901 -3,06737 0	2,487562 -7,61606 9,950249 0 -5,03123 1,601017 0 -40,8783	9,515393 0,014028 -13,4291 -2,48756 10,64695 4,353234 -10,0905 -5,03123	-0,01403 17,30146 -2,48756 -37,7077 4,353234 22,66188 -4,91901 1,601017	-13,4291 2,487562 17,88052 0 -13,4291 -2,48756 0 9,950249	2,487562 -37,7077 0 72,40789 -2,48756 -37,7077 9,950249	10,64695 -4,35323 -13,4291 2,487562 9,515393 -0,01403 -7,86481 2,487562	-4,35323 22,66188 2,487562 -37,7077 0,014028 17,30146 2,487562 -7,61606
X3e = 10 24,07137 -10,5721 -24,1649 4,919014 11,3156 0,014028 -10,0905 5,031235 -7,86481	0^6* -10,5721 51,23555 5,031235 -74,5131 -0,01403 27,037 4,919014 1,601017 2,487562	-24,1649 5,031235 57,30745 0 -24,1649 -5,03123 0 -9,95025	4,919014 -74,5131 0 152,0338 -4,91901 -74,5131 -9,95025 0 9,950249	11,3156 -0,01403 -24,1649 -4,91901 24,07137 10,57214 -7,86481 -2,48756 -10,0905	0,014028 27,037 -5,03123 -74,5131 10,57214 51,23555 -2,48756 -7,61606 -4,91901	-10,0905 4,919014 0 -9,95025 -7,86481 -2,48756 38,97804 0 -3,06737	5,031235 1,601017 -9,95025 0 -2,48756 -7,61606 0 52,90839	-7,86481 2,487562 0 9,950249 -10,0905 -4,91901 -3,06737 0 38,97804	2,487562 -7,61606 9,950249 0 -5,03123 1,601017 0 -40,8783	9,515393 0,014028 -13,4291 -2,48756 10,64695 4,353234 -10,0905 -5,03123 -7,86481	-0,01403 17,30146 -2,48756 -37,7077 4,353234 22,66188 -4,91901 1,601017 -2,48756	-13,4291 2,487562 17,88052 0 -13,4291 -2,48756 0 9,950249	2,487562 -37,7077 0 72,40789 -2,48756 -37,7077 9,950249 0 -9,95025	10,64695 -4,35323 -13,4291 2,487562 9,515393 -0,01403 -7,86481 2,487562 -10,0905	-4,35323 22,66188 2,487562 -37,7077 0,014028 17,30146 2,487562 -7,61606 4,919014
X3e = 10 24,07137 -10,5721 -24,1649 4,919014 11,3156 0,014028 -10,0905 5,031235 -7,86481 2,487562	0^6* -10,5721 51,23555 5,031235 -74,5131 -0,01403 27,037 4,919014 1,601017 2,487562 -7,61606	-24,1649 5,031235 57,30745 0 -24,1649 -5,03123 0 -9,95025 0 9,950249	4,919014 -74,5131 0 152,0338 -4,91901 -74,5131 -9,95025 0 9,950249	11,3156 -0,01403 -24,1649 -4,91901 24,07137 10,57214 -7,86481 -2,48756 -10,0905 -5,03123	0,014028 27,037 -5,03123 -74,5131 10,57214 51,23555 -2,48756 -7,61606 -4,91901 1,601017	-10,0905 4,919014 0 -9,95025 -7,86481 -2,48756 38,97804 0 -3,06737	5,031235 1,601017 -9,95025 0 -2,48756 -7,61606 0 52,90839 0 -40,8783	-7,86481 2,487562 0 9,950249 -10,0905 -4,91901 -3,06737 0 38,97804	2,487562 -7,61606 9,950249 0 -5,03123 1,601017 0 -40,8783 0 52,90839	9,515393 0,014028 -13,4291 -2,48756 10,64695 4,353234 -10,0905 -5,03123 -7,86481 -2,48756	-0,01403 17,30146 -2,48756 -37,7077 4,353234 22,66188 -4,91901 1,601017 -2,48756 -7,61606	-13,4291 2,487562 17,88052 0 -13,4291 -2,48756 0 9,950249 0 -9,95025	2,487562 -37,7077 0 72,40789 -2,48756 -37,7077 9,950249 0 -9,95025	10,64695 -4,35323 -13,4291 2,487562 9,515393 -0,01403 -7,86481 2,487562 -10,0905 5,031235	-4,35323 22,66188 2,487562 -37,7077 0,014028 17,30146 2,487562 -7,61606 4,919014 1,601017
x3e = 10 24,07137 -10,5721 -24,1649 4,919014 11,3156 0,014028 -10,0905 5,031235 -7,86481 2,487562 9,515393	0^6* -10,5721 51,23555 5,031235 -74,5131 -0,01403 27,037 4,919014 1,601017 2,487562 -7,61606 0,014028	-24,1649 5,031235 57,30745 0 -24,1649 -5,03123 0 -9,95025 0 9,950249 -13,4291	4,919014 -74,5131 0 152,0338 -4,91901 -74,5131 -9,95025 0 9,950249 0 -2,48756	11,3156 -0,01403 -24,1649 -4,91901 24,07137 10,57214 -7,86481 -2,48756 -10,0905 -5,03123 10,64695	0,014028 27,037 -5,03123 -74,5131 10,57214 51,23555 -2,48756 -7,61606 -4,91901 1,601017 4,353234	-10,0905 4,919014 0 -9,95025 -7,86481 -2,48756 38,97804 0 -3,06737 0 -10,0905	5,031235 1,601017 -9,95025 0 -2,48756 -7,61606 0 52,90839 0 -40,8783 -5,03123	-7,86481 2,487562 0 9,950249 -10,0905 -4,91901 -3,06737 0 38,97804 0 -7,86481	2,487562 -7,61606 9,950249 0 -5,03123 1,601017 0 -40,8783 0 52,90839 -2,48756	9,515393 0,014028 -13,4291 -2,48756 10,64695 4,353234 -10,0905 -5,03123 -7,86481 -2,48756 24,07137	-0,01403 17,30146 -2,48756 -37,7077 4,353234 22,66188 -4,91901 1,601017 -2,48756 -7,61606 10,57214	-13,4291 2,487562 17,88052 0 -13,4291 -2,48756 0 9,950249 0 -9,95025 -24,1649	2,487562 -37,7077 0 72,40789 -2,48756 -37,7077 9,950249 0 -9,95025 0 -4,91901	10,64695 -4,35323 -13,4291 2,487562 9,515393 -0,01403 -7,86481 2,487562 -10,0905 5,031235 11,3156	-4,35323 22,66188 2,487562 -37,7077 0,014028 17,30146 2,487562 -7,61606 4,919014 1,601017 -0,01403
x3e = 10 24,07137 -10,5721 -24,1649 4,919014 11,3156 0,014028 -10,0905 5,031235 -7,86481 2,487562 9,515393 -0,01403	0^6* -10,5721 51,23555 5,031235 -74,5131 -0,01403 27,037 4,919014 1,601017 2,487562 -7,61606 0,014028 17,30146	-24,1649 5,031235 57,30745 0 -24,1649 -5,03123 0 -9,95025 0 9,950249 -13,4291 -2,48756	4,919014 -74,5131 0 152,0338 -4,91901 -74,5131 -9,95025 0 9,950249 0 -2,48756 -37,7077	11,3156 -0,01403 -24,1649 -4,91901 24,07137 10,57214 -7,86481 -2,48756 -10,0905 -5,03123 10,64695 4,353234	0,014028 27,037 -5,03123 -74,5131 10,57214 51,23555 -2,48756 -4,91901 1,601017 4,353234 22,66188	-10,0905 4,919014 0 -9,95025 -7,86481 -2,48756 38,97804 0 -3,06737 0 -10,0905 -4,91901	5,031235 1,601017 -9,95025 0 -2,48756 -7,61606 52,90839 0 -40,8783 -5,03123 1,601017	-7,86481 2,487562 0 9,950249 -10,0905 -4,91901 -3,06737 0 38,97804 0 -7,86481 -2,48756	2,487562 -7,61606 9,950249 0 -5,03123 1,601017 0 -40,8783 0 52,90839 -2,48756 -7,61606	9,515393 0,014028 -13,4291 -2,48756 10,64695 4,353234 -10,0905 -5,03123 -7,86481 -2,48756 24,07137 10,57214	-0,01403 17,30146 -2,48756 -37,7077 4,353234 22,66188 -4,91901 1,601017 -2,48756 -7,61606 10,57214 51,23555	-13,4291 2,487562 17,88052 0 -13,4291 -2,4875 0 9,950249 0 -9,95025 -24,1649 -5,03123	2,487562 -37,7077 0 72,40789 -2,48756 -37,7077 9,950249 0 -9,95025 0 -4,91901 -74,5131	10,64695 -4,35323 -13,4291 2,487562 9,515393 -0,01403 2,487562 -10,0905 5,031235 11,3156 0,014028	-4,35323 22,66188 2,487562 -37,7077 0,014028 17,30146 2,487562 -7,61600 4,919014 1,601017 -0,01403 27,037
x3e = 10 24,07137 -10,5721 -24,1649 4,919014 11,3156 0,014028 -10,0905 5,031235 -7,86481 2,487562 9,515393 -0,01403 -13,4291	0^6* -10,5721 51,23555 5,031235 -74,5131 -0,01403 27,037 4,919014 1,601017 2,487562 -7,61606 0,014028 17,30146 2,487562	-24,1649 5,031235 57,30745 0 -24,1649 -5,03123 0 -9,950249 -13,4291 -2,48756 17,88052	4,919014 -74,5131 0 152,0338 -4,91901 -74,5131 -9,95025 0 9,950249 0 -2,48756 -37,7077	11,3156 -0,01403 -24,1649 -4,91901 24,07137 10,57214 -7,86481 -2,48756 -10,0905 -5,03123 10,64695 4,353234 -13,4291	0,014028 27,037 -5,03123 -74,5131 10,57214 51,23555 -2,487566 -7,61606 -4,91901 1,601017 4,353234 22,66188 -2,48756	-10,0905 4,919014 0 -9,95025 -7,86481 -2,48756 38,97804 0 -3,06737 0 -10,0905 -4,91901	5,031235 1,601017 -9,95025 0 -2,48756 -7,61606 0 52,90839 0 -40,8783 -5,03123 1,601017 9,950249	-7,86481 2,487562 0 9,950249 -10,0905 -4,91901 -3,06737 0 38,97804 0 -7,86481 -2,48756	2,487562 -7,61606 9,950249 0 -5,03123 1,601017 0 -40,8783 0 52,90839 -2,48756 -7,61606 -9,95025	9,515393 0,014028 -13,4291 -2,48756 10,64695 4,353234 -10,0905 -5,03123 -7,86481 -2,48756 24,07137 10,57214 -24,1649	-0,01403 17,30146 -2,48756 -37,7077 4,353234 22,66188 -4,91901 1,601017 -2,48756 -7,61606 10,57214 51,23555 -5,03123	-13,4291 2,487562 17,88052 0 -13,4291 -2,48756 0 9,950249 0 -9,95025 -24,1649 -5,03123 57,30745	2,487562 -37,7077 0 72,40789 -2,48756 -37,7077 9,950249 0 -9,95025 0 -4,91901 -74,5131	10,64695 -4,35323 -13,4291 2,487562 9,515393 -0,01403 -7,86481 2,487562 -10,0905 5,031235 11,3156 0,014028 -24,1649	-4,35323 22,66188 2,487562 -37,7077 0,014028 17,30146 2,487562 -7,61606 4,919014 1,601017 -0,01403 27,037 5,031235
K3e = 10 24,07137 -10,5721 -24,1649 4,919014 11,3156 0,014028 -10,0905 5,031235 -7,86481 2,487562 9,515393 -0,01403 -13,4291 2,487562	0^6* -10,5721 51,23555 5,031235 -74,5131 -0,01403 27,037 4,919014 1,601017 2,487562 -7,61606 0,014028 17,30146	-24,1649 5,031235 57,30745 0 -24,1649 -5,03123 0 -9,950249 -13,4291 -2,48756 17,88052 0	4,919014 -74,5131 0 152,0338 -4,91901 -74,5131 -9,95025 0 9,950249 0 -2,48756 -37,7077 0 72,40789	11,3156 -0,01403 -24,1649 -4,91901 24,07137 10,57214 -7,86481 -2,48756 5,03123 10,64695 4,353234 -13,4291 -2,48756	0,014028 27,037 -5,03123 -74,5131 10,57214 51,23555 -2,48756 -7,61606 4,91901 1,601017 4,353234 22,66188 -2,48756 -37,7077	-10,0905 4,919014 0 -9,95025 -7,86481 -2,48756 38,97804 0 -3,06737 0 -10,0905 -4,91901 0 9,950249	5,031235 1,601017 -9,95025 0 -2,48756 -7,61606 0 52,90839 0 -40,8783 -5,03123 1,601017 9,950249	-7,86481 2,487562 0 9,950249 -10,0905 -4,91901 -3,06737 0 38,97804 0 -7,86481 -2,48756 0 -9,95025	2,487562 -7,61606 9,950249 0 -5,03123 1,601017 0 -40,8783 0 52,90839 -2,48756 -7,61606 -9,95025	9,515393 0,014028 -13,4291 -2,48756 10,64695 4,353234 -10,0905 -5,03123 -7,86481 -2,48756 24,07137 10,57214 -24,1649 -4,91901	-0,01403 17,30146 -2,48756 -37,7077 4,353234 22,66188 -4,91901 1,601017 -2,48756 -7,61606 10,57214 51,23555 -5,03123 -74,5131	-13,4291 2,487562 17,88052 0 -13,4291 -2,48756 0 9,950249 0 -9,95025 -24,1649 -5,03123 57,30745	2,487562 -37,7077 0 72,40789 -2,48756 -37,7077 9,950249 0 -9,95025 0 -4,91901 -74,5131 0 152,0338	10,64695 -4,35323 -13,4291 2,487562 9,515393 -0,01403 -7,86481 2,487562 -10,0905 5,031235 11,3156 0,014028 -24,1649 4,919014	-4,35323 22,66188 2,487562 -37,7077 0,014028 17,30146 2,487562 -7,61606 4,919014 1,601017 -0,01403 27,037 5,031235 -74,5131

```
 8\# - \text{Montar a Matriz de Rigidez Global: } K_{36x36} = \begin{bmatrix} K_{ele1} & \dots & 0 \\ 0 & K_{ele2} & 0 \\ 0 & \dots & K_{ele3} \end{bmatrix}   \text{Ke1=[Ke\_element1 sym(zeros(16,20));}   \text{sym(zeros(20,36))];}   \text{Ke2=[sym(zeros(10,36));}   \text{sym(zeros(16,10)) Ke\_element2 sym(zeros(16,10));}   \text{sym(zeros(10,36))];}   \text{Ke3=[sym(zeros(20,36));}   \text{sym(zeros(16,20)) Ke\_element3];}   \text{K = Ke1+Ke2+Ke3;}
```

 $K_global = double(K)*10^6$



AE 245 - Finite Element I

Prof. Alfredo - 2º. Semester of 2015

																				-4,35323	10,64695	2,487562	-13,4291	-0,01403	9,515393	2,487562	-7,86481	5,031235	-10,0905	0,014028	11,3156	4,919014	-24,1649	-10,5721	24,07137
0	0	0		0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	3 22,66188	5 -4,35323	2 -37,7077	1 2,487562	3 17,30146	3 0,014028	2 -7,61606	1 2,487562	5 1,601017	5 4,919014	8 27,037	6 -0,01403	4 -74,5131	9 5,031235	1 51,23555	7 -10,5721
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			077					562		014			131			
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2,487562 -	-13,4291 2	0 7	17,88052	-2,48756 -	-13,4291 -	9,950249	0 9	-9,95025	0	-5,03123 -	-24,1649 -	0 1	57,30745	5,031235 -	-24,1649 4
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-37,7077	2,487562	72,40789	0	-37,7077	-2,48756	0	9,950249	0	-9,95025	-74,5131	-4,91901	152,0338	0	-74,5131	4,919014
																				0,014028	9,515393	-2,48756	-13,4291	4,353234	10,64695	-5,03123	-10,0905	-2,48756	-7,86481	10,57214	24,07137	-4,91901	-24,1649	-0,01403	11,3156
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8 17,30146	3 -0,01403	6 -37,7077	1 -2,48756	4 22,66188	5 4,353234	3 1,601017	5 -4,91901	6 -7,61606	1 -2,48756	4 51,23555	7 10,57214	1 -74,5131	9 -5,03123		6 0,014028
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	146 2,48	103 -7,8		756			017		906			-	-	123		
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2,487562 -7	-7,86481 2,	9,950249	0 9,	-4,91901 1,	-10,0905 -5	0	-3,06737	0 52	38,97804	-2,48756 -7	-7,86481 -2	-9,95025	0 -5		-10,0905 5,
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-7,61606	2,487562	0	9,950249	1,601017	-5,03123	-40,8783	0	52,90839	0	-7,61606	-2,48756	0	-9,95025		5,031235
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4,919014	-10,0905	-9,95025	0	-2,48756	-7,86481	0	38,97804	0	-3,06737	-4,91901	-10,0905	9,950249	0	2,487562	-7,86481
Ī	Ī	Ī	Ī	Ī	Ī	Ī	Ī	Ī	Ī	Ī	Ī	Ī	Ī	Ī	Ī	Ī	Ī	Ī	Ī	1,601017	5,031235		-9,95025	-7,61606	-2,48756	52,90839		-40,8783		1,601017	-5,03123	Ī	9,950249	-7,61606	2,487562
0	0	0	0	0	0	0	0	0	0	0 -4,3	0 10,6	0 2,487562	0 -13,4291	0 -0,01403	0 9,515393	0 2,487562	0 -7,86481	0 5,031235	0 -10,0905	17		0	25 -48,3298	90	56 48,14274	39 -2,48756	0 -7,86481	83 -5,03123	0 -10,0905	17 4,353234	23 10,64695	0 -2,48756	49 -13,4291	06 0,014028	62 9,515393
0	0	0	0	0	0	0	0	0	0	-4,35323 22	10,64695 -4								0905 4,9	0 54	22,6312	0 -1	3298	0 10	4274		_								
0	0	0	0	0	0	0	0	0	0	22,66188 2	-4,35323	-37,7077	2,487562 1	17,30146	0,014028	-7,61606 9	2,487562	1,601017	4,919014	54,07399	0	-149,026	0	102,4711	0	-7,61606	-2,48756	1,601017 9	-4,91901	22,66188	4,353234	-37,7077	-2,48756 1	17,30146 2	-0,01403
0	0	0	0	0	0	0	0	0	0	2,487562	-13,4291	0	17,88052	-2,48756	-13,4291	9,950249	0	-9,95025	0	0	-48,3298	0	114,6149	0	-48,3298	-9,95025	0	9,950249	0	-2,48756	-13,4291	0	17,88052	2,487562	-13,4291
											2,487562	72,40789		-37,7077	-2,48756		9,950249		-9,95025	-149,026		304,0676		-149,026			-9,95025		9,950249	-37,7077	-2,48756	72,40789		-37,7077	2,487562
0	0	0	0	0	0	0	0	0	0	-37,7077 0,014028	2 9,515393	9 -2,48756	0 -13,4291	77 4,353234	6 10,64695	0 -5,03123	10,0905	0 -2,48756	-7,86481	6	0 48,14274	6	0 -48,3298	96	0 22,6312	0 5,031235	-10,0905	0 2,487562	19 -7,86481	77 -0,01403	6 9,515393	9 2,487562	0 -13,4291	-4,35323	2 10,64695
0	0	0	0	0	0	0	0	0	0											0 102	274	0 -14	298	0 54,0	312										
0	0	0	0	0	0	0	0	0	0	17,30146 2	-0,01403 -	-37,7077 9	-2,48756	22,66188 -	4,353234 -	1,601017	-4,91901 -	-7,61606	-2,48756 3	102,4711 -	0	-149,026	0	54,07399 4	0 -	1,601017	4,919014	-7,61606	2,487562	17,30146	0,014028	-37,7077	2,487562	22,66188	-4,35323
0	0	0	0	0	0	0	0	0	0	2,487562	-7,86481	9,950249	0	-4,91901	-10,0905	0	-3,06737	0	38,97804	-2,48756	-7,86481	-9,95025	0	4,919014	-10,0905	0	0	0	0	0	0	0	0	0	0
	_	0	0	0	0	0	0	0	_	-7,61606	2,487562		9,950249	1,601017	-5,03123	-40,8783		52,90839	_	-7,61606	-2,48756		-9,95025	1,601017	5,031235	0	0	_	_	0	0	0		_	
										4,919014	-10,0905	-9,95025		-2,48756	-7,86481		38,97804		-3,06737	-4,91901	-10,0905	9,950249		2,487562	-7,86481										
0	0	0	0	0	0	0	0	0	0	14 1,601017	05 5,031235	25	0 -9,95025	56 -7,61606	81 -2,48756	0 52,90839	2	0 -40,8783	37	01 1,601017	05 -5,03123	49	0 9,950249	62 -7,61606	81 2,487562	0	0	0	0	0	0	0	0	0	0
0 4,	0 10,	0 2,4	0 -13	0-0,	0 9,5	0 2,4	0 -7,	0 5,0	0 -10	017		0		909			0 -7;		0 -10			0 -2,			562 9,5	0	0	0	0	0	0	0	0	0	0
-4,35323 2	10,64695 -	2,487562 -	-13,4291 2	-0,01403 1	9,515393 0	2,487562 -	-7,86481 2	5,031235 1	-10,0905 4	0 5	22,6312	0	-48,3298	0 1	48,14274	-2,48756 -	-7,86481 -	-5,03123 1	-10,0905 -	4,353234 2	10,64695 4	-2,48756 -	-13,4291 -	0,014028 1	9,515393 -	0	0	0	0	0	0	0	0	0	0
22,66188	-4,35323	-37,7077	2,487562	17,30146	0,014028	-7,61606	2,487562	1,601017	4,919014	54,07399	0	-149,026	0	102,4711	0	-7,61606	-2,48756	1,601017	-4,91901	22,66188	4,353234	-37,7077	-2,48756	17,30146	-0,01403	0	0	0	0	0	0	0	0	0	0
2,487562	-13,4291	0	17,88052	-2,48756	-13,4291	9,950249		-9,95025			-48,3298		114,6149		-48,3298	-9,95025		9,950249		-2,48756	-13,4291		17,88052	2,487562	-13,4291										
2 -37,70		0 72,40789	2	6 -37,7077	1 -2,48756	9	0 9,950249	U	0 -9,95025	0 -149,026		0 304,0676	9	0 -149,026	00	u	0 -9,95025	9	0 9,950249	6 -37,7077	1 -2,48756	0 72,40789	2	2 -37,7077	1 2,487562	0	0	0	0	0		0	0	0	
-37,7077 0,014028			0 -13,429	177 4,35323	56 10,6469	0 -5,0		0 -2,4	125 -7,8648:)26	0 48,1	576	0 -48,	126	0 22,	0 5,03123	10,090	0 2,48756	49 -7,8648	0,0140	756 9,51539	89 2,48756	0 -13,429)77 -4,3532	62 10,6469	0	0	0	0	0	0	0	0	0	0
	w	-2,48756 -3	_	-4	01	-5,03123 1,0	-10,0905 -4	-2,48756 -7		0 10	48,14274	0 -1	48,3298	0 54	22,6312	U		2	3-4		3 (2				0	0	0	0	0	0	0	0	0	0
30146	-0,01403	-37,7077 9	-2,48756	22,66188	1,353234	1,601017	4,91901	-7,61606	2,48756	102,4711	0	-149,026	0	54,07399 4	0	1,601017	4,919014	-7,61606	2,487562	7,30146),014028	-37,7077	2,487562	22,66188	4,35323	0	0	0	0	0	0	0	0	0	0
2,487562	-7,86481	9,950249	0	-4,91901	-10,0905	0	-3,06737	0	38,97804	-2,48756	-7,86481	-9,95025	0	4,919014	-10,0905	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17,30146 2,487562 -7,61606	2,487562		9,950249	1,601017	-5,03123	-40,8783	ĺ	52,90839		-7,61606	-2,48756	0	-9,95025	1,60101	5,031235																				
06 4,919	62 -10,0905	0 -9,95025	19	17 -2,48756	23 -7,86481	£	0 38,97804	39	0 -3,06737	06 -4,91901	56 -10,0905	0 9,950249	25	1,601017 2,487562 -7,61606 0,014028	35 -7,86481	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4,919014 1,601017	905 5,03	025	0 -9,5	756 -7,		0 52,9	804	0 -40	737	901 1,60		249	0 9,950249 -13,4291	562 -7,1	481 2,41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21017 4	5,031235	0	-9,95025 -:	-7,61606 10	-2,48756 24	52,90839 -	0	-40,8783 -	0	1,601017 4,353234	-5,03123 10	0	50249 -	51606 0,	2,487562 9,	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-0,01403	11,3156	-4,91901	-24,1649	10,57214	24,07137	-2,48756	-7,86481	-5,03123	-10,0905		10,64695	-2,48756			9,515393	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27,03	0,014028	-74,5131	-5,03123	51,23555	10,57214	-7,61606	-2,48756	1,601017	-4,91901	22,66188	4,353234	-37,7077	-2,48756	17,30146	-0,01403	_			_							_	_	_	_					_	_
7 5,0312	8 -24,1649	_	3 57,30745	5 -5,03123	4 -24,1649	6 -9,95025	5	7 9,950249		8 -2,48756	4 -13,4291		6 17,88052	6 2,487562	3 -13,4291	ت	٥	ی	ی	ی	٥	٥	ی	ی	٥	ں	ی	ی	ی	ی	٥	٥	ی	U	J
35 -74,		0 152,	745			125	0 -9,9	7.49	0 9,95	756 -37,			152			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5131 -1		152,0338 4,5	0 -2	-74,5131 0,0	-4,91901 1	0 5,0	-9,95025 -1	0 2,	9,950249 -7	-37,7077 -0	-2,48756 9,5	72,40789 2,4	0 -1	-37,7077 -4	2,487562 10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27,037 5,031235 -74,5131 -10,5721 51,23555		4,919014	-24,1649 5	0,014028	11,3156	5,031235 1	-10,0905 4	2,487562	-7,86481 2	-0,01403 1	9,515393 (2,487562	-13,4291 2,48756	-4,35323 2	10,64695	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51,23555	-10,5721	-74,5131	5,031235	27,037	-0,01403	1,601017	4,919014	-7,61606	2,487562	17,30146	0,014028	-37,7077	2,487562	22,66188	-4,35323	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			-	•			_				.,	•	•			_	_	_	_	_	_	_	_	_	_	_	_	_	_	_	_	_	_	_	



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9# - Zerar a linhas e colunas que possuem os constraints: 1, 3, 4 e 5 da Matriz Global de Rigidez

```
%Cancelando as linhas que tem reacoes
nr = 5;% Numero de Reacoes
nn = 18; %Numero de Nós
F = zeros(36,1);
F(16*2,1) = (1/6)*10000
F(17*2,1) = (4/6)*10000
F(18*2,1) = (1/6)*10000
K global(1,:)=0;
K_{global(:,1)=0}
K_{global(3,:)=0};
K global(:,3)=0;
K_{global(4,:)=0};
K global(:,4)=0;
K_global(5,:)=0;
K_global(:,5)=0;
K global(1,1)=1;
K_global(3,3)=1;
K_global(4,4)=1;
K_global(5,5)=1;
```

10# - Determinar os deslocamento, strass e strains: u = Nd, $\sigma = DBd$ utilizando o vetor de carregamento igual a $F = \left[0...0...F / 6^{n\delta=32}...F * 4 / 6^{n\delta=34}...F / 6^{n\delta=36}\right]$

```
K_global
Disp = inv(K_global)*F

%Element 1:
sigma_e1 = D*Ble*Disp(1:16,1)
sigma_e2 = D*B2e*Disp(11:26,1)
sigma_e3 = D*B3e*Disp(21:36,1)
```

	Deslocamento (in)												
0	1	-0,03124	11	-0,04998	21	-0,05628	31						
0,001224	2	0,084927	12	0,293588	22	0,564827	32						
0	3	-1,27E-17	13	-5,99E-17	23	-8,64E-17	33						
0	4	0,084096	14	0,293186	24	0,564787	34						
0	5	0,031238	15	0,049979	25	0,056278	35						
0,001224	6	0,084927	16	0,293588	26	0,564827	36						
-0,01671	7	-0,04171	17	-0,05421	27								
0,023552	8	0,177544	18	0,425271	28								
0,016715	9	0,041715	19	0,054215	29								
0,023552	10	0,177544	20	0,425271	30								

Folha: 10 / 17



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	sigma_element1(:,:,1) =												
Nó	1	2	3	4	5	6	7	8					
	ξ=-1, η=1	ξ=-1, η=0	ξ=-1, η=-1	ξ=0, η=-1	ξ=-1, η=1	ξ=-1, η=1	ξ=-1, η=1	ξ=-1, η=1					
Coord.	1.0e+05	1.0e+04	1.0e+05	1.0e+05	1.0e+05	1.0e+05	1.0e+04	1.0e+05					
	*	*	*	*	*	*	*	*					
σх	53.126	-0.0000	-53.126	46.875	-46.875	40.624	-0.0000	-40.624					
σу	-0.0791	-0.0000	0.0791	0.0037	-0.0037	0.0865	-0.0000	-0.0865					
Тху	-0.3194	-34.023	-0.3194	-0.0070	-0.0070	-0.3194	-34.023	-0.3194					
			sigm	na_element2	2(:,:,1) =								
Nó	1	2	3	4	5	6	7	8					
	ξ=-1, η=1	ξ=-1, η=0	ξ=-1, η=-1	ξ=0, η=-1	ξ=-1, η=1	ξ=-1, η=1	ξ=-1, η=1	ξ=-1, η=1					
Coord.	1.0e+05	1.0e+04	1.0e+05	1.0e+05	1.0e+05	1.0e+05	1.0e+04	1.0e+05					
	*	*	*	*	*	*	*	*					
σх	34.353	-0.0000	-34.353	28.125	-28.125	21.897	-0.0000	-21.897					
σу	-0.1016	-0.0000	0.1016	0.0040	-0.0040	0.1096	-0.0000	-0.1096					
Тху	-0.3198	-34.228	-0.3198	-0.0051	-0.0051	-0.3198	-34.228	-0.3198					
			sign	na_element3	3(:,:,1) =								
Nó	1	2	3	4	5	6	7	8					
	ξ=-1, η=1	ξ=-1, η=0	ξ=-1, η=-1	ξ=0, η=-1	ξ=-1, η=1	ξ=-1, η=1	ξ=-1, η=1	ξ=-1, η=1					
Coord.	1.0e+05	1.0e+05	1.0e+05	1.0e+04	1.0e+04	1.0e+04	1.0e+04	1.0e+04					
	*	*	*	*	*	*	*	*					
σх	-15.650	-15.650	-15.650	-93.750	-93.750	-30.999	-30.999	-30.999					
σу	0.0778	0.0778	0.0778	0.2020	0.2020	-0.3738	-0.3738	-0.3738					
Тху	-0.3191	-0.3191	-0.3191	-0.0906	-0.0906	-31.905	-31.905	-31.905					

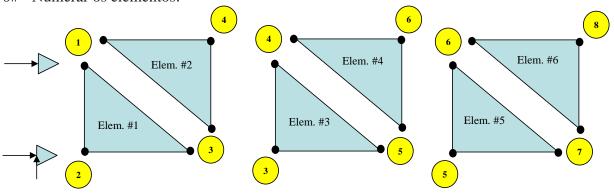


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Triangular Elements: 10 steps

0# - Numerar os elementos:



1# - Coordenadas dos Nós dos Elementos: $P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8$

2# - Funções de Interpolação: $\begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ \xi & \xi & \xi \\ \eta & \eta & \eta \end{bmatrix} \begin{bmatrix} 1 \\ \xi & \eta & \eta \end{bmatrix}$ onde $\xi = [-1,1], \eta = [-1,1]$

%Interpolation Functions of 8-noded Serendipity element - DEPENDE DA INCIDENCIA DE NOS

N = [1; qsi; eta;]

 $N_{int} = N_{c*N}$

N1(qsi,eta) = N_int(1,1) N2(qsi,eta) = N_int(2,1) N3(qsi,eta) = N_int(3,1)

%Derivation Interpolation Functions of 8-noded Serendipity element

dN1_dE= diff(N1,qsi)
dN2_dE= diff(N2,qsi)
dN3 dE= diff(N3,qsi)

dN1_dn= diff(N1,eta)
dN2_dn= diff(N2,eta)
dN3 dn= diff(N3,eta)

%%Determinant. Jacobian:

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```
3# - Calcular: x(\xi, \eta) = \sum_{i=1}^{m} Ni(\xi, \eta) * x_i e y(\xi, \eta) = \sum_{i=1}^{m} Ni(\xi, \eta) * x_i
%Isoparametric Formulation - DEPENDE DA INCIDENCIA DE NOS
syms X1(qsi,eta) Y1(qsi,eta) X2(qsi,eta) Y2(qsi,eta) X3(qsi,eta) Y3(qsi,eta)...
     X4(qsi,eta) Y4(qsi,eta) X5(qsi,eta) Y5(qsi,eta) X6(qsi,eta) Y6(qsi,eta)
%Element 1#
X1(qsi,eta) = vec_x(1)*N3(qsi,eta) + vec_x(3)*N2(qsi,eta) + vec_x(2)*N1(qsi,eta)
Y1(qsi,eta) = vec_y(1)*N3(qsi,eta) + vec_y(3)*N2(qsi,eta) + vec_y(2)*N1(qsi,eta)
%Element 2#
X2(qsi,eta) = vec_x(2)*N1(qsi,eta) + vec_x(4)*N2(qsi,eta) + vec_x(3)*N3(qsi,eta)

Y2(qsi,eta) = vec_y(2)*N1(qsi,eta) + vec_y(4)*N2(qsi,eta) + vec_y(3)*N3(qsi,eta)
%Element 3#
 Y3 (qsi,eta) = vec_y(4)*N1 (qsi,eta) + vec_y(3)*N3 (qsi,eta) + vec_y(5)*N2 (qsi,eta) 
%Element 4#
X4(qsi,eta) = vec_x(4)*N1(qsi,eta) + vec_x(6)*N2(qsi,eta) + vec_x(5)*N3(qsi,eta)
Y4(qsi,eta) = vec_y(4)*N1(qsi,eta) + vec_y(6)*N2(qsi,eta) + vec_y(5)*N3(qsi,eta)
%Element 5#
X5(qsi,eta) = vec x(6)*N1(qsi,eta) + vec x(5)*N3(qsi,eta) + vec x(7)*N2(qsi,eta)
Y5(qsi,eta) = vec_y(6)*N1(qsi,eta) + vec_y(5)*N3(qsi,eta) + vec_y(7)*N2(qsi,eta)
%Element 6#
X6(qsi,eta) = vec x(6)*N1(qsi,eta) + vec x(8)*N2(qsi,eta) + vec x(7)*N3(qsi,eta)
Y6(qsi,eta) = vec_y(6)*N1(qsi,eta) + vec_y(8)*N2(qsi,eta) + vec_y(7)*N3(qsi,eta)
%Derivation Interpolation Functions of 8-noded Serendipity element
dN1_dE= diff(N1,qsi)
dN2_dE= diff(N2,qsi)
dN3_dE= diff(N3,qsi)
```

4# - Determinante da matriz Jacobiana: $J = \begin{bmatrix} X, & Y, \\ X, & Y, \end{bmatrix}$

```
det_J1 = (x1(1)-x1(3))*(y1(2)-y1(3))-(x1(2)-x1(3))*(y1(1)-y1(3));
det_J2 = (x2(1)-x2(3))*(y2(2)-y2(3))-(x2(2)-x2(3))*(y2(1)-y2(3));
det_J3 = (x3(1)-x3(3))*(y3(2)-y3(3))-(x3(2)-x3(3))*(y3(1)-y3(3));
det_J4 = (x4(1)-x4(3))*(y4(2)-y4(3))-(x4(2)-x4(3))*(y4(1)-y4(3));
det_J5 = (x5(1)-x5(3))*(y5(2)-y5(3))-(x5(2)-x5(3))*(y5(1)-y5(3));
det_J6 = (x6(1)-x6(3))*(y6(2)-y6(3))-(x6(2)-x6(3))*(y6(1)-y6(3));
det_J2 = 1.6000
det_J2 = 1.6000
det_J3 =-1.6000
det_J4 =1.6000
det_J5 =-1.6000
det_J6 =1.6000
```

5# - Calcular Matriz:
$$B = \sum_{i=1}^{3} \begin{bmatrix} N_{,x} & 0 \\ 0 & N_{,y} \\ N_{,y} & N_{,x} \end{bmatrix}$$

B1e =

```
21,01648 10,71429 -14,4231 -4,94505 -6,59341 -5,76923
10,71429 43,51648 -5,76923 -41,2088 -4,94505 -2,30769
-14,4231 -5,76923 21,01648 0 0 10,71429
```



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B2e =					
-0,5	0	0	0	0,5	0
0	0	0	-1,25	0	1,25
0	-0,5	-1,25	0	1,25	0,5
B3e =					
-0,5	0	0	0	0,5	0
0	-1,25	0	1,25	0	0
-1,25	-0,5	1,25	0	0	0,5
B4e =					
-0,5	0	0	0	0,5	0
0	0	0	-1,25	0	1,25
0	-0,5	-1,25	0	1,25	0,5
B5e =					
-0,5	0	0	0	0,5	0
	-1,25	0	1,25	0	0
-1,25	-0,5	1,25	0	0	0,5
B6e =					
-0,5	0	0	0	0,5	0
0	0	0	-1,25	-	1,25
0	-0,5	-1,25	0	1,25	0,5

6# - Calcular a Matriz de Rigidez por elemento:

K1 = 10^6*					
21,01648	10,71429	-14,4231	-4,94505	-6,59341	-5,76923
10,71429	43,51648	-5,76923	-41,2088	-4,94505	-2,30769
-14,4231	-5,76923	14,42308	0	0	5,769231
-4,94505	-41,2088	0	41,20879	4,945055	0
-6,59341	-4,94505	0	4,945055	6,593407	0
-5,76923	-2,30769	5,769231	0	0	2,307692
K2 = 10^6*					
6,593407	0	0	4,945055	-6,59341	-4,94505
•	0 2,307692		•	-6,59341 -5,76923	•
•	2,307692		0	-5,76923	•
0	2,307692	5,769231 14,42308	0	-5,76923 -14,4231	-2,30769 -5,76923

-4,94505 -2,30769 -5,76923 -41,2088 10,71429 43,51648



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K3 = 10^6*					
21,01648	10,71429	-14,4231	-4,94505	-6,59341	-5,76923
		-5,76923			
		14,42308			
-4,94505	-41,2088	0	41,20879	4,945055	0
-6,59341	-4,94505	0	4,945055	6,593407	0
-5,76923	-2,30769	5,769231	0	0	2,307692
K4 = 10^6*					
6,593407	0	0	4,945055	-6,59341	-4,94505
0	2,307692	5,769231	0	-5,76923	-2,30769
0	5,769231	14,42308	0	-14,4231	-5,76923
4,945055	0	0	41,20879	-4,94505	-41,2088
-6,59341	-5,76923	-14,4231	-4,94505	21,01648	10,71429
-4,94505	-2,30769	-5,76923	-41,2088	10,71429	43,51648
K5 = 10^6*					
21,01648	10,71429	-14,4231	-4,94505	-6,59341	-5,76923
10,71429	43,51648	-5,76923	-41,2088	-4,94505	-2,30769
		14,42308			
-4,94505	-41,2088	0	41,20879	4,945055	0
		0			
-5,76923	-2,30769	5,769231	0	0	2,307692
K6 = 10^6*					
6,593407	0	0	4,945055	-6,59341	-4,94505
0	2,307692	5,769231	0	-5,76923	-2,30769
0	5,769231	14,42308	0	-14,4231	-5,76923
		0			
-6,59341	-5,76923	-14,4231	-4,94505	21,01648	10,71429
-4,94505	-2,30769	-5,76923	-41,2088	10,71429	43,51648

	K_{ele1}	0	•••	0	0	0	
	0	K_{ele2}	0	0	0	0	
7# - Montar a Matriz de Rigidez Global: $K_{16x16} =$	0	0	K_{ele3}	0	0	0	
7# - Montar a Matriz de Rigidez Global: **\frac{16x16}{} = \frac{1}{2} =	0	0	0	K_{ele4}	0	0	
	0	0	0	0	K_{ele5}	0	
	0	0	•••	0	0	K_{ele6}	

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$K_Global = 10^6*$

21,01648	10,71429	-14,4231	-4,94505	-6,59341	-5,76923	0	0	0	0	0	0	0	0	0	0
10,71429	43,51648	-5,76923	-41,2088	-4,94505	-2,30769	0	0	0	0	0	0	0	0	0	0
-14,4231	-5,76923	21,01648	0	0	10,71429	-6,59341	-4,94505	0	0	0	0	0	0	0	0
-4,94505	-41,2088	0	43,51648	10,71429	0	-5,76923	-2,30769	0	0	0	0	0	0	0	0
-6,59341	-4,94505	0	10,71429	42,03297	10,71429	-28,8462	-10,7143	-6,59341	-5,76923	0	0	0	0	0	0
-5,76923	-2,30769	10,71429	0	10,71429	87,03297	-10,7143	-82,4176	-4,94505	-2,30769	0	0	0	0	0	0
0	0	-6,59341	-5,76923	-28,8462	-10,7143	42,03297	10,71429	0	10,71429	-6,59341	-4,94505	0	0	0	0
0	0	-4,94505	-2,30769	-10,7143	-82,4176	10,71429	87,03297	10,71429	0	-5,76923	-2,30769	0	0	0	0
0	0	0	0	-6,59341	-4,94505	0	10,71429	42,03297	10,71429	-28,8462	-10,7143	-6,59341	-5,76923	0	0
0	0	0	0	-5,76923	-2,30769	10,71429	0	10,71429	87,03297	-10,7143	-82,4176	-4,94505	-2,30769	0	0
0	0	0	0	0	0	-6,59341	-5,76923	-28,8462	-10,7143	42,03297	10,71429	0	10,71429	-6,59341	-4,94505
0	0	0	0	0	0	-4,94505	-2,30769	-10,7143	-82,4176	10,71429	87,03297	10,71429	0	-5,76923	-2,30769
0	0	0	0	0	0	0	0	-6,59341	-4,94505	0	10,71429	21,01648	0	-14,4231	-5,76923
0	0	0	0	0	0	0	0	-5,76923	-2,30769	10,71429	0	0	43,51648	-4,94505	-41,2088
0	0	0	0	0	0	0	0	0	0	-6,59341	-5,76923	-14,4231	-4,94505	21,01648	10,71429
0	0	0	0	0	0	0	0	0	0	-4,94505	-2,30769	-5,76923	-41,2088	10,71429	43,51648

8# - Zerar a linhas e colunas que possuem os constraints: 1, 3 e 4 da Matriz Global de Rigidez

$K_Global = 10^6*$

1,00E-06	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	43,51648	0	0	-4,94505	-2,30769	0	0	0	0	0	0	0	0	0	0
0	0	1,00E-06	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1,00E-06	0	0	0	0	0	0	0	0	0	0	0	0
0	-4,94505	0	0	42,03297	10,71429	-28,8462	-10,7143	-6,59341	-5,76923	0	0	0	0	0	0
0	-2,30769	0	0	10,71429	87,03297	-10,7143	-82,4176	-4,94505	-2,30769	0	0	0	0	0	0
0	0	0	0	-28,8462	-10,7143	42,03297	10,71429	0	10,71429	-6,59341	-4,94505	0	0	0	0
0	0	0	0	-10,7143	-82,4176	10,71429	87,03297	10,71429	0	-5,76923	-2,30769	0	0	0	0
0	0	0	0	-6,59341	-4,94505	0	10,71429	42,03297	10,71429	-28,8462	-10,7143	-6,59341	-5,76923	0	0
0	0	0	0	-5,76923	-2,30769	10,71429	0	10,71429	87,03297	-10,7143	-82,4176	-4,94505	-2,30769	0	0
0	0	0	0	0	0	-6,59341	-5,76923	-28,8462	-10,7143	42,03297	10,71429	0	10,71429	-6,59341	-4,94505
0	0	0	0	0	0	-4,94505	-2,30769	-10,7143	-82,4176	10,71429	87,03297	10,71429	0	-5,76923	-2,30769
0	0	0	0	0	0	0	0	-6,59341	-4,94505	0	10,71429	21,01648	0	-14,4231	-5,76923
0	0	0	0	0	0	0	0	-5,76923	-2,30769	10,71429	0	0	43,51648	-4,94505	-41,2088
0	0	0	0	0	0	0	0	0	0	-6,59341	-5,76923	-14,4231	-4,94505	21,01648	10,71429
0	0	0	0	0	0	0	0	0	0	-4,94505	-2,30769	-5,76923	-41,2088	10,71429	43,51648

```
%Displacement
d = K_Global\F
%Stress:
sigma_e1 = vpa(D*B_e1*d(1:6,1),2)
sigma_e2 = vpa(D*B_e2*d(3:8,1),2)
sigma_e3 = vpa(D*B_e3*d(5:10,1),2)
sigma_e4 = vpa(D*B_e4*d(7:12,1),2)
sigma_e5 = vpa(D*B_e5*d(9:14,1),2)
sigma_e6 = vpa(D*B_e6*d(11:16,1),2)
```



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	Deslocan	nento (in)	
0	1	-0,03368	10
-0,00097	2	0,004872	11
0	3	-0,03385	12
0	4	-0,00569	13
-0,00357	5	-0,06259	14
-0,01073	6	0,005595	15
0,002961	7	-0,06276	16
-0,0109	8		_
-0,00523	9		

Sigmas	Sigma 1	Sigma 2	Sigma 3	Sigma 4	Sigma 5	Sigma 6
σx	-4.7e4	4.7e4	-2.9e4	2.9e4	-9.8e3	9.8e3
σγ	2.3e4	7.8e3	-1.5e4	2.4e3	-9.3e3	-3.4e3
Тху	-5.6e4	3.1e4	-3.8e4	1.3e4	-2.1e4	-3.9e3

b) Classical beam theory in bending.

$$u = \left(\frac{P}{EI}\right)\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right)$$
 onde $I = \frac{b * h^3}{12} = \frac{8^3 * 1}{12}$

```
E = 30e6; % Unit: [PSI]
v = 0.3;
t = 1; % Unit: [in]
P=10000
L=6
h = 0.8
I = t*(h^3)/12
j=1
for i=1:6
    x=i;
    u(j) = (P/E*I)*(L*(x^2)/2 - (x^3)/6)
    j = j + 1;
```

Results:

Deslocamento (in)												
	1		2		3		4		5		6	
Х	у	х	У	х	У	Х	У	х	У	х	У	
0	4,03E-05	0	-0,0001517	0	-0,00032	0	-0,00053096	0	-0,00077037	0	-0,001024	