Design of Experiments (DOE) Using the Taguchi Approach

This document contains brief reviews of several topics in the technique. For summaries of the recommended steps in application, read the published article attached.

(Available for free download and review.)

TOPICS:

Other References:
1. DOE Demystified..   : http://manufacturingcenter.com/tooling/archives/1202/1202qmdesign.asp
2. 16 Steps to Product... http://www.qualitydigest.com/june01/html/sixteen.html
3. Read an independent review of Qualitek-4:
http://www.qualitydigest.com/jan99/html/body_software.html
4. A Strategy for Simultaneous Evaluation of Multiple Objectives, A journal of the Reliability Analysis Center, 2004, Second quarter, Pages 14 - 18. http://rac.alionscience.com/pdf/2Q2004.pdf
5. **Design of Experiments Using the Taguchi Approach : 16 Steps to Product and Process Improvement** by Ranjit K. Roy **Hardcover** - 600 pages Bk&Cd-Rom edition (January 2001) John Wiley & Sons; ISBN: 0471361011

6. **Primer on the Taguchi Method -** Ranjit Roy (**ISBN:087263468X** Originally published in 1989 by Van Nostrand Reinhold. Current publisher/source is Society of Manufacturing Engineers). The book is available directly from the publisher, Society of Manufacturing Engineers (SME) P.O. Box 6028, Dearborn, Michigan 48121, USA.256 Pages. 50 Illustrations. Order code: 2436-2487.

For open enrollment seminar on Taguchi DOE technique, visit http://Nutek-us.com/wp-s4d.html .

Subject Overview (The Taguchi Approach)

Design Of Experiments (DOE) is a powerful statistical technique introduced by R. A. Fisher in England in the 1920's to study the effect of multiple variables simultaneously. In his early applications, Fisher wanted to find out how much rain, water, fertilizer, sunshine, etc. are needed to produce the best crop. Since that time, much development of the technique has taken place in the academic environment, but did help generate many applications in the production floor.

As a researcher in Electronic Control Laboratory in Japan, Dr. Genechi Taguchi carried out significant research with DOE techniques in the late 1940's. He spent considerable effort to make this experimental technique more user-friendly (easy to apply) and applied it to improve the quality of manufactured products. Dr. Taguchi's standardized version of DOE, popularly known as the Taguchi method or Taguchi approach, was introduced in the USA in the early 1980's. Today it is one of the most effective quality building tools used by engineers in all types of manufacturing activities.

The DOE using Taguchi approach can economically satisfy the needs of problem solving and product/process design optimization projects. By learning and applying this technique, engineers, scientists, and researchers can significantly reduce the time required for experimental investigations.

DOE can be highly effective when yow wish to:

- Optimize product and process designs, study the effects of multiple factors (i.e.- variables, parameters, ingredients, etc.) on the performance, and solve production problems by objectively laying out the investigative experiments. (Overall application goals).
- Study Influence of individual factors on the performance and determine which factor has more influence, which ones have less. You can also find out which factor should have tighter tolerance and which tolerance should be relaxed. The information from the experiment will tell you how to allocate quality assurance resources based on the objective data. It will indicate whether a supplier's part causes problems or not (ANOVA data), and how to combine different factors in their proper settings to get the best results (Specific Objectives).

Further, the experimental data will allow you determine:

- How to substitute a less expensive part to get the same performance improvement you propose
- How much money you can save the design
- How you can determine which factor is causing most variations in the result
- How you can set up your process such that it is insensitive to the uncontrollable factors
- Which factors have more influence on the mean performance
- What you need to do to reduce performance variation around the target
- How your response varies proportional to signal factor (Dynamic response)
- How to combine multiple criteria of evaluation into a single index
- How you can adjust factor for overall satisfaction of criteria an adjust factors for a system whose of evaluations
- How the uncontrollable factors affect the performance
  etc.,

Advantage of DOE Using Taguchi Approach

The application of DOE requires careful planning, prudent layout of the experiment, and expert analysis of results. Based on years of research and applications Dr. Genechi Taguchi has standardized the methods for each of these DOE application steps described below. Thus, DOE using theTaguchi approach has become a much more attractive tool to practicing engineers and scientists.

Experiment planning and problem formulation - Experiment planning guidelines are consistent with modern work disciplines of working as teams. Consensus decisions about experimental objectives and factors make the projects more successful.

Experiment layout -High emphasis is put on cost and size of experiments... Size of the experiment for a given number of factors and levels is standardized... Approach and priority for column assignments are established... Clear guidelines are available to deal with factors and interactions (interaction tables)... Uncontrollable factors are formally treated to reduce variation... Discrete prescriptions for setting up test conditions under uncontrollable factors are described... Guidelines for carrying out the experiments and number of samples to be tested are defined

Data analysis -Steps for analysis are standardized (main effect, NOVA and Optimum)... Standard practice for determination of the optimum is recommended... Guidelines for test of significance and pooling are defined...

Interpretation of results - Clear guidelines about meaning of error term... Discrete indicator about confirmation of results (Confidence interval)... Ability to quantify improvements in terms of dollars (Loss function)

Overall advantage - DOE using Taguchi approach attempts to improve quality which is defined as the consistency of performance. Consistency is achieved when variation is reduced. This can be done by moving the mean performance to the target as well as by reducing variations around the target. The prime motivation behind the Taguchi experiment design technique is to achieve reduced variation (also known as ROBUST DESIGN). This technique, therefore, is focused to attain the desired quality objectives in all steps. The classical DOE does not specifically address quality .

"The primary problem addressed in classical statistical experiment design is to model the response of a product or process as a function of many factors called model factors. Factors, called nuisance factors, which are not included in the model, can also influence the response... The primary problem addressed in Robust Design is how to reduce the variance of a product's function in the customer's environment." -Madhav Phadke, Quality Engineering using Robust Design.

WHAT'S NEW?

1. NEW PHILOSOPHY
   - Building quality in the product design.
   - Measuring quality by deviation from target (not by rejection).

2. NEW DISCIPLINE
   - Complete planning of experiments and evaluation criteria before conducting experiments.
   - Determining a factor's influence by running the complete experiment.

3. SIMPLER AND STANDARDIZED EXPERIMENT DESIGN FORMAT
   - Orthogonal arrays for experimental design.
   - Outer array design for robust product design.
   - More clear and easier methods for analysis of results.

QUALITY: DEFINITION and OBJECTIVE

- Reduced variation around the target with least cost.

APPROACH: ROBUST DESIGN

- Reduce variation without actually removing the cause of variation. Achieve consistent performance by making product/process insensitive to the influence of uncontrollable factors.

WHAT DOES IT DO? - Optimize design, solve problems, build robust products, etc.

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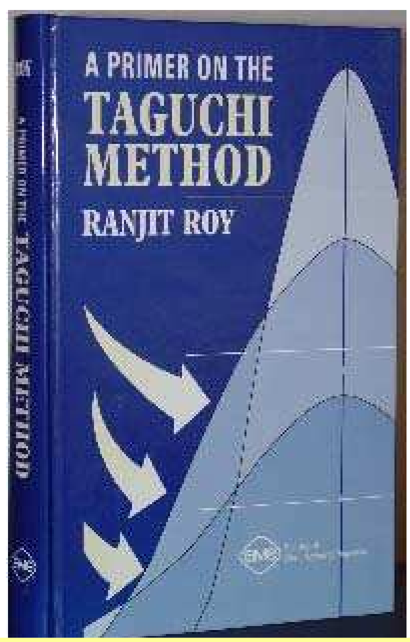
WHY DO IT?

- Save cost (Reduce warranty, rejection and cost of development).

AREAS OF APPLICATION:

- Analytical simulation (in early stages of design).
- Development testing (in design and development).
- Process development.
- Manufacturing.
- Problem solving in all areas of manufacturing and production.



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TAGUCHI METHOD REVIEW

APPLICATION STEPS

The Taguchi method is used to improve the quality of products and processes. Improved quality results when a higher level of performance is consistently obtained.

The highest possible performance is obtained by determining the optimum combination of design factors. The consistency of performance is obtained by making the product/process insensitive to the influence of the uncontrollable factor. In Taguchi's approach, the optimum design is determined by using design of experiment principles, and consistency of performance is achieved by carrying out the trial conditions under the influence of the noise factors.

1. **BRAINSTORMING**

This is a necessary first step in any application. The session should include individuals with first hand knowledge of the project. All matters should be decided based on group consensus, (One person -- One vote).

- Determine what you are after and how to evaluate it. When there is more than one criterion of evaluation, decide how each criterion is to be weighted and combined for the overall evaluation.
- Identify all influencing factors and those to be included in the study.
- Determine the factor levels.
- Determine the noise factor and the condition of repetitions.

2. **DESIGNING EXPERIMENTS**

Using the factors and levels determined in the brainstorming session, the experiments now can be designed and the method of carrying them out established. To design the experiment, implement the following:

- Select the appropriate orthogonal array.
- Assign factor and interaction to columns.
- Describe each trial condition.
- Decide order and repetiting trials.

3. **RUNNING EXPERIMENT**

Run experiments in random order when possible.

4. **ANALYZING RESULTS**

Before analysis, the raw experimental data might have to be combined into an overall evaluation criterion. This is particularly true when there are multiple criteria of evaluation. Analysis is performed to determine the following:

- The optimum design.
- Influence of individual factors.
- Performance at the optimum condition & confidence interval (**C. I.**).

BRAINSTORMING

SUGGESTED TOPICS OF DISCUSSIONS:

1. OBJECTIVES AND EVALUATION CRITERIA

- What are the criteria of evaluation?
- How are each of these criteria measured?
- How are these criteria combined into a single number?
- What is the common characteristic of these criteria?
- What is the relative influence these criteria exhibit?

2. FACTORS

- What are the factors that influence the performance criteria?
- Which factors are more important than others?

3. NOISE FACTORS

- Which factors can't be controlled in real life?
- Is the performance dependent on the application environment?

4. FACTOR LEVELS

- What are the ranges of values the factors can assume within practical limits?
- How many levels of each factor should be used for the study?
- What is the tradeoff for a higher level?

5. INTERACTION BETWEEN FACTORS

- Which factors are most likely to interact?
- How many interactions can be studied?

6. SCOPES OF STUDIES

- How many experiments can we run?
- When do we need the results?
- How much does each experiment cost?

7. ADDITIONAL ITEMS

- What do we do with factors that are not included in the study?
- In what order do we run these experiments?
- Who will do these experiments?

etc.

QUALITY CHARACTERISTICS

Quality Characteristic (QC) generally refers to the measured results of the experiment. The QC can be single criterion such as pressure, temperature, efficiency, hardness, surface finish, etc. or a combination of several criteria together into a single index. QC also refers to the nature of the performance objectives such as "bigger is better", "smaller is better" or "nominal is the best".

In most industrial applications, QC consists of multiple criteria. For example, an experiment to study a casting process might involve evaluating cast specimens in terms of (a) hardness, (b) visual inspection of surface and (c) number of cavities. To analyze results, readings of evaluation under each of these three criteria for each test sample can be used to determine the optimum. The optimum conditions determined by using the results of each criterion may or may not yield the same factor combination for the optimum. Therefore, a weighted combination of the results under different criteria into a single quantity may be highly desirable. While combining the results of different criteria, they must first be normalized and then made to be of type 'smaller is better' or 'bigger is better'.

When quality characteristic (QC) consists of, say, three criteria, an overall evaluation criteria (OEC) can be constructed as:

OEC = (X1/X1ref.)W1 + (X2/X2ref.)W2

+ (X3/X3ref.)W3

where X = evaluation under a criterion
Xref = a reference (maximum) value of reading
W = weighting factor of the criterion (in %)

Use of OEC as the result of an experimental sample instead of several readings from all criteria, offers an objective method of determining the optimum condition based on overall performance objectives.

When there are multiple criteria of evaluation, the experimenter can analyze the experiments based on readings under one category at a time as well as by using the OEC. If the individual outcomes differ from each other, the optimum obtained by using OEC as a result should be preferred.

FACTORS AND LEVELS

FACTORS ARE:

- design parameters that influence the performance.
- input that can be controlled.
- included in the study for the purpose of determining their influence and control upon the most desirable performance.

Example: In a cake baking process the factors are; Sugar, Flour, Butter, Egg, etc.

LEVELS ARE:

- Values that a factor assumes when used in the experiment

Example: As in the above cake baking process the levels for sugar and flour could be:
2 pounds, 5 pounds, etc. (Continuous level)
type 1, type 2, etc. (Discrete level)

LIMITS: Number of factors: 2 -63, number of levels: 2, 3, and 4.

INTERACTION BETWEEN FACTORS

Two factors (A and B) are considered to have interaction between them when one has influence on the effect of the other factor respectively.

Consider the factors "temperature" and "humidity" and their influence on comfort level. If the temperature is increased by, say 20 degrees F, the comfort level decreases by, say 30% when humidity is kept at 90%. On a different day, if the temperature is raised the same amount at a humidity level of 70%, the comfort level is reported to drop only by 10%. In this case, the factors "temperature" and "humidity" are interacting with each other.

Interaction:

- is an effect (output) and does not alter the trial condition.
- can be determined even if no column is reserved for it.
- can be fully analyzed by keeping appropriate columns empty.
- affects the optimum condition and the expected result.

NOISE FACTORS AND OUTER ARRAYS

Noise factors are those factors:
- that are not controllable.
- whose influences are not known.
- which are intentionally not controlled.

To determine robust design, experiments are conducted under the influence of various noise factors.

An "Outer Array" is used to reduce the number of noise conditions obtained by the combination of various noise factors.

For example:
Three 2-level noise factors can be combined using an L-4 into four noise conditions(4 repetitions). Likewise seven 2-level noise factors can be combined into eight conditions(8 repetitions) using an L8 as an outer array.

When trial conditions are repeated without the formal "Outer Array" design, the noise conditions are considered random.

SCOPE AND SIZE OF EXPERIMENT

The scope of the study, i.e., cost and time availability, are factors that help determine the size of the experiment. The number of experiments that can be accomplished in a given period of time, and the associated costs are strictly dependent on the type of project under study.

The total number of samples available divided by the number of repetitions yields the size of the array for design. The array size dictates the number of factors and their appropriate levels included in the study.

Example: A number of factors are identified for an optimization study.
- Time available is two weeks during which only 25 tests can be run.
- Three repetitions for each trial condition is desired.
- Array size 25/3 --> 8 L-8 array.
- Seven from the identified 2-level factors can be studied.

ORDER OF RUNNING EXPERIMENTS

There are two common ways of running experiments. Suppose an experiment uses an L-8 array and each trial is repeated 3 times. How are the 3x8=24 experiments carried out?

REPLICATION - The most desirable way is to run these 24 in random order.

REPETITION - The most practical way may be to select the trial condition in random order then complete all repetitions in that trial.

NOTE: In developing conclusions from the results of designed experiments and assigning statistical significance, it is assumed that the experiments were unbiased in any way, thus randomness is desired and should be maintained when possible.

MINIMUM REQUIREMENT - A minimum of one experiment per trial condition is required. Avoid running an experiment in an upward or downward sequence of trial numbers.

REPETITIONS AND REPLICATIONS

REPETITION: Repeat a trial condition of the experiment with/without a noise factor (outer array).

Example: L-8 inner array and L-4 outer array. 8x4 = 32 samples. Select a trial condition randomly and complete all 4 samples.
Take the next trial at random and continue.

REPLICATION: Conduct all the trials and repetitions in a completely randomized order.

In the above example, select one sample at a time in random order from among the 24 (8x4).

NOTE: Results from replication contain more information than those from repetition. Since replication requires resetting the the same trial condition, it captures variation in results due to experimental set up.

AVAILABLE ORTHOGONAL ARRAYS

The following Standard Orthogonal Arrays are commonly used to design experiments:

2-Level Arrays: L-4 L-8 L-12 L-16 L-32 L-64

3-Level Arrays: L-9 L-18 L-27 (L-18 has one 2-level column)

4-Level Arrays: L-16 & L-32 Modified

TRIANGULAR TABLE/LINEAR GRAPHS

TRIANGULAR TABLE OF INTERACTIONS
(2-LEVEL COLUMNS)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(1)	3	2	5	4	7	6	9	8	11	10	13	12	15	14
	(2)	1	6	7	4	5	10	11	8	9	14	15	12	13
		(3)	7	6	5	4	11	10	9	8	15	14	13	12
			(4)	1	2	3	12	13	14	15	8	9	10	11
				(5)	3	2	13	12	15	14	9	8	11	10
					(6)	1	14	15	12	13	10	11	8	9
						(7)	15	14	13	12	11	10	9	8
							(8)	1	2	3	4	5	6	7
								(9)	3	2	5	4	7	6
									(10)	1	6	7	4	5
										(11)	7	6	5	4
											(12)	1	2	3
												(13)	3	2
													(14)	1
														(15)

(Interaction tables for 3-level and 4-level factors are not shown here)

LINEAR GRAPHS - Linear graphs are graphical representations of certain readings of the Triangular table for convenience of experiment designs.
The graphs consist of combination of a line with circles/balls at the ends. The end points represent the columns where the interacting factors are assigned and the number associated with the line indicate the column number for the interaction.

Example: For L-4 Orthogonal array, 1 x 2 => 3, which will be shown in graph form as

```
                    3
1  o-------------------------o 2
```

Complicated Linear Graphs for higher order arrays are not shown here.

UPGRADING A COLUMN

COLUMN MODIFICATIONS:

PREPARING A 4-LEVEL COLUMN - Select 3 2-level columns that are naturally interacting. Pick two and discard the third.
Use the two columns to generate a new column.

Follow these rules to combine the new columns:

Old Columns			New Column
1	1	------->	1
1	2	------->	2
2	1	------->	3
2	2	------->	4

Example: Suppose factor A is a 4-level factor. Using columns 1 2 3 of an L-16, a new 4-level column can be prepared and factor A assigned.

PREPARING AN 8-LEVEL COLUMN - An 8-level column can be prepared from three of the seven interacting columns of an L-16. (Use columns 1 2 & 4, discard 3 5 6 & 7.)

Follow these rules:

Old Columns				New Column
1	1	1	------->	1
1	1	2	------->	2
1	2	1	------->	3
1	2	2	------->	4
2	1	1	------->	5
2	1	2	------->	6
2	2	1	------->	7
2	2	2	------->	8

Note: An eight level factor/column is not supported by QUALITEK-4 software. The above information is for user reference only.

DUMMY TREATMENT

This method allows a 3-level column to be made into a 2 -level column or a 4-level column into a 3-level column (e.g. levels 1 2 3 to 1 2 1').
The notation 1' is used to keep track of the changed status only. For level assignment 1'=1. The selection of the level to be treated is arbitrary.

Example: Three 3-level factors and one 2-level factor.
- Use an L-9. Dummy treat any column and assign the 2-level factor.

RESULTS OF MULTIPLE CRITERIA

Frequently, your experiment may involve evaluating results in terms of more than one criteria of evaluations. For example, in a cake baking experiment, the cakes baked under different recipes (trial conditions) may be evaluated by taste, looks and moistness. These criteria may be subjective and objective in nature. The best recipes can be determined by analyzing results of each criterion separately. The recipes for optimum conditions determined this way may or may not be the same. Thus, it may be desirable to combine the evaluations under different criteria into one single overall criteria and use them for analysis.

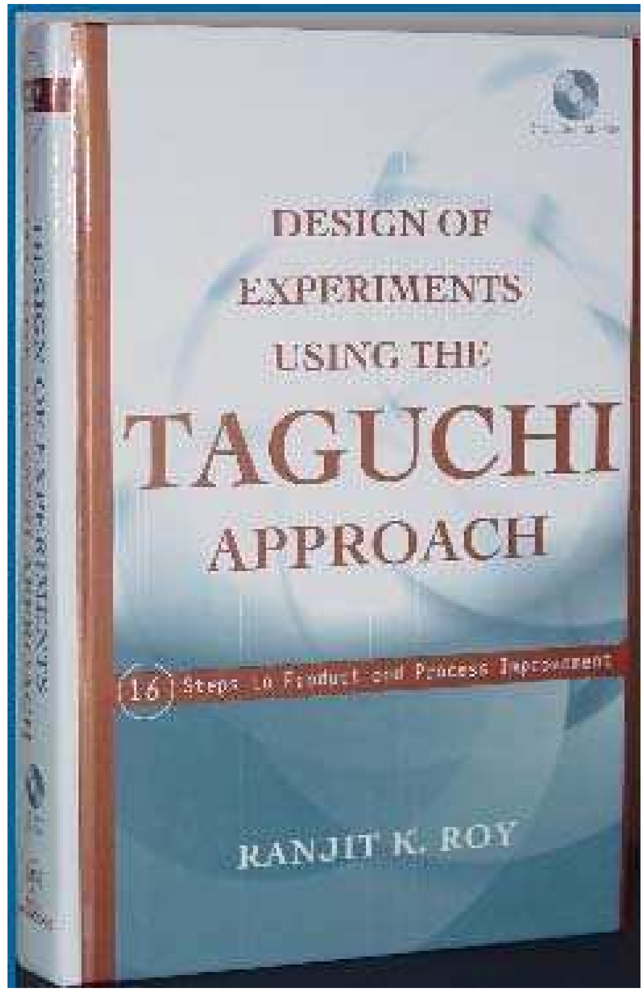
To combine readings under different evaluation criteria, they must first be normalized (unitless), then combined with proper weighting. Furthermore, all evaluations must be of the same quality characteristic, i.e., either bigger or smaller is a better type. When an evaluation is of the opposite it can be subtracted from a larger constant to conform to the desired characteristic [(X2ref. - X2) instead of X2].

For the purpose of combining all evaluations into a single criterion,

Assume:
X1 = Numeric evaluation under criterion 1
X1ref = Highest numerical value X1 can assume
Wt1 = Relative weighting of criterion 1

Then an Overall Evaluation Criterion (OEC) can be defined as:

OEC = (X1/X1ref)xWt1 + (X2/X2ref)xWt2 + ...



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SIGNAL TO NOISE RATIOS (S/N) FOR STATIC AND DYNAMIC SYSTEMS

MSD AND S/N RATIOS

NOTES AND RECOMMENDATION ON USE OF S/N RATIOS (Static condition)

Recommendation: If you are not looking for a specific objective, then SELECT S/N ratio based on Mean Squared Deviation (MSD).

MSD expression combines variation around the given target and is consistent with Taguchi's quality objective.

S/N based on variance is independent of target value and points to variation around the target.

S/N based on variance and mean combines the two effects with target at 0. The purpose is to increase this ratio ((Vm-Ve)/(nxVe)) and thus
a + sign is used in front of Log() for S/N. Also, since for an arbitrary target value, (Vm-Ve) may be negative, target=0 is used for calculation of Vm. Expressions for all types of S/N ratios are shown on the next screen.

RELATIONSHIPS AMONG OBSERVED RESULTS, MSD AND S/N RATIOS
(Static condition)

MSD = ((Y1-Y0)^2 + (Y2-Y0)^2 + ... (Yn-Y0)^2)/n for NOMINAL IS BEST
Variance: Ve = (SSt - SSm)/(n-1) .... for NOMINAL IS BEST
Variance and Mean = (SSm - Ve)/(n*Ve) (with TARGET=0)
where SSt = Y1^2 + Y2^2 and SSm = (Y1 + Y2 +..)^2/n

MSD = (Y1^2 + Y2^2 + ... Yn^2)/n for SMALLER IS BETTER
MSD = (1/Y1^2 + 1/Y2^2 + ... 1/Yn^2)/n for BIGGER IS BETTER

S/N = - 10 x Log(MSD)................. for all characteristics
S/N = + 10 x Log(Ve or Ve and Mean) .. for NOMINAL IS BEST only.

Note: Symbol (^2) indicates the value is SQUARED.

DYNAMIC CHARACTERISTIC
(Conduct of experiments and analysis of results)

Reference text: *Taguchi Methods* by Glen S. Peace, Addison Wesley Publishing Company, Inc. NY, 1992 (Pages 338-363)

WHAT IS DYNAMIC CHARACTERISTIC?
A system is considered to exhibit dynamic characteristics when the strength of a particular factor has a direct effect on the response. Such a factor with a direct influence on the result is called a SIGNAL factor.

SIGNAL FACTOR- is an input to the system. Its value/level may change.

CONTROL FACTOR - is also an input to the system. Values/level is fixed at the optimum level for the best performance.

NOISE FACTOR - is an uncontrollable factor. Its level is random during actual performance.

STATIC SYSTEM GOAL - is to determine combination of control factor levels which produces the best performance when exposed to the influence of the varying levels of noise factors.

DYNAMIC SYSTEM GOAL - is to find the combination of control factor levels which produces different levels of performances in direct proportion to the signal factor, but produces minimum variation due to the noise factors at each level of the signal.

Example: Fabric dyeing process
Control factor: Types of dyes, Temperature, PH number, etc.
Signal factor: Quantity of dye
Noise factor: Amount of starch

CONDUCTING EXPERIMENTS WITH DYNAMIC CHARACTERISTICS

When carrying out experiments, proper order and sequence of samples tested under each trial condition must be maintained. The number of samples required for each trial condition, will depend on the number of levels of signal factor, noise conditions and repetitions for each cell (a fixed condition of noise and signal factor).

Step 1. Design experiment with control factors by selecting your design type (manual or automatic design) from the main screen menu.
Step 2. Print description of trial conditions by selecting the PRINT option.
Step 3. Enter in your descriptions and experiment notes on the DYNAMIC CHARACTERISTICS screen.

* You will need to describe signal and noise factors and their levels. You will also have to decide on the number of levels of signal and noise factors. BUT MOST IMPORTANTLY, you will have to choose the nature of the ideal function (Straight line representing the behavior Response vs. Signal) applicable to your system.

Step 4. Strictly follow the prescribed test conditions.
Step 5. Enter results in the order and locations (run#) prescribed using the RESULTS option from the main menu.

SIGNAL-TO-NOISE RATIO EQUATIONS (alternate dynamic characteristic equations)
Signal factor may not always be clearly defined or known. For common industrial experiments, one or more attributes may be applicable:

* TRUE VALUE KNOWN
* INTERVAL BETWEEN FACTOR LEVELS KNOWN
* FACTOR LEVEL RATIOS KNOWN
* FACTOR LEVEL VALUES VAGUE

Depending on the circumstances of the input signal values and the resulting response data, different signal-to-noise (S/N) ratio equations are available.

ZERO POINT PROPORTIONAL - Select this response type of equation when response line passes through the origin. The signal may be known, unknown or vague.

REFERENCE POINT PROPORTIONAL - This response type of equation should be the choice when the response line does not go through the origin but through a known value of the signal or when signal values are wide apart or far away from origin. When the signal values are known, zero point or reference point proportional should be considered first. If neither is appropriate, the linear equation should be used.

LINEAR EQUATION - is based on the equation represnting the least squares of response fit and should be used where neither zero and reference point proportional equation are appropriate. Use it when signal values are close together and response does not pass through the origin.

WHEN IN DOUBT plot the response as a function of the signal factor values on a linear graph and examine the y-intercept. If it passes through origin, use ZERO POINT. If the intercept is not through origin but the line passes through a fixed point, use REF. POINT. In all other situation use LINEAR EQUATION.

S/N Ratio Equation and Calculation Steps

y = m + Beta (M - Mavg) + e Linear Eqn. (L)
y = Beta M Zero Point (Z)
y = Beta (M - Mstd.) + ystd Ref. Point (R)

Where y = system response (QC), M = Signal factor
Beta = slope of the ideal Eqn. Mavg = Average of signals
ystd. = avg. response under reference/standard signal
Mstd = reference/standard value of the signal strength

Notations
* = multiplication, ^ = raised to the power
/ = division by

Response Components for Each Trial Condition

(Layout shown only for trial#1 below)

SIGNAL FACTOR

Signal lev 1 Signal lev 2 Signal lev 3

N1_______N2___N1______N2____N1_______N2_
Trl#1| y11, y12, y13, y14.. y21, y22, y23, y24.. y31, y32, y33, y34.

Step 1: Determine r (Start with trial# 1)

ro = Number of samples tested under each SIGNAL LEVEL (Number of NOISE LEVELSxSAMPLES per CELL)

M1, M2, M3,.. Mk. Signal levels (strengths)
N1 & N2 are two levels of the noise factor
k = number of signal levels
Mavg = (M1 + M2 + ... Mk)/k

r = ro [(M1-Mavg)^2 + (M2-Mavg)^2 +... + (Mk-Mavg)^2] ... (L)
r = ro [(M1-Mstd)^2 + (M2-Mstd)^2 +... + (Mk-Mstd)^2] ... (R)
r = ro (M1^2 + M2^2 + M3^2 ... + Mk^2) .... (Z)

Step 2: Calculate of Slope Beta

Beta = (1/r) [y1*(M1-Mavg) +y2*(M2-Mavg) +... + yk*(Mk-Mavg)] .. (L)

Beta = (1/r) [y1*(M1-Mstd) +y2*(M2-Mstd) +... + yk*(Mk-Mstd)] .. (R)

Beta = (1/r) (y1*M1 + y2*M2 + ... + yk*Mk) .. (Z)

Step 3: Determine Total Sum of Squares

St = Sum [Sum (yij - yavg)] i= 1,2 .. k. j=1,2,.. ro .. (L)

yavg = ystd for (R), yavg = 0 for (Z)

Step 4: Calculate Variation Caused by the Linear Effect

Sbeta = r Beta^2 .... for all equations

Sbeta = (1/r) [y1*(M1-Mavg) +y2*(M2-Mavg) +.. + yk*(Mk-Mavg)]^2 .. (L)

Sbeta = (1/r) [y1*(M1-Mstd) +y2*(M2-Mstd) +.. + yk*(Mk-Mstd)]^2 .. (R)

Sbeta = (1/r) (y1*M1 + y2*M2 + ... + yk*Mk)^2 ... (Z)

Step 5: Calculate Error Variance

Ve = Se / [k*ro - 2] ...... (L)

Ve = Se / [k*ro - 1] ...... (R and Z)

Step 6: Calculate S/N Ratio

Eta = 10 Log (Sbeta - Ve) / (r*Ve) ... for all Eqns.

Step 7: Repeat calculations for all other trials in the same manner.

Example calculations: Case of LINEAR EQUATION (Expt. file: DC-AS400.QT4)

The results of samples tested for trial#1 of an experiment with dynamic characteristic. There are three signal levels, two noise levels, and two repetitions per cell.

M1		M2		M3	
Noise 1	Noise 2	Noise 1	Noise 2	Noise 1	Noise 2
5.2 5.6	5.9 5.8	12.3 12.1	12.4 12.5	22.4 22.6	22.5 22.2

Signal strengths: M1 = 1/3, M2 = 1, M3 = 3

CALCULATIONS FOR S/N

Mavg = (1/3 + 1 + 3) / 3 = 1.444

ro = 4 (2 simple/cell * 2 noise levels)

r = 4[(1/3 - 1.444)^2 + (1 - 1.444)^2 + (3 - 1.444)^2] ... (L)
= 4(1.2343 + 0.1971 + 2.421)
= 15.41

y1 = 5.2 + 5.6 + 5.9 + 5.8 = 22.5

y2 = 12.3 + 12.1 + 12.4 + 12.5 = 49.3

y3 = 22.4 + 22.6 + 22.5 + 22.2 = 89.7

Beta = (1/r)[22.5*(1/3-1.444) + 49.3*(1-1.444) + 89.7*(3-1.444)]
= (1/15.41) [-24.9975 - 21.692 + 139.5732]
= 92.8842/15.4101
= 6.01

Sbeta = r*Beta^2 = 15.4101 * 6.0274^2 = 556.82

yavg = [5.2 + 5.6 + ... + 22.2]/12 = 161.5/12 = 13.46

St = (5.2 - yavg)^2 + (5.6 - yavg)^2 + ...+ (22.2 - yavg)^2

= 68.23 + 61.78 + 57.15 + 58.67 + 1.346 + 1.85 + 1.123 + .921
+ 79.92 + 83.54 + 81.72 + 76.387
= 572.65

Se = St - Sb = 572.65 - 556.82 = 15.83

Ve = Se / (12 - 2) = 15.83 /10 = 1.583

Eta = 10 Log (Sbeta - Ve) / (r*Ve) ... for all Eqns.

= 10 Log [(556.82 - 1.583)/(15.41*1.583)]

= 10 Log(22.76)

= 13.572 (S/N for the trial# 1 results)

Similarly, S/N ratios for all other trial conditions are calculated and analysis performed using NOMINAL IS THE BEST quality characteristic as normally done for the static systems.

LOSS FUNCTION

The Loss Function offers a way to quantify the improvement from the optimum design determined from an experimental design study.

Definitions:

L = K (Y - Yo)^2 .... for a single sample.

L = K (MSD) .... for the whole population.

where L = Loss in dollar.
K = Proportionality constant.
Yo = Target value of the quality characteristic.
Y = Measured value of the quality characteristic.

THE COST SAVINGS WHEN THE MEAN VALUE IS HELD AT A TARGET VALUE CAN BE CALCULATED
WHEN THE FOLLOWING INFORMATION IS AVAILABLE :

- TARGET VALUE OF QUALITY CHARACTERISTIC.
- TOLERANCE OF QUALITY CHARACTERISTIC.
- COST OF REJECTION AT PRODUCTION (PER UNIT).
- UNITS OF PRODUCTION PER MONTH (TOTAL).
- S/N RATIO OF THE OLD DESIGN.
- S/N RATIO OF THE IMPROVED DESIGN.

: Since the S/N ratio is a direct product of ANOVA, it is conveniently used for calculation of loss.
However, the loss function requires MSD and must be calculated from the S/N ratio.

ATTRACTIVENESS OF THE TAGUCHI APPROACH

YOU CAN:
Do it yourself
Solve many production problems
Improve product/process designs
Reduce variation and save costs
Expect to get good results over 95% of the time

YOU DO NOT:
Need not be an expert
Necessarily need to do a lot of experiments

YOU NEED TO:
Be willing to work together as teams
Take initiative to improve designs before release

TAGUCHI VS. CLASSICAL DESIGN OF EXPERIMENTS (DOE)

Taguchi approach and classical design of experiments (DOE) were developed to achieve separate objectives and are different in many ways. Some of differences are:

TAGUCHI DOE

- OBJECTIVES: Obtain reproducible results and robust products.

GENERAL ATTRIBUTES:
* Standard or "cook book" approach.
* Methods are not standardized.
* Smaller number of experiments.
* Larger number of experiments.
* Standard method of noise factor
* No standardized method of noise treatment.
* Seeks to find stable condition
* Develops models by separation in the face of an error.
* Used to solve engineering problems.
* Used to solve scientific experiments.

CLASSICAL DOE

- Objective: Gather scientific knowledge about factor effects and their interactions.
- Weak main effects for random error.

GENERAL NOTES AND COMMENTS: HELPFUL TIPS ON APPLICATIONS

COMBINATION DESIGN
This is a method to fit two "2-level" factors in a "3-level" column. Suppose you have factors A and B at two levels and factors C, D, and E at three levels. An L-9 has four "3-level" columns. Factors C, D & E can occupy three columns leaving one column for A and B. A and B form A1B1 A2B1 A1B2 & A2B2. Select any three of these four and assign them to the three levels of the respective columns reserved for A and B.

DESIGNS TO INCLUDE NOISE FACTORS (OUTER ARRAY)

This version (version 4.7) of the program simultaneously handles inner and outer arrays. The noise conditions for repetitions can be studied by describing the outer array following completion of experiment design (inner array). Whether an outer array is present or not, up to 35 repetitions of results (columns) can be entered and an analysis performed using this software.

Comments from Expert Users (Why or why not use the Taguchi Approach)

"To me, Taguchi is attractive because of two reasons:

1. *It confines the experimental space*
2. *Economics*

The down side is that at some point of time, you should be bold enough to make the giant leap (or at least what seems like a giant leap) to implement the findings." - Sogal, E-mail: sogal@ix.netcom.com

"Having done some extensive research in the area of Experimental Design, there are no hard and fast rules for the choice of experimental design for a particular problem. It is not a good practice to stick to one approach for solving all process optimization problems using Taguchi methods of Experimental Design. However Taguchi approach is the best approach for those organizations who are new to experimental design area due to its statistical or mathematical simplicity (degree of statistics involved). It provides a systematic approach to experimentation so that you can study a large number of variables in a minimum number of experimental trials. This will have a knock-on effect on experimental budget and resources. Another reason why Taguchi approach is better over Classical approach is the concept of achieving robustness in the functional performance by inducing the presence of noise factors during the experiment. It is a good starting point towards continuous improvement of process/product performance. However it is simply not the best optimization technique available today. Taguchi would not be able to provide us the true optimal value of a factor setting. It merely tells us which is the best level for a factor setting from the levels chosen for experimentation. In my view, the choice of experimental design is based on :

1. the degree of optimization required for the response or quality characteristic of interest
2. statistical robustness and validity
3. complexity of understanding the choice of designs
4. cost and time constraints
5. ease of implementation
6. design resolution

Hope this helps. You may add my contact name and e-mail address for further discussion.". - Dr. Jiju Antony, International Manufacturing Center, University of Warwick

"The classical DOE is more concerned with statistics and model creating. Engineering solution is rather on behind. For this reasons, C-DOE is not generally accepted in industrial environment. In other words, engineers consider C-DOE as a too difficult tool for practice.

Taguchi DOE does not require deep and rigorous scientific and statistical background (knowledge), instead engineering solution is preferred. So this approach is more understandable for practical engineers. Method is relatively easy to implement and understand. Method gives good results in practice." - **Dr. Pavel Blecharz**

"Response Surface Methods (RSM) and other approaches are quite suitable for eg. research studies where often the influence of the various factors to be investigated are not well known. Here often quite a number of experimental trials need to be done as one ventures into somewhat uncharted territory. On the other hand in practical engineering problems the problem under investigation often relates to "fine tuning" of a process where the people involved have a reasonable "feel" for the process. The Taguchi approach is quite suitable for this purpose. Often researchers make use of Taguchi Methods for screening a large number of factors to narrow it down for more intensive study by RSM.

Taguchi Methods are relatively easy to grasp by co-workers on the shop floor as compared to the more statistically intense alternatives and aids their buy-in."

- **Dr. Wim Richter,** South Africa

When approaching a comparison of two viable alternatives, you should always "appear" to take the high road while serving your own interests. Expound on areas where "both" methods are viable and comparable, but then identify areas where the "preferred" element is clearly MORE advantageous to the user, creating the both very good, but one obviously better illusion.

Use two different "obviously better" scenarios:

1. *When the field of use is outside of the "common" area where both products are viable; in essence, this method illustrates a better solution for your specific use; "Both can be viable in THAT type of application, but in these areas, the Taguchi method offers much more"*

2. *When the field of use is within the "common" viable application arena; this identifies only specific advantages within the field of use.*

"Both are viable in this arena, but the specific advantages in this area are.....". By playing the odds, one half of the prospects will fall within the common field of use (the advantages there must be very specific - showing your expertise), but the other half falling outside of the portrayed common field of use, making the "assumptive" decision obvious to the reader/receiver when presented in this manner. - Cliff Veach,

"As a consultant and trainer in the areas of Statistics and Statistical Process Control I am confronted, on a regular basis, with this question of whether to suggest a Classical Design of Experiments or to use the Taguchi methods . The question becomes quit easy to answer. If the customer has minimal knowledge of their process with a large number of factors to investigate or has more than two levels of each factor to examine the answer is Taguchi.

Three reasons: *Reduces Time - analyze only the interactions that you believe truly exist, Reduces cost - reduction of all but necessary interactions,and Classical DOE does not (normally) accommodate more than two levels of each factor nor lend itself to mixed level designs whereas Taguchi does.* There are more reasons but I'll

Article

A well-run DOE method leads the way

to better products and processes.

by Ranjit K. Roy, Ph.D.

Improvement can be achieved in one or more of the many characteristics of any given product or process. In most situations, however, improvement primarily implies that performance is enhanced. Experimental design is one technique that can be learned and applied to determine product or process design for improved performance.

Figure 1: Performance Before Experimental Study

Figure 1: Performance Before Experimental Study

For a high-volume manufactured part, the two statistical performance characteristics that manufacturers typically aim to achieve are improving the mean (average) and reducing the variability around the mean. For improvement, our goal is to move the performance of a population of parts to the target and minimize the variability around it (see figures 1 and 2). No matter the application, performance consistency is a desirable characteristic to achieve. Performance consistency is achieved when the performance is on-target most of the time.

Figure 1: Performance Before Experimental Study

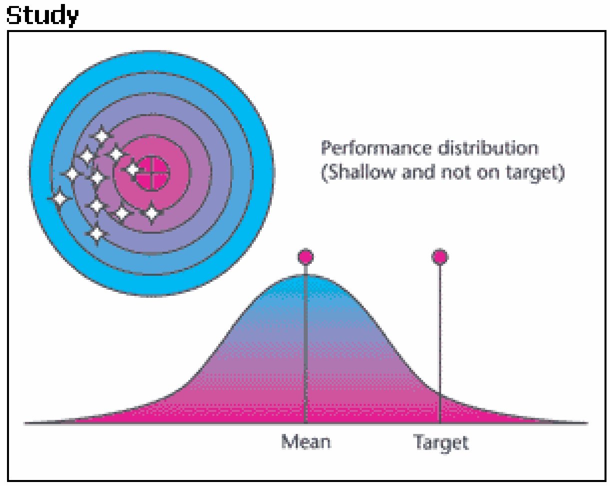
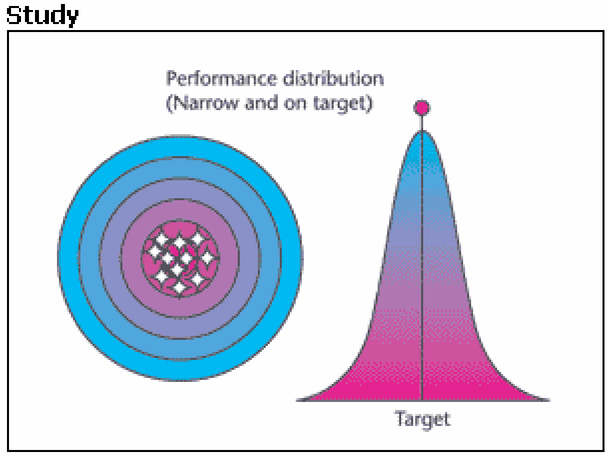


Figure 1: Performance Before Experimental Study



An effective way to improve performance is to optimize the engineering designs of products or processes by experimental means. A structured and economical way to study projects whose performance depends on many factors is to apply the experimental method known as "design of experiments," a statistical technique introduced in the 1920s by Ronald A. Fisher in England. In the 1950s, Genichi Taguchi of Japan proposed a much-standardized version of the technique for engineering applications. His prescription for experiment designs, a new strategy to incorporate the effects of uncontrollable factors and ability to quantify the performance improvement in terms of dollars by use of a loss function, made the DOE technique much more attractive to the practicing engineers and scientists in all kinds of industries.

In January, John Wiley & Sons Inc. published my book on DOE/Taguchi technique. Intended primarily for the self-learner, the book takes the reader through the entire application and analysis process in 16 different steps. One who learns the topics covered in these 16 steps well will be able to handle more than 99 percent of the situations common to manufacturing and production activities. Following are the 16 steps you will need to master DOE using the Taguchi approach for your own product and process design improvement.

Step 1: Design of experiments and the Taguchi approach

A quick review and understanding of the Taguchi version of DOE is essential before diving into the subject. The purpose here is to gather a clear understanding of what DOE is and understand how Taguchi standardized the experiment design process to make the technique easier to apply.

Step 2: Definition and measurement of improvement

No experiment that lacks the means to measure its results is complete or useful. A clear definition of objectives and measurement methods allows us to compare two individual performances, but a separate yardstick is needed to compare performances of one population (multiple products or processes) with another. In general, individual performance measures are different for different projects, but consistency is the means by which we measure population performance. Consistent performance produces reduced variations around the target (when present) and results in reduction of scrap, rejection and warranty. In this step, you learn how population performances are measured and compared.

Step 3: Common experiments and analyses methods

A common practice for studying single or multiple factors is to experiment with one factor at a time while holding all others fixed. This practice is attractive, as it's simple and supported by common sense. However, the results are often misleading and fail to reproduce conclusions drawn from such an exercise. A more effective method for these situations is to study their effect simultaneously by setting up experiments following the DOE technique. This step should lead to some understanding of basic DOE principles.

Step 4: Designing experiments using orthogonal arrays

The word "design" in "design of experiments" implies a formal layout of the experiments that contains information about how many tests are to be carried out and the combination of factors included in the study. Once the project is identified, the objectives and factors and their levels are determined by following a recommended sequence of discussion in a planning meeting. There are many possible ways to lay out the experiment; the best method depends on the project. A number of standard orthogonal arrays (number tables) have been constructed to facilitate designs of experiments. Each of these arrays can be used to design experiments to suit several experimental situations. This step should be devoted to learning about the different orthogonal arrays and understanding how easy it is to design experiments by using them.

Step 5: Designing experiments with two-level factors

Experiments that involve studies of factors with two levels are both simple and common. There are a set of orthogonal arrays (designated as L-4, L-8, L-12, L-16, L-32, L-64, etc.) created specifically for two-level factors. Experiments of all sizes can be easily designed using these arrays, as long as all factors involved are tested at two levels. By completing this step, you will learn how quickly experiments involving two-level factors can be designed and analyzed using the standard orthogonal arrays.

Step 6: Designing experiments with three-level and four-level factors

When only two levels of factors are studied, the factors' behavior is necessarily assumed to be linear. When nonlinear effects are suspected, more than two larger two-level orthogonal arrays can be modified to accommodate three-level and four-level factors, a set of standard arrays such as L-9, L-18, L-27, modified L-16 and modified L-32 are also available for this purpose. This step should help you learn the design and analysis of these more complex experiments.

Step 7: Analysis of variance (ANOVA)

Calculations of result averages and averages for factor-level effects, which only involve simple arithmetic operations, produce answers to major questions that were unconfirmed in the earlier steps about the project. However, questions concerning the influence of factors on the variation of results --in terms of discrete proportion --can only be obtained by performing analysis of variance. In this step, you'll learn how all analysis of variance terms are calculated. Utilize this step to review a number of example analyses to build your confidence in interpreting the experimental results.

Step 8: Designing experiments to study interactions between factors

Interaction among factors, which is one factor's effect on another, is quite common in industrial experiments. When experiments with factors don't produce satisfactory results, or when interactions among factors are suspected, the experiment must accommodate interaction studies. In this step, your objective will be learning how to design experiments to include interaction and how to analyze the results to determine if interaction is present. You will also learn how to determine the most desirable condition in cases in which interaction is found to be significant. Although interactions among several factors, and between factors at three or four levels, are also present, studies and corrections for interaction between two two-level factors will suffice for most situations.

The materials in steps 1-8 prepare you for many applications in the production floor. As long as the factors you want to study are all at the same level, you're able to design experiments using one of the available orthogonal arrays. You're also able to analyze the results of such experiments following the standard method of analysis, which uses the averages (means) of the multiple sample test results of individual experiments, and determine the optimum design conditions. With the knowledge you should gather in these steps, you can indeed apply the DOE to solve most production problems whose solutions lie in finding the proper combination of the controllable factors, instead of some special causes.

The reality, however, is that you will often have factors at mixed levels; some will be at three-level, some at four-level, and many at two-level. You also need to learn how to analyze the results for variability. Recall that it's the reduction of variability, which instills performance consistency, that we're after. The following additional steps address these items and prepare you to handle most every type of experimental situation.

If your applications always involve production problem solving, you may find that your knowledge up to this point is quite adequate for the job. Nevertheless, you may want to sharpen your application skills before proceeding to learn about the advanced concepts in the technique described in the eight steps that follow.

Step 9: Experiments with mixed-level factors

Experiment designs with all of the factors at one level are easily handled using one of the available standard arrays. But these standard arrays can't always accommodate many mixed-factor situations that you might find in industrial settings. Most mixed-level designs, however, can be accomplished by altering the standard orthogonal arrays. Your goal will be to learn the procedure by which columns of an array are modified to upgrade and downgrade the number of levels in creating a new column. This way, a two-level array can be modified to have three-level and four-level columns. Conversely, to accommodate a factor with a lesser number of levels, a four-level column can be reduced to a three-level, and a three-level column to two-level, by a method known as "dummy treatment."

Step 10: Combination designs

For some applications, the factors and levels are such that standard use of the orthogonal array doesn't produce an economical experimental strategy. In such situations, a special experiment design technique such as a combination design might offer a significant savings in number of samples. This step will familiarize you with the necessary assumptions that must be made in order to lay out experiments using combination design. With this technique, generally, two two-level factors are studied by assigning them to a three-level column.

Step 11: Robust design strategy

Variations among parts manufactured to the same specifications are common even when attempts are made to keep all factors at their desired levels.

Remember, variation reduction is our ultimate goal. When performance is consistently on-target (the desired value), the customer perceived quality of the product is favorably affected. Variation is most often due to factors that are not controllable or are too expensive to control. These are called the "noise factors." In robust design methodology, the approach is not to control the noise factors, but to minimize their influence by adjusting the controllable factors that are included in the study. This new strategy, promoted by Taguchi, reduces variability without actually removing the cause of variation.

Step 12: Analysis using signal-to-noise (S/N) ratios

The traditional method of calculating average factor effects and thereby determining the desirable factor levels (optimum condition) is to look at the simple averages of the results. Although average calculation is relatively simple, it doesn't capture the variability of results within a trial condition. A better way to compare the population behavior is to use the mean-squared deviation, which combines effects of both average and standard deviation of the results. For convenience of linearity and to accommodate wide-ranging data, a logarithmic transformation of MSD (called the signal-to-noise ratio) is recommended for analysis of results. This step will teach you how MSD is calculated for different quality characteristics and how analysis using S/N ratios differs from the standard practice. When the S/N ratio is used for results analysis, the optimum condition identified from such analysis is more likely to produce consistent performance.

Step 13: Results analysis using multiple evaluation criteria

Often, a product (or process) is expected to satisfy multiple objectives. The result in this case comprises multiple evaluation criteria, which represents performance in each of the objectives. It's common practice, however, to analyze only one criteria at a time because different objectives are likely to be evaluated by different criteria, each of which has different units of measurement and relative weighting. When the results are analyzed separately for different criteria and the desirable design conditions are determined, there is no guarantee that the factor combination will all be alike. An objective way to analyze the results is to combine the multiple evaluations into a single criterion, which incorporates the units of measurements and the relative weights of the individual criterion of evaluation. You should devote your time during this step to learning the principles involved in formulation of an overall evaluation criterion for analysis of multiple objectives, when present.

Step 14: Quantification of variation reduction and performance improvement

Most of your DOE applications allow you to determine optimum design that is expected to produce an overall better performance. The improvement of performance often means that either the average or the variations (or both) have improved. When the new design is put into practice (i.e., the recommended design is incorporated), it's expected to reduce scrap and warranty costs. In turn, this reduction more than offsets the cost of the new design. The expected monetary savings from the improved design can be calculated by using Taguchi's loss function. In this step, you'll learn how to estimate the expected savings from the improvement predicted by the experimental results. Further, you'll also learn how the expected improvement in performance from the new design is expressed in terms of capability improvement indexes such as Cp and Cpk.

Step 15: Effective experiment planning

As far as the benefits from the technique are concerned, experiment planning is the most important among the different application activities. Therefore, it's a required first and necessary step in the application process. Planning for DOE/Taguchi requires structured brainstorming with project team members. The nature of discussions in the planning session is likely to vary from project to project and is best facilitated by one who is well-versed in the technique. Your effort in this step will be to learn the structure of proven planning sessions documented by experienced application specialists.

Step 16: Review of example case studies

The application knowledge gained in steps 1-15 could be overwhelming if you didn't have immediate projects on which to practice. One way to build more confidence and extend your application expertise is by familiarizing yourself with numerous types of case studies with complete experiment design and results analysis. In this final step, you should seek out and thoroughly review complete project application reports. Complete case studies should contain discussions under most of the following topics:

- Project title or problem definition
- Project objective(s)
- Evaluation criteria and quality characteristic
- Identified factors and levels and those that are included in the study
- Suspected interactions and those that are selected for the initial study
- Uncontrollable factors (noise factors) and how they were treated
- Sequence of running of the experimental conditions
- Measured results, which represent evaluation of different objectives
- Main effects indicating the trend of factors' influence
- Analysis of variance for relative influence of the factor to the variation of results
- Optimum condition and the expected performance
- Improvement and expected monetary savings
- Graphical representation of variation reduction expected from the improved design

Now that you have an idea about the topics and the study sequence, one question remains: How do you actually go about learning them?

To get yourself comfortable with DOE application knowledge, you will need to understand four phases in the application process: (1) experiment planning, (2) experiment design, (3) results analysis and (4) interpretation of results. Of these, you need not --and may not be able to afford the time --to be too good with experiment design and number crunching. These are mundane tasks, so feel comfortable letting computer program do the work for you; your focus should be to learn the practiced and proven discipline of how to plan an experiment following a structured sequence of discussion. The experiment planning process requires more the art of teamwork than experimental science. Only the experienced can describe and share methods that have worked. Look for references that describe and teach the technique through application examples.

Both experiment planning and interpretation analysis are areas you'll want to gain control over. The nature of discussions and findings in these areas are always project-specific. As the experimenter, you'll know far more about these two areas than anyone else. Good knowledge of the project objectives, how objectives are evaluated, how the factors included in the study were selected, and so on will help you confidently interpret results from the routine analysis. You will benefit most when your reference book stresses application rather than theory.

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About the author

Ranjit K. Roy, Ph.D., P.E., PMP is an engineering consultant specializing in Taguchi approach of quality improvement. Roy has achieved international recognition as a consultant and trainer for his down-to-earth teaching style of the Taguchi experimental design technique. He is the author of the texts Design of Experiments Using Taguchi Approach: 16 Steps to Product, Process Improvement *and* A Primer on the Taguchi Method, *and of* Qualitek-4 *software for design and analysis of Taguchi experiments. rkr@nutek-us.com*

Experiment Design Solutions

Notations:

3-2LF = Three 2-level factors, 1-4LF = One 4-level factor,

AxB = Interaction between two 2-level factors A and B, etc.

AxB = 4 x 8 => 12 should be read as "Assign factors A to col. 4, B to col. 8, and reserve col. 12 for interaction AxB", etc.

Design solutions for a number of experimental situations are presented below. Using the notations described above, the experimental requirements are first stated, followed by a common experiment design strategy. The design shown are not necessarily unique, alternative solutions may exist. Notations A, B, C, etc. represent factor descriptions.

For most convenient use of the design recommendations, list your factors and interactions by first assigning the character notations to each factor, then selecting the REQUIREMENTS that best match your situation. As always double check your interaction and upgraded columns with the applicable TRIANGULAR TABLE.

Recommended Array Selection and Column Assignments

EXPERIMENT DESIGNS USING STANDARD ARRAYS *(no interaction or column upgrading)*

1. REQUIREMENTS: 2-2LF or 3-2LF
   DESIGN: L-4, factors assigned to columns arbitrarily

2. REQUIREMENTS: 4, 5, 6 or 7 -2LF
   DESIGN: L-8, factors cols. 1, 2, 4 & 6.Remaining columns left empty.

3. REQUIREMENTS: 8, 9, 10 or 11 -2LF, interaction present but ignored
   DESIGN: L-12, assign factors to columns arbitrarily (DO NOT USE L-12 TO STUDY INTERACTION)

4. REQUIREMENTS: 12, 13, 14 or 15 -2LF
   DESIGN: L-16, assign factors to columns arbitrarily

5. REQUIREMENTS: 16, 17, ... or 31 -2LF
   DESIGN: L-32, assign factors to columns arbitrarily.

6. REQUIREMENTS: 32, 33, ... or 63 -2LF
   DESIGN: L-64, assign factors to columns arbitrarily.

7. REQUIREMENTS: 2, 3 or 4 3LF
   DESIGN: L-9, factors assigned arbitrarily

8. REQUIREMENTS: 1 or 2-2LF and 2-3LF

DESIGN: L-9, Dummy treat columns for 2-level factors.

9. REQUIREMENTS: 1-2LF and 4, 5, 6 or 7 -3LF

DESIGN: L-18, assign the 2-level factor to col. 1 and all other factors to cols. 2 - 8. (DO NOT USE L-18 TO STUDY INTERACTIONS)

10. REQUIREMENTS: 2 -2LF(A & B) and 4, 5 or 6 -3LF

DESIGN: L-18, assign factor A to col. 1, dummy treat and assign factor B to col. 2. Assign other factors to cols. 3 - 8.

11. REQUIREMENTS: 8, 9, 10, 11, 12 or 13 -3LF

DESIGN: L-27, assign factors to columns arbitrarily.

12. REQUIREMENTS: 3, 4 or 5 -4LF

DESIGN: Modified L-16, assign factors to columns arbitrarily.

13. REQUIREMENTS: 6, 7, 8 or 9 -4LF

DESIGN: Modified L-32, Leave col. 1 empty and assign factors to the other columns arbitrarily.

14. REQUIREMENTS: 1-2LF and 5, 6, 7, 8 or 9 -4LF

DESIGN: Modified L-32, assign 2-level factor to col. 1 and assign other factors to the remaining columns arbitrarily.

DESIGNS WITH MULTIPLE INTERACTIONS *(dependent and independent pairs)*

15. REQUIREMENTS: 2-2LF(A&B) and AxB

DESIGN: L-4, factors A in col. 1,B in col. 2 and interaction AxB in col. 3

16. REQUIREMENTS: 3, 4, 5 or 6 - 2LF and one interaction, AxB

DESIGN: L-8, factor A in col.1, B in col. 2 and interaction AxB in col. 3. Other 2-level factors in the remaining column.

17. REQUIREMENTS: 3, 4 or 5 -2LF and two dependent interactions, AxB and BxC

DESIGN: L-8, Factors A in col. 1, B in col. 2 and C in col. 4, Interactions AxB in col. 3 and BxC in col. 6

18. REQUIREMENTS: 3 or 4 -2LF and 3 dependent interactions AxB, BxC and CxA

DESIGN: L-8, Factors A in col. 1, B in col. 2 and C in col. 4. Interactions AxB in col. 3, BxC in col. 6, and CxA in col. 5.

19. REQUIREMENTS: 4 - 2LF and 3 interactions AxB, AxC and AxD
DESIGN: L-8, Factors A in col. 1, B in col. 2, C in col. 4 and Din col. 7. Interactions AxB in col. 3 and AxC in col. 5 and AxD in col. 6

20. REQUIREMENTS: 2-2LF(A&B) and interaction (AxB)
DESIGN: L-9, assign factors A to col. 1, and reserve cols. 3 & 4 to study interaction between the two 3-level factors, AxB.

21. REQUIREMENTS: 4 or 5 -2LF and 2 interactions AxB and CxD
DESIGN: L-16, factor A in col. 1, B in col. 2 and int. AxB in col. 3. Factors C in col. 4, D in col. 8 and int. CxD in col. 12.

22. REQUIREMENTS: 8, 9, 10,.... or 14 -2LF and 1 interaction (AxB)
DESIGN: L-16, assign factors A to col. 1, B to col2, and AxB to col. 3.

23. REQUIREMENTS: 8, 9, 10,.... or 13 -2LF and 2 interactions (AxB and BxC)
DESIGN: L-16, assign factors A to col. 1, B to col. 2, C to col. 4, AxB to col. 3 and BxC to col.6. Assign other factors to the remaining columns.

24. REQUIREMENTS: 8, 9, 10,.... or 13 -2LF and 2 interactions (AxB and AxC)
DESIGN: L-16, assign factors A to col. 1, B to col. 2, C to col. 4, AxB to col. 3 and AxC to col.5.

25. REQUIREMENTS: 8, 9, 10,.. or 13 -2LF and 2 interactions (independent, AxB and CxD)
DESIGN: L-16, assign factors A to col. 1, B to col. 2, C to col. 4, D to col. 8, AxB to col. 3 and CxD to col.12.

26. REQUIREMENTS: 8, 9, 10, 11 or 12 -2LF and 3 interactions (AxB, BxC and CxA)
DESIGN: L-16, assign factors A to col. 1, B to col. 2, C to col. 4, AxB to col. 3, BxC to col. 6 and CxA to col. 5.

27. REQUIREMENTS: 8, 9, 10, 11 or 12 -2LF and 3 interactions (AxB, AxC and AxD)
DESIGN: L-16, assign factors A to col. 1, B to col. 2, C to col. 4, D to col. 7, AxB to col. 3 and AxC to col. 5 and AxD to col. 6.

28. REQUIREMENTS: 8, 9, 10, 11 or 12 -2LF and 3 interactions (AxB, AxC and CxD)
DESIGN: L-16, assign factors A to col. 1, B to col. 2, C to col. 4, D to col. 8, AxB to col. 3 and AxC to col. 5 and CxD to col.12.

29. REQUIREMENTS: 8, 9, 10, 11 or 12 -2LF and 3 interactions (AxB, BxC and CxD)
DESIGN: L-16, assign factors A to col. 1, B to col. 2, C to col. 4, D to col. 8, AxB to col. 3 and BxC to col.6 and CxD to col.12.

30. REQUIREMENTS: 8, 9, 10, 11 or 12 -2LF and 3 interactions (AxB, CxD and ExF)
DESIGN: L-16, assign factors A to col. 1, B to col. 2, C to col. 4, D to col. 8, E to col. 7, F to col. 9, AxB to col. 3, CxD to col.12 and ExF to col. 14.

31. REQUIREMENTS: 10 or 11 -2LF(A,B,C,...J) and 4 interactions (AxB, BxC, CxA and DxE)
DESIGN: L-16, assign factors A to col. 1, B to col. 2, C to col. 4, D to col. 7 and E to col. 9. Place interactions AxB to col. 3, BxC to col. 6 CxA to col. 5 and DxE to col. 14.

32. REQUIREMENTS: 10 or 11 -2LF(A,B,C,...J) and 4 interactions (AxB, BxC, CxD and ExF)
DESIGN: L-16, assign factors A to col. 1, B to col. 2, C to col. 4, D to col. 8, E to col. 7 and F to col. 9. Place interactions AxB to col. 3, BxC to col. 6 CxD to col. 12 and ExF to col. 14.

33. REQUIREMENTS: 10 or 11 -2LF(A,B,C,...J) and 4 interactions (AxB,AxC, AxD and ExF)
DESIGN: L-16, assign factors A to col. 1, B to col. 2, C to col. 4, D to col. 8, E to col. 7 and F to col. 9. Place interactions AxB to col. 3, AxC to col. 5, AxD to col. 9 and ExF to col. 14.

34. REQUIREMENTS: 10 or 11 -2LF(A,B,C,...J) and 4 interactions(AxB,AxC, AxD and AxE)
DESIGN: L-16, assign factors A to col. 1, B to col. 2, C to col. 4, D to col. 8 and E to col. 15. Place interactions AxB to col. 3, AxC to col. 5, AxD to col. 9 and ExF to col. 14.

35. REQUIREMENTS: 10 or 11 -2LF(A,B,C,...J) and 4 interactions (AxB, CxD, ExF and GxH)
DESIGN: L-16, assign factors A to col. 1, B to col. 2, C to col. 4, D to col. 8, E to col. 7, F to col. 9, G to col. 5 and H to col. 10. Place interactions AxB to col. 3, CxD to col.12, ExF to col. 14 and GxH to col. 15.

36. REQUIREMENTS: 10 -2LF(A,B,C,...J) and 5 interactions (AxB, CxD, ExF, GxH and IxJ)
DESIGN: L-16, assign factors A to col. 1, B to col. 2, C to col. 4, D to col. 8, E to col. 7, F to col. 9, G to col. 5, H to col. 10, I to col. 6 and J to col. 11. Place interactions AxB to col. 3, CxD to col.12 and ExF to col. 14, GxH to col. 15 and IxJ to col13 (Note: the five interacting groups in L-16 are 1x2=>3, 4x8=>12, 7x9=>14, 5x10=>15 and 6x11=>13) .

37. REQUIREMENTS: 10 -2LF(A,B,C,..) and 5 interactions (AxB, BxC, CxA, DxE and DxF)
DESIGN: L-16, assign factors A to col. 1, B to col. 2, C to col. 4, D to col. 7, E to col. 9 an F to col. 8. Place interactions AxB to col. 3, BxC to col.6, CxA to col. 5, DxE to col. 14 and DxE to col. 15.

38. REQUIREMENTS: 10 -2LF(A,B,C,..) and 5 interactions (AxB, AxC, AxD, AxE and AxF)
DESIGN: L-16, assign factors A to col. 1, B to col. 2, C to col. 4, D to col. 8, E to col. 10 an F to col. 12. Place interactions AxB to col. 3, AxC to col. 5, AxD to col. 9, AxE to col. 11 and AxF to col. 13.

MIXED-LEVEL FACTOR DESIGNS *(2, 3 and 4-level factors only)*

39. REQUIREMENTS: 4-2LF and 1-4LF(A)
DESIGN: L-8, assign factor A in col. 1, all other factors in cols. 4, 5, 6 & 7

40. REQUIREMENTS: 1, 2, or 3 -2LF and 1-4LF(A)
DESIGN: L-8, assign factor A in col. 1, all other factors in cols. 4, 5, 6 & 7 as appropriate

41. REQUIREMENTS: 1, 2, 3 or 4 -2LF and 1-3LF(A)
DESIGN: L-8, assign factor A in col. 1, all other factors in cols. 4, 5, 6 & 7 as appropriate

42. REQUIREMENTS: 2-2LF(A & B) and 3-3LF (AxB is considered absent)
DESIGN: L-9 used for COMBINATION DESIGN. Assign the two 2-level factor combinations(3 levels) to any of the four columns of the array.

43. REQUIREMENTS: 12, 11, 10 ..or 5 -2LF, 1-4LF(A)
DESIGN: L-16. Upgrade the interacting groups of cols., 1 2 3 to a 4-level columns(1). Assign factor A to col. 1 and the 2-level factors to the remaining columns.

44. REQUIREMENTS: 12, 11, 10 ..or 5 -2LF, 1-3LF(A)
DESIGN: L-16. Upgrade the interacting groups of cols., 1 2 3 to a 4-level columns(1). Dummy treat this 4-level columns to a 3-level (col. 1). Assign factor A to col. 1 and the 2-level factors to the remaining columns.

45. REQUIREMENTS: 9, 8, 7,..or 5 -2LF, 1-3LF(A) and 1-4LF(B)
DESIGN: L-16. Upgrade two interacting groups of cols., 1 2 3 and 4 8 12 to two 4-level columns(1 and 4). Dummy treat the first 4-level columns to a 3-level (col. 1). Assign factor A to col. 1, B to col. 4 and the 2-level factors to the remaining columns.

46. REQUIREMENTS: 9, 8, 7, ..or 2 -2LF, 2-3LF(A & B)
DESIGN: L-16. Upgrade two interacting groups of cols., 1 2 3 and 4 8 12 to two 4-level columns(1 and 4). Dummy treat the two 4-level columns to 3-level (cols. 1 and 4). Assign factor A to col. 1, B to col. 4 and the 2-level factors to the remaining columns.

47. REQUIREMENTS: 6, 5, 4, 3 or 2 -2LF, 2-4LF(A & B) and 1-4LF(C)
DESIGN: L-16. Upgrade two interacting groups of cols., 1 2 3 and 4 8 12 to two 4-level columns(1and 4). Assign factor A to col. 1, B to col. 4 and the 2-level factors to the remaining columns.

48. REQUIREMENTS: 6, 5, 4, 3 or 2 -2LF, 2-3LF(A & B) and 1-4LF(C)
DESIGN: L-16. Upgrade three interacting groups of cols., 1 2 3, 4 8 12 and 7 9 14, to three 4-

level columns(1, 4 and 7). Dummy treat the first two 4-level columns to 3-level (cols. 1 and 4). Assign factor A to col. 1, B to col. 4 and C to col. 7. Assign the 2-level factors to the remaining columns.

49. REQUIREMENTS: 6, 5, 4, 3 or 2 -2LF, 1-3LF(A) and 2-4LF(B & C)
DESIGN: L-16. Upgrade three interacting groups of cols., 1 2 3, 4 8 12 and 7 9 14, to three 4-level columns(1, 4 and 7). Dummy treat the first 4-level column to a 3-level (col. 1). Assign factor A to col. 1, B to col. 4 and C to col. 7. Assign the 2-level factors to the remaining columns.

50. REQUIREMENTS: 6, 5, 4, 3 or 2 -2LF and 3-4LF(A,B & C)
DESIGN: L-16. Upgrade three interacting groups of cols., 1 2 3, 4 8 12 and 7 9 14, to three 4-level columns(1, 4 and 7). Assign factor A to col. 1, B to col. 4 and C to col. 7. Assign the 2-level factors to the remaining columns.

51. REQUIREMENTS: 6, 5, 4, 3 or 2 -2LF and 3-3LF(A,B & C)
DESIGN: L-16. Upgrade three interacting groups of cols., 1 2 3, 4 8 12 and 7 9 14, to three 4-level columns(1, 4 and 7). Dummy the upgraded 4-level columns to 3-level columns. Assign factor A to col. 1, B to col. 4 and C to col. 7. Assign the 2-level factors to the remaining columns.

OUTER ARRAY DESIGN FOR ROBUSTNESS *(Static System win noise factors)*

52. REQUIREMENTS: 2-2LF or 3-2LF Noise factors
DESIGN: L-4, Noise factors assigned to columns arbitrarily

53. REQUIREMENTS: 4, 5, 6 or 7 -2LF Noise factors
DESIGN: L-8, Noise factors cols. 1, 2, 4 & 6.Remaining columns left empty.

54. REQUIREMENTS: 8, 9, 10 or 11 -2LF, interaction present but ignored
DESIGN: L-12, assign factors to columns arbitrarily (DO NOT USE L-12 TO STUDY INTERACTION)

55. REQUIREMENTS: 12, 13, 14 or 15 -2LF Noise factors
DESIGN: L-16, assign factors to columns arbitrarily

56. REQUIREMENTS: 2, 3 or 4 3LF Noise factors
DESIGN: L-9, Noise factors assigned arbitrarily

57. REQUIREMENTS: 1 or 2-2LF and 2-3LF Noise factors
DESIGN: L-9, Dummy treat columns for 2-level Noise factors.

58. REQUIREMENTS: 1-2LF and 4, 5, 6 or 7 -3LF Noise factors

DESIGN: L-18, assign the 2-level Noise factor to col. 1 and all other factors to cols. 2 - 8.

59. REQUIREMENTS: 2 -2LF(A & B) and 4, 5 or 6 -3LF Noise factors
DESIGN: L-18, assign factor A to col. 1, dummy treat and assign factor B to col. 2. Assign other factors to cols. 3 - 8.

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(Look for More Design Tips in Future Updates)

Application Steps (How to apply the DOE/Taguchi technique):

1. **Select Project**: Identify a design optimization or production problem solving project . Define project clearly based on function you intend to improve. For complex systems/process, review subsystems/sub-processes and select activities responsible for the function. Lead if it's your own project, suggest DOE if it's some one else's.
2. **Plan Experiment:** Conduct or Arrange the planning/brainstorming session. If it's your own project, you will benefit more if some one else facilitated the session. Determine:
   - Evaluation criteria and establish a scheme to combine them
   - Control factors and their levels.
   - Interaction (if any)
   - Noise factors (if any)
   - Number of samples to be tested.
   - Experiment resources and logistics
3. **Designing experiments:** Design experiment & describe trial conditions. Also:
   - Determine the order of running the experiment
   - Describe noise conditions for testing samples if the design includes an outer array
4. **Conduct Experiments:** Carry out experiments by selecting the trial condition in random order, and:
   - Note readings, calculate and record averages if multiple readings of the same criteria are taken.

- Calculate OEC using the formula defined in the planning session.

5. **Analyze Results:** Reduce observations (in case of multiple objectives) into results and perform analysis to:

   - Determine factor influence (Main Effect)
   - Identify significant factors (ANOVA)
   - Determine optimum condition and estimate performance
   - Calculate confidence interval of optimum performance
   - Adjust design tolerances based on ANOVA

6. **Confirm Expected Performance:** Test one or more samples at the optimum condition to:

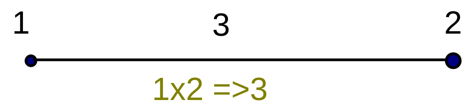
   - Establish performance at the optimum condition
   - Compare the average performance with the confidence interval determined from DOE

Common Orthogonal Arrays

Interactions (Linear Graphs)

$L_4(2^3)$ Array

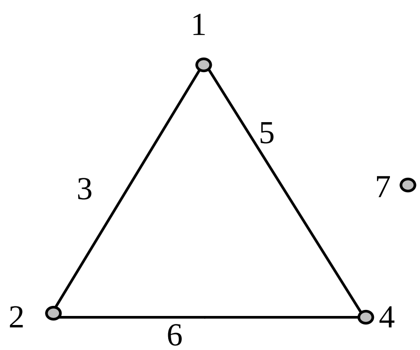
Trial#\	1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

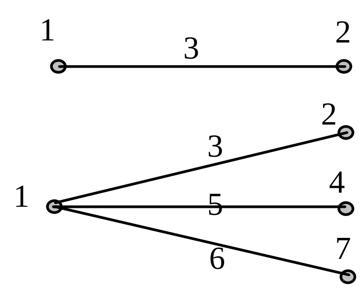


$L_8(2^7)$ Array

COL.>>

Trial#	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2





L_{12}

Column =>

Cond.	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	2	2	2	2	2	2
3	1	1	2	2	2	1	1	1	2	2	2
4	1	2	1	2	2	1	2	2	1	1	2
5	1	2	2	1	2	2	1	2	1	2	1
6	1	2	2	2	1	2	2	1	2	1	1
7	2	1	2	2	1	1	2	2	1	2	1
8	2	1	2	1	2	2	2	1	1	1	2
9	2	1	1	2	2	2	1	2	2	1	1
10	2	2	2	1	1	1	1	2	2	1	2
11	2	2	1	2	1	2	1	1	1	2	2
12	2	2	1	1	2	1	2	1	2	2	1

NOTE:
The L-12 is a special array designed to investigate main effects of 11 2-level factors.
THIS ARRAY IS NOT RECOMMENDED FOR ANALYZING INTERACTIONS

L_{16}

Column

Cond.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
3	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
4	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
5	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
6	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
7	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
8	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
10	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
11	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
12	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
13	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
14	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
15	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
16	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1

2- Level Orthogonal Arrays(Contd.)

(2-LEVEL, 31 FACTORS)

$L_{32}\ (2^{31})$

Col => Cond	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
4	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1
5	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
6	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1
7	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
8	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2
9	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
10	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1
11	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
12	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2
13	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
14	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2
15	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
16	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1
17	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
18	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1
19	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
20	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2
21	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
22	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2
23	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
24	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1
25	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
26	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2
27	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
28	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1
29	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
30	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1
31	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1
32	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2

$L_9(3^4)$

COL==>

COND	1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

$L_{18}\ (2^1\ 3^7)$

Col==>

Trial	1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1	1
2	1	1	2	2	2	2	2	2
3	1	1	3	3	3	3	3	3
4	1	2	1	1	2	2	3	3
5	1	2	2	2	3	3	1	1
6	1	2	3	3	1	1	2	2
7	1	3	1	2	1	3	2	3
8	1	3	2	3	2	1	3	1
9	1	3	3	1	3	2	1	2
10	2	1	1	3	3	2	2	1
11	2	1	2	1	1	3	3	2
12	2	1	3	2	2	1	1	3
13	2	2	1	2	3	1	3	2
14	2	2	2	3	1	2	1	3
15	2	2	3	1	2	3	2	1
16	2	3	1	3	2	3	1	2
17	2	3	2	1	3	1	2	3
18	2	3	3	2	1	2	3	1

$L_{27}(3^{13})$

Column =>

Cond.	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	2	2	2	2	2	2	2	2	2
3	1	1	1	1	3	3	3	3	3	3	3	3	3
4	1	2	2	2	1	1	1	2	2	2	3	3	3
5	1	2	2	2	2	2	2	3	3	3	1	1	1
6	1	2	2	2	3	3	3	1	1	1	2	2	2
7	1	3	3	3	1	1	1	3	3	3	2	2	2
8	1	3	3	3	2	2	2	1	1	1	3	3	3
9	1	3	3	3	3	3	3	2	2	2	1	1	1
10	2	1	2	3	1	2	3	1	2	3	1	2	3
11	2	1	2	3	2	3	1	2	3	1	2	3	1
12	2	1	2	3	3	1	2	3	1	2	3	1	2
13	2	2	3	1	1	2	3	2	3	1	3	1	2
14	2	2	3	1	2	3	1	3	1	2	1	2	3
15	2	2	3	1	3	1	2	1	2	3	2	3	1
16	2	3	1	2	1	2	3	3	1	2	2	3	1
17	2	3	1	2	2	3	1	1	2	3	3	1	2
18	2	3	1	2	3	1	2	2	3	1	1	2	3
19	3	1	3	2	1	3	2	1	3	2	1	3	2
20	3	1	3	2	2	1	3	2	1	3	2	1	3
21	3	1	3	2	3	2	1	3	2	1	3	2	1
22	3	2	1	3	1	3	2	2	1	3	3	2	1
23	3	2	1	3	2	1	3	3	2	1	1	3	2
24	3	2	1	3	3	2	1	1	3	2	2	1	3
25	3	3	2	1	1	3	2	3	2	1	2	1	3
26	3	3	2	1	2	1	3	1	3	2	3	2	1
27	3	3	2	1	3	2	1	2	1	3	1	3	2

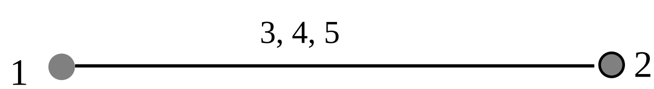
4-Level Orthogonal Arrays

This array is called the modified L-16 array which is made by combining the 5 interacting groups in the original 16 2-level columns.

$L_{16}\ (4^5)$

Col. => Trial	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	3	3	3	3
4	1	4	4	4	4
5	2	1	2	3	4
6	2	2	1	4	3
7	2	3	4	1	2
8	2	4	3	2	1
9	3	1	3	4	2
10	3	2	4	3	1
11	3	3	1	2	4
12	3	4	2	1	3
13	4	1	4	2	3
14	4	2	3	1	4
15	4	3	2	4	1
16	4	4	1	3	2

Linear Graph of L_{16}



To study interaction between two 4-level factors we must set aside three 4-level columns.

4- Level Orthogonal Arrays(contd.)

(2-Level and 4-Level)

$L_{32}\,(2^1 x 4^9)$

Trial\Column==>	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1
2	1	1	2	2	2	2	2	2	2	2
3	1	1	3	3	3	3	3	3	3	3
4	1	1	4	4	4	4	4	4	4	4
5	1	2	1	1	2	2	3	3	4	4
6	1	2	2	2	1	1	4	4	3	3
7	1	2	3	3	4	4	1	1	2	2
8	1	2	4	4	3	3	2	2	1	1
9	1	3	1	2	3	4	1	2	3	4
10	1	3	2	1	4	3	2	1	4	3
11	1	3	3	4	1	2	3	4	1	2
12	1	3	4	3	2	1	4	3	2	1
13	1	4	1	2	4	3	3	4	2	1
14	1	4	2	1	3	4	4	3	1	2
15	1	4	3	4	2	1	1	2	4	3
16	1	4	4	3	1	2	2	1	3	4
17	2	1	1	4	1	4	2	3	2	3
18	2	1	2	3	2	3	1	4	1	4
19	2	1	3	2	3	2	4	1	4	1
20	2	1	4	1	4	1	3	2	3	2
21	2	2	1	4	2	3	4	1	3	2
22	2	2	2	3	1	4	3	2	4	1
23	2	2	3	2	4	1	2	3	1	4
24	2	2	4	1	3	2	1	4	2	3
25	2	3	1	3	3	1	2	4	4	2
26	2	3	2	4	4	2	1	3	3	1
27	2	3	3	1	1	3	4	2	2	4
28	2	3	4	2	2	4	3	1	1	3
29	2	4	1	3	4	2	4	2	1	3
30	2	4	2	4	3	1	3	1	2	4
31	2	4	3	1	2	4	2	4	3	1
32	2	4	4	2	1	3	1	3	4	2

Triangular Table for
2-Level Orthogonal Arrays

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(1)	3	2	5	4	7	6	9	8	11	10	13	12	15	14
	(2)	1	6	7	4	5	10	11	8	9	14	15	12	13
		(3)	7	6	5	4	11	10	9	8	15	14	13	12
			(4)	1	2	3	12	13	14	15	8	9	10	11
				(5)	3	2	13	12	15	14	9	8	11	10
					(6)	1	14	15	12	13	10	11	8	9
						(7)	15	14	13	12	11	10	9	8
							(8)	1	2	3	4	5	6	7
								(9)	3	2	5	4	7	6
									(10)	1	6	7	4	5
										(11)	7	6	5	4
											(12)	1	2	3
												(13)	3	2
													(14)	1
														(15)

ETC...

To find the interaction column between factor a placed in column 4 and factor b placed in column 7, look for the number at the intersection of the horizontal line through (4) and the vertical line through (7), which is 3**.**

4 x 7 => 3

A x B => AxB

Likewise 1 x 2 => 3

3 x 5 => 6

Etc.

The set of three columns (4, 7, 3), (1, 2, 3), etc. are called interacting groups of columns.