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Modeling the uncertainty in response surface methodology through optimization and Monte Carlo simulation: An application in stamping process

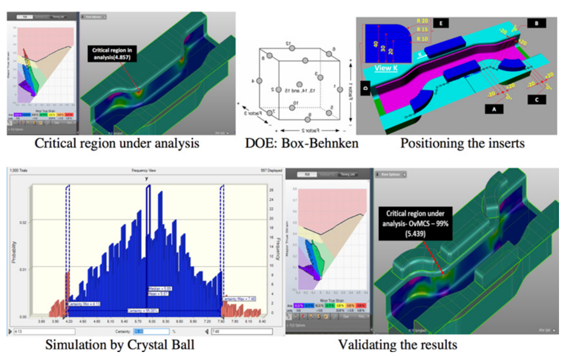
Aneirson Francisco da Silva *, Fernando Augusto Silva Marins, Erica Ximenes Dias, Jose Benedito da Silva Oliveira

São Paulo State University, Department of Production, Brazil

HIGHLIGHTS

- The advantages by this innovative procedure including the statistical validation
- The main contribution is insertion of uncertainties in the objective function.
- The new procedure was applied to a real case related to a stamping process.
- The results were compared to those obtained by applying classic RSM.

GRAPHICAL ABSTRACT



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ABSTRACT

Among the most frequently used experimental design techniques is the response surface methodology (RSM), which uses an approximation of the real objective function, in the form of an empirical quadratic function. RSM allows the identification of the relations between independent variables (or factors) and a (dependent) response variable. The main contribution of this article is to propose a new procedure that considers the insertion of uncertainties in the coefficients of this empirical function, which is what generally occurs, in practical experimental problems. The new procedure was applied to a real case related to a stamping process in an automotive company, and the results were compared to those obtained by applying classic RSM. The advantages offered by this innovative procedure are presented and discussed, including the statistical validation of the results. The proposed procedure reduces, and sometimes eliminates, the need for additional confirmatory experiments in the laboratory, and allows getting a better adjustment of the factor values and the optimized response variable value compared to the results calculated by classic RSM. It was possible to determine that the proposed procedure outperforms the use of (deterministic) optimization, using the generalized reduced gradient (GRG) algorithm, which is traditionally employed in RSM applications.

1. Introduction

A way to improve and adequately manage the productivity of manufacturing processes or services is through the use of design of

* Corresponding author.
E-mail address: aneirson.silva@unesp.br (A.F. da Silva).

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experiments (DOE) statistical techniques, which aim at understanding the impact of key factors identified by the companies involved in these processes and services [14]. Through the use of DOE, it is possible to identify in the response variable y (dependent variable), which is the characteristic investigated in the experimental problem, the linear effect, squared effects and interaction effects of independent variables x_i [3].

The development of this study was primarily motivated by the demand presented by companies in Brazil, in different sectors, which use DOE, mainly response surface methodology (RSM), to improve their manufacturing processes and services. The situation reported by analysts and engineers from the evaluated companies was that they frequently observed a discrepancy between the results generated by traditional (deterministic) optimization with RSM and the real results. In fact, they saw that although the RSM models used were statistically significant, the adjustment of the optimized factors, or decision variables, (x_i), according to the RSM solution generated a very different value for the response variable (y).

This situation ended up creating difficulties regarding the adjustments that needed to be made in the company's processes to improve performance, as well as in the confidence that the optimized values of the factors shown by the RSM were, in fact, appropriate for implementation, since the investments in these changes can be high. There are operations research methods that have the potential to help companies improve their productive processes or services, notably in cases in which there is a relevant influence of uncertainties on the parameters and the performance of the process.

In this paper, to verify the occurrence of uncertainties of the process parameters, we chose optimization via Monte Carlo simulation (OvMCS) [12], combined with RSM [1]. Note that according to [6,12], the OvMCS method is adequate to solve problems with objective functions and complex restrictions, which involve continuous variables as well as discrete variables, and which have various optimal local solutions.

Monte Carlo simulation (MCS) is used to solve complex engineering problems because it can deal with a large number of random variables, various distribution types, and highly nonlinear engineering models [8].

In addition to this introduction, this article is organized in more five sections. Section 2 presents scientific relevance, questions research and the objectives of work. Section 3 presents definitions and concepts related to the response surface methodology and optimization via Monte Carlo simulation. Section 4 describes the proposed methodology. Section 5 presents the results of the proposed procedure applied to a real stamping process. Finally, Section 6 presents the conclusions and suggestions for further research.

2. Scientific relevance and objectives

To verify the importance and identify research opportunities in the area of experiment planning, we conducted some bibliometric analyses, and the results are presented as follows. Fig. 1 shows that, in the period of 1992–2018, using combinations of the keywords "Response Surface Methodology", "Uncertainty" and "Monte Carlo Simulation" in the Web of Science database, 23 articles were identified, of which 19 were published between 2010 and 2018, evidencing the contemporaneity of this theme.

Finally, Table 1 presents the results of a search carried out in the Web of Science database up to 2019, with the keywords "Design of Experiments", "Response Surface Methodology", "Polynomial Function", "Uncertainty" "Optimization", "Monte Carlo Simulation", and "Optimization via Monte Carlo Simulation". The filters used in this search included "Articles" and "Proceedings Papers", where the keywords were cited in titles or abstracts.

It should be observed that only 24 publications included DOE and Uncertainty and RSM together, but the number of citations (365) in the period analyzed is high, with 2017 standing out with >40 citations.

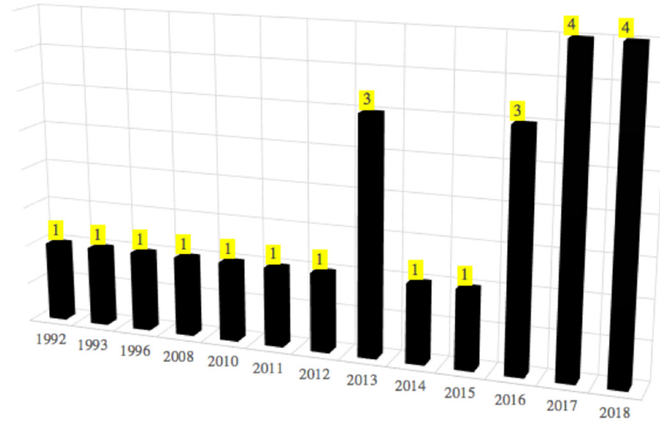

Fig. 1. Number of publications with keywords "Response Surface Methodology", "Uncertainty", and "Monte Carlo Simulation".
Source: Web of Science

These results indicate that there is still a good opportunity for research involving these subjects. Besides that, it should be pointed out that when including the keyword "Optimization via Monte Carlo Simulation", along with the three previous keywords, with the filters used, we found no papers in the database consulted.

In this context, we considered the following research questions:

- How should one consider the occurrence of uncertainties in the coefficients of the objective function of experimental problems modeled in RSM?
- What statistical approaches can be used to represent the occurrence of uncertainties in the coefficients of the objective function of experimental problems modeled in RSM?

Table 1
Number of publications and citations with keywords - "Design of Experiments", "Response Surface Methodology", "Polynomial Function", "Optimization", and "Monte Carlo Simulation".
Source: Web of Science

Keywords	Citations	Publications
"Design of Experiments" and "Optimization"	40,954	4146
"Design of Experiments" and "Response Surface Methodology"	14,117	1168
"Design of Experiments" and "Uncertainty"	4303	429
"Response Surface Methodologys" and "Uncertainty" and "Monte Carlo Simulation"	507	23
"Design of Experiments" and "Polynomial Function"	229	12
"Response Surface Methodology" and "Polynomial Function"	636	38
"Monte Carlo Simulation" and "Polynomial Function"	227	18
"Design of Experiments" and "Monte Carlo Simulation"	774	85
"Response Surface Methodology" and "Monte Carlo Simulation"	1016	91
"Design of Experiments" and "Uncertainty" and "Response Surface Methodology"	365	24
"Response Surface Methodology" and "Optimization" and "Monte Carlo Simulation"	292	42
"Response Surface Methodology" and "Uncertainty" and "Optimization" and "Monte Carlo Simulation"	33	10
"Deterministic Model" and "Monte Carlo Simulation"	1605	97
"Deterministic Model" and "Monte Carlo Simulation" and "Optimization"	215	19
"Design of Experiments" and "Uncertainty" and "Optimization via Monte Carlo Simulation"	0	0
"Response Surface Methodology" and "Uncertainty" and "Optimization via Monte Carlo Simulation"	0	0
"Deterministic Model" and "Monte Carlo Simulation" and "Optimization" and "Design of Experiments"	0	0
"Stamping Process" and "Design of Experiments"	73	15
"Stamping Process" and "Design of Experiments" and "Monte Carlo Simulation"	0	0

– How can one optimize an experimental problem modeled in RSM with the occurrence of uncertainties in the coefficients of the objective function?

To answer these questions, the overall objective of this study is to develop a procedure to consider the occurrence of uncertainties in the coefficients of the objective function of experimental problems modeled by RSM. Our specific objectives include applying the proposed procedure to a stamping process in a company in the automotive sector, aiming:

- To identify the advantages of the proposed procedure compared to the traditional RSM, which adopts deterministic optimization.
- To statistically validate the proposed procedure.

As previously mentioned, the method selected to insert uncertainty in the coefficients of the objective function was optimization via Monte Carlo simulation (OvMCS).

Finalizing this section, we highlight that this paper makes an interesting academic and practical contribution, since as found in the bibliographic search, there is a need for new knowledge about experimental problems considering uncertainty. In this context, we incorporate a proposal of an innovative approach for DOE involving RSM, OvMCS and uncertainty, and statistically validate it by a real application to a stamping process.

3. Background of response surface methodology and optimization via Monte Carlo simulation

The design of experiments (DOE) is adopted to identify the important variables (or factors), in a process and what their values (or conditions) should be to optimize the studied process's performance. For each factor, based on the result of experiments, limit values are selected, and in general within them two levels are tested for each factor. Therefore, the total number of experiments for a complete factorial design is given by 2^n, with n being the number of factors studied. Observe that in this complete factorial experimental scheme, when the number of factors increases, the number of experiments increases exponentially, and it can become infeasible to use it when there are many factors to be considered [14].

In this context, a useful DOE technique is the Response Surface Methodology - RSM, which substitutes a complex optimization problem with a sequence of simpler problems, with objective functions approximated by response surfaces (usually a second degree polynomial), and enabling a faster resolution of large real problems [2].

In general, a response variable of interest (y) is related to the factors (x_i) of a process in the form:

$$y = f(x_1, x_2, \ldots, x_n) + \epsilon \tag{1}$$

where ϵ is a random error, which includes the variations of the response variable that are not explained by the factors $x_1, x_2, \ldots, x_n$.

Since in the majority of practical problems addressed by the RSM function (1) is not known, this method uses an approximation of the real function in the form of a second-order polynomial function [5,20], as expressed in Eq. (2):

$$y = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n} \beta_{ii} x_i^2 + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \beta_{ij} x_i x_j + \epsilon \tag{2}$$

where y is an empirical function, x_i is the ith independent variable, x_j is the jth independent variable, β_0 is the offset term, β_i is the linear effect and β_{ii} is the squared effect, β_{ij} is the interaction effect, and ϵ is a random error.

A confidence interval (CI), considering a significance level α, can be constructed for the parameters of an empirical function, using Eq. (3):

$$CI(\beta, 1-\alpha) = \left[\hat{\beta} \pm z_{1-\frac{\alpha}{2}} \times \mathrm{SD}\left(\hat{\beta}\right)\right] \tag{3}$$

where $\hat{\beta}$ is an estimated value for parameter β, SD is the associated standard deviation, and e $z_{1-\frac{\alpha}{2}}$ is obtained on a Normal Distribution Table associated with the value of α.

There are two traditional ways to model problems in RSM: Central Composite Designs – CCD [14] and Box- Behnken Designs – BBD [10].

We now briefly describe the two alternatives:

– **Central Composite Designs – CCD** are the most commonly used in response surface designed experiment, and are a factorial or fractional factorial designs augmented with a group of axial points (also called star points) that allow estimating curvature [14].
If the validation tests with a CCD are positive (the response measures at the center of the field are statistically equal to the predicted value at the same point), the study is generally completed [20]. If the tests are negative, supplementary trials are undertaken to establish a second-degree model.
The additional trials are represented by the design points located on the axes of the coordinates and by new central points [10]. The points located on the coordinate axes are called star points. As shown in Fig. 2, CCD has three parts:

- **Factorial design**: These are full- or fractional-factorial designs with two levels per factor. The experimental points are at the corners of the study domain. They are the black points Fig. 2.
- **Star designs**: The points of a star design are on the axes and are, in general, all located at the same distance from the center of the study domain. They are the gray points in Fig. 2.
- **Center points**: There are usually center points, at the center of the study domain, for both the factorial design and the star design. They are the white points.

– **Box- Behnken Designs** - They allow us to directly implement second-degree models, where all the factors have three levels: −1, 0, and 1. These designs are easy to carry out and have a sequential property. With this procedure, it is possible to study x factors and

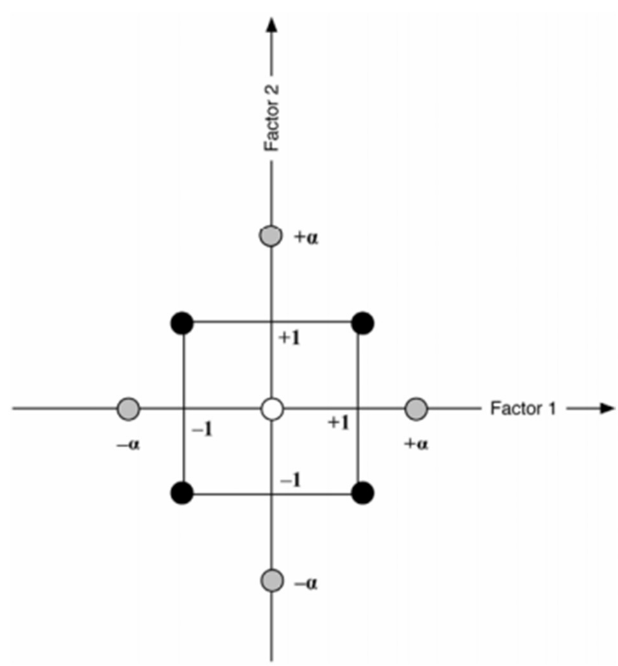

Fig. 2. Central Composite Designs for two factors.
Source: [10]

still have the option to add new ones without losing the results from the trials previously carried out [10].

This arrangement means that all experimental points are placed equidistant from the center of the study domain, which is on a sphere or hypersphere, depending on the number of dimensions [10].

So, the Box-Behnken design for three factors is constructed on a cube with 12 edges. The experimental points are not placed at the corners of the hypercube, but in the middle of the edges, in the center of the faces (squares), or in the center of the cube [15]. Therefore, the Box Behnken design for three factors has 15 (= 12 + 3) trials, associated with twelve experimental points at the center of each edge, and three points at the center of the cube, as shown in Fig. 3.

An operations research technique utilized in the procedure proposed her is Monte Carlo simulation (MCS), which is the common name for a wide variety of probability techniques, and it is a powerful statistical analysis tool. It is based on the use of random numbers (sampling) and probability statistics to investigate problems in fields as diverse as material science, economics, chemistry and biophysics, statistical physics, nuclear physics, flow of traffic flow and many others [13].

MCS is also used to solve complex engineering problems since it can deal with a large number of random variables, various distribution types, and highly nonlinear engineering models [8,11]. In industrial problems, the manager must optimize systems or processes that are frequently influenced by uncertainties, for instance, in the coefficients of the objective function and in the coefficients of the restrictions [7]. As pointed out by [12], the MCS is a way to evaluate the possible consequences of uncertainties in the optimization problems [9].

According to [19], an optimization via Monte Carlo simulation - OvMCS problem can be formulated as being:

$$\max g(x) = E_P[G(x,\omega)], x \in X \tag{4}$$

where G: $\mathbb{R}^n \times \Omega \rightarrow \mathbb{R}$ and the expectation is taken with respect to probability P, defined in a sample space (Ω, F) and $X \subset \mathbb{R}^n$. In the following section, we present the classification of this study, the materials and methods used, and describe the procedure for optimizing experimental problems under uncertainty, using RSM and OvMCS.

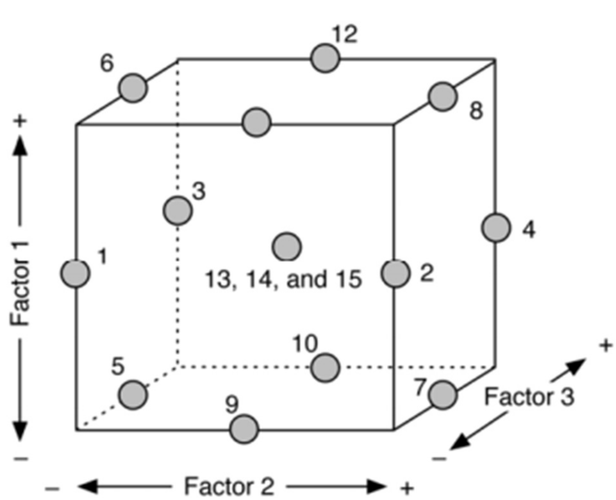

Fig. 3. Box-Behnken design for three factors.
Source: [10]

4. Methodology

This research can be classified as being applied in nature, with normative empirical objectives, and is the way to address a quantitative problem, and the procedures are modeling, simulation and experimental [4].

In Fig. 4 there is a flow chart with the steps for applying the systematic proposal to optimize the development of an experiment, which includes the combination of RSM, Uncertainty and OvMCS.

The techniques used in the steps to apply of the proposed procedure (see Fig. 4) were:

- Generation of the empirical functions representing the objective functions of RSM was performed with the ordinary least squares (OLS) algorithm [14].
- The traditional (deterministic) optimization was done with the generalized reduced gradient – GRG algorithm, available in Solver from Ms-Excel™.
- We used the OptQuest optimizer from the Crystal-Ball™ Trial Version software [17] to complete the optimizations in three stages. The OptQuest incorporates metaheuristics to guide its search algorithm toward better solutions. At a more advanced level, OptQuest does a much better job of finding optimal solutions than is possible with manual calculations. OptQuest surpasses the limitations of genetic algorithm optimizers because it uses multiple, complimentary search methods, including advanced tabu search and scatter search, to help find the best global solutions [16].

As shown in the flow chart in Fig. 5, the search process continues until OptQuest reaches some termination criterion, either a limit on the amount of time devoted to the search or a maximum number of simulations.

After the statistical validation, it will be possible to use different configurations in the experiment and accurately predict the confidence interval of the response variable investigated and, thus carry out more realistic experiments for different experimental configurations, without the need to carry out the confirmation experiment, which is often time-consuming and costly.

For the purposes of illustration and validation of the proposed general procedure, including the occurrence of uncertainties in the coefficients of the objective function of experimental problems modeled by RSM, we applied it to a real case involving a stamping process in a company in the automotive sector. The example, has five factors, and RSM with Box- Behnken design was also used to illustrate how to statistically to validate the proposed procedure.

To solve the example, we used an Intel Core i7, GHZ processor 2.8 with 16 GB RAM, and IOS operational system. The computational time was about 3 h. Additionally, we employed the optimizer Optquest of the software Crystal Ball™ was utilized.

In the next section, we describe the use of the proposed procedure in the studied real problem.

5. A stamping problem solved by the proposed methodology

In the sequence, considering a real stamping problem, the results obtained in each step (see Fig. 8) of the proposed procedure are presented.

5.1. Step 1 - identify the experimental problem

The object of study was a Brazilian multinational automotive company with 31 units in 14 countries and about 15 thousand employees. It produces side rails meeting the wide range of specifications on the market, stamped on the newest steel HSDP (High Strengths Dual Phases). Its great expertise in supplying chassis for trucks, buses and pick-ups for the leading automakers contributes to the development of projects that are focused on reducing cost and weight, combined with high resistance and durability. The studied product is a transmission crossmember, with thickness of 6.8 mm, using LNE 380 steel. This company has to solve a

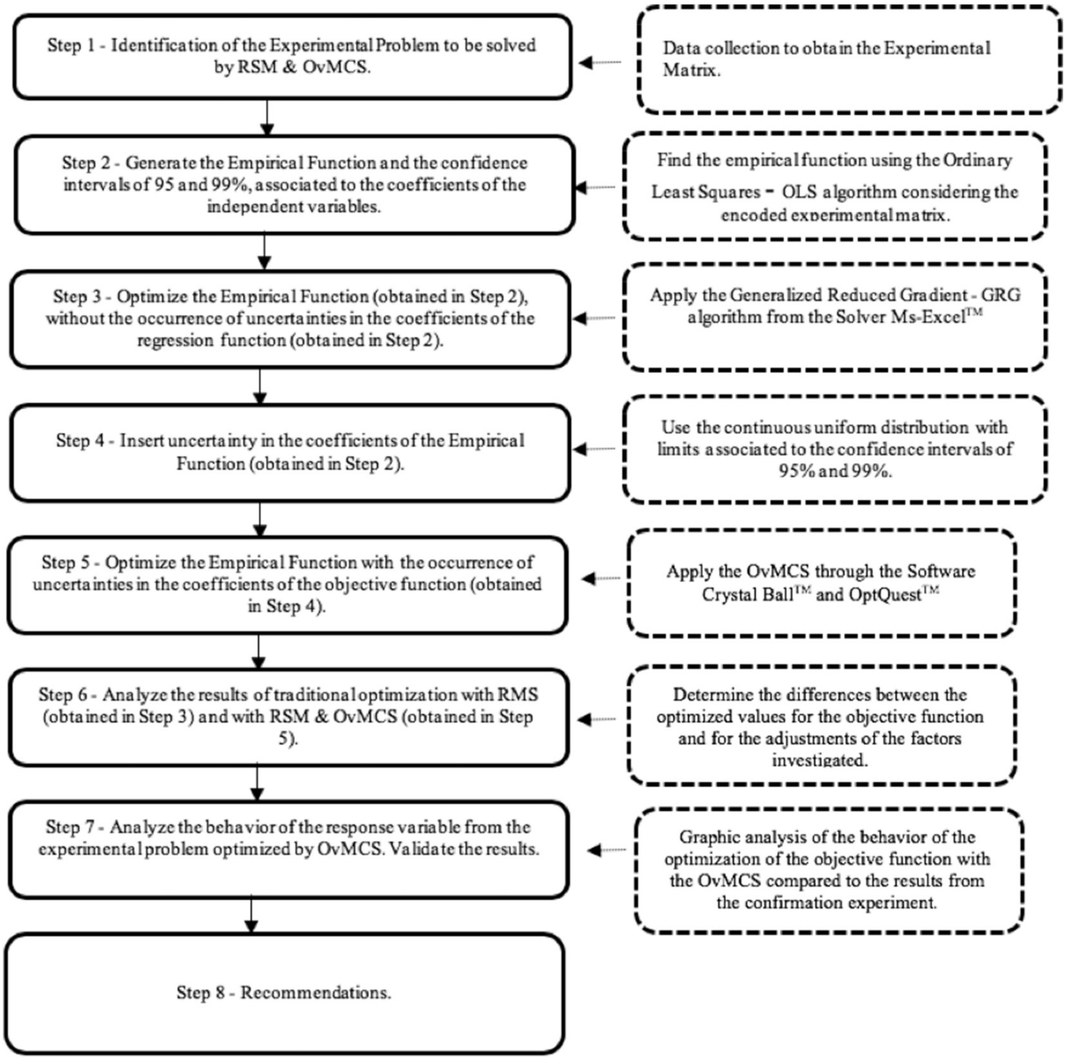

Fig. 4. Methodological approach.

complex problem associated with the occurrence of cracks in stamped parts, as highlighted in red in Fig. 6, which shows the reduction of the thickness of the crossbeam, from 6.8 mm to 4.857 mm, obtained with the current configuration of the mold during the stamping process.

Fig. 6 was constructed with the Autoform™ Software, which plots the stress–strain curve, which is an important tool to evaluate the fracture strength of a material. In fact, in the stamping industry, finite element simulation is a critical step to optimize of the sheet metal forming processes [21], and an indispensable input to the finite element model is the flow stress curve of the sheet material, also known as the true stress-strain curve [21].

In summary, the problems detected by the engineers were:

– Reduction of thickness in the critical region of the stamped components during the forming process.
– Incidence of cracks during the forming operation and possibility of cracks during use of the product.
– Lack of parameters to define compensations, in the tool project planning phase.

As an experiment in planning strategy, before using the Behnken - Design Box, a Plackett Burman Design [14] was used, since initially there were many (in a total of 23) x_s factors (positioning the inserts in the mold) to be investigated. The company's engineers suggested to placing inserts of different heights along the mold to decrease the appearance of cracks. Fig. 7 shows the configuration of the original mold, and the mold with the inserts (associated with the letters A - W), with different heights, which was used in the Plackett Burman design N24. The response y was obtained through finite element simulation performed by Autoform™ software. Appendix A shows the levels of each factor and the statistical treatment performed.

From the practical experience of the company's engineers, and based on the Pareto chart built by the Minitab™ software [14], it was possible to reduce the number of x_s factors from 23 to 15, and a Plackett Burman design N16 experiment was performed. Fig. 8 illustrates the new inserts' configuration in the mold. Appendix A shows the levels of each factor and the statistical treatment performed.

After using the Plackett Burman design N16 the number of factors was reduced to five, which was the number used in the application of the RSM with Box - Behnken design, as shown in Table 2.

Fig. 9 shows the adjustments of the inserts made in the mold for the execution of the RSM with Box - Behnken design experiment, and in the Appendix A presented all the $243 = 3^5$ possible combinations used in this experiment. As an illustration, a small part of it is reproduced in Table 3.

5.2. Step 2 - generate an empirical function associated with the objective function of RSM

The encoded experimental matrix associated with the process studied, obtained in Step 1, it is used to generate the empirical function and

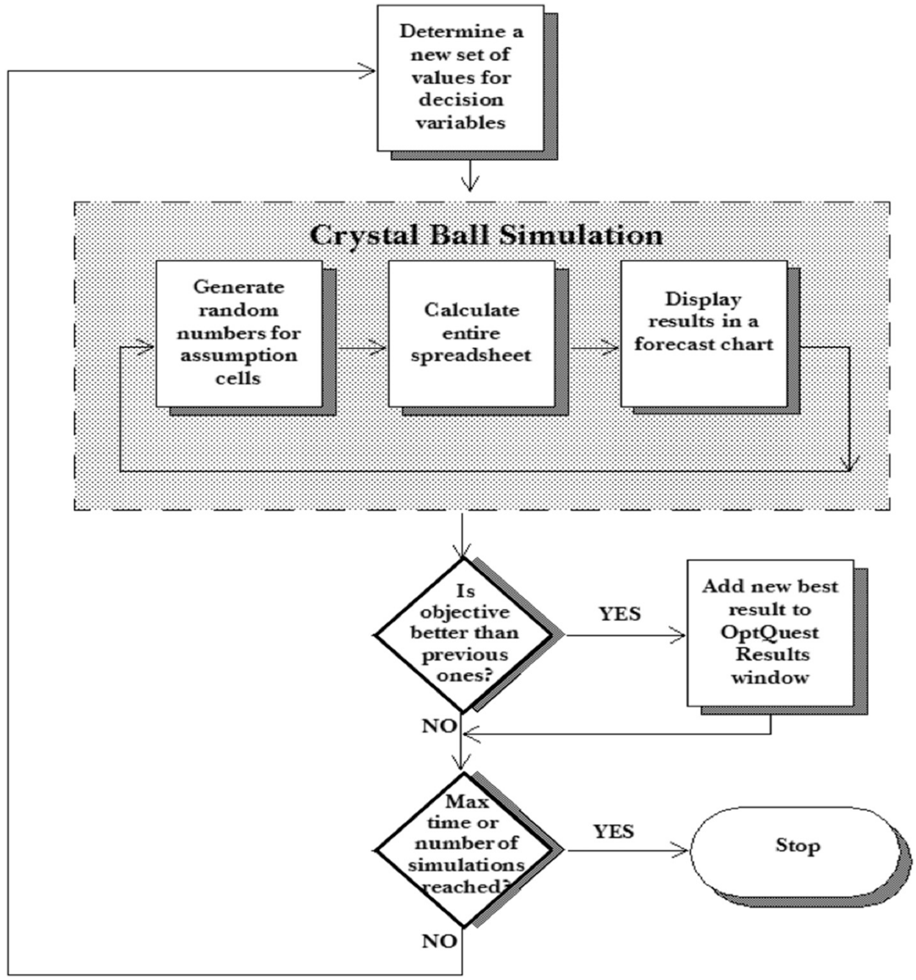

Fig. 5. OptQuest Flow.
Source: [16].

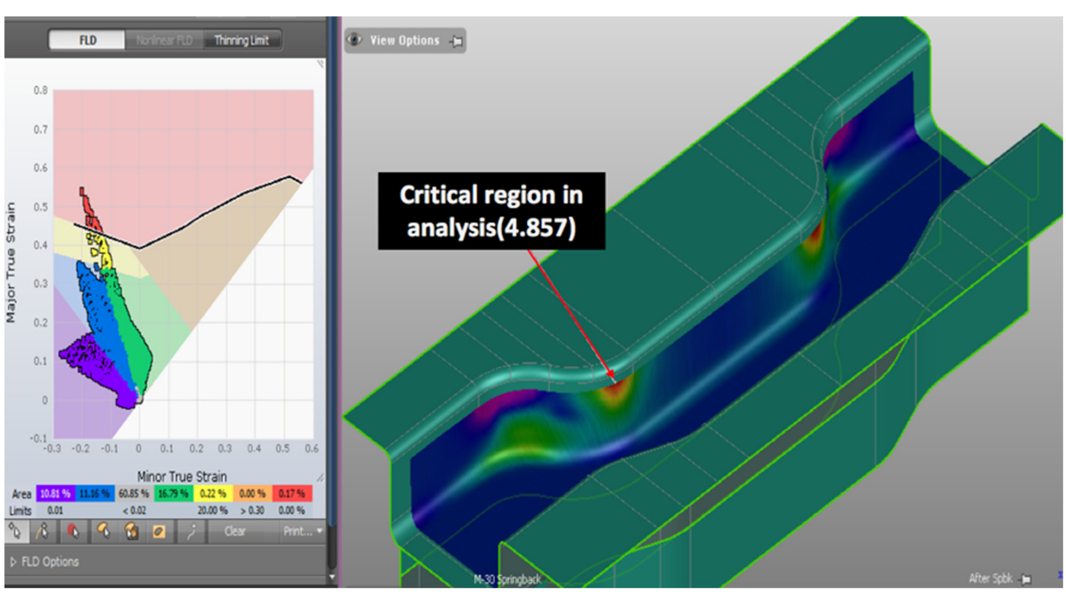

Fig. 6. Critical region in the analysis. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
Source: Autoform™ Software.

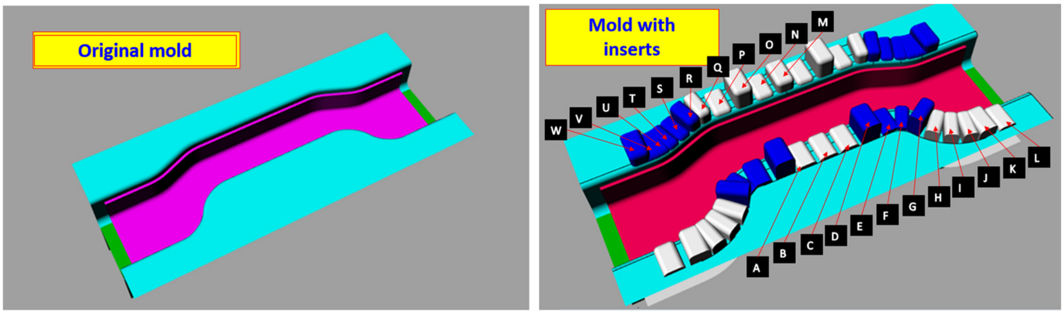

Fig. 7. Geometry of the mold of the stamped product without and with the inserts.

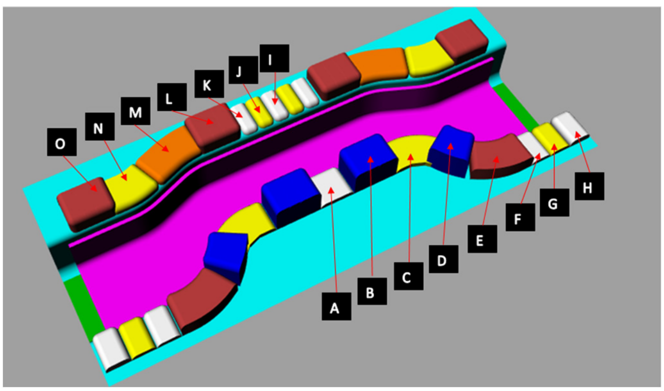

Fig. 8. Plackett Burman design N16.

the confidence intervals (CI), of 95% and 99%, for all the coefficients of the independent variables. Based on the results from the F and *t*-Student statistical tests [14], in expression (2), the terms that are not significant with $\alpha = 5\%$ are disregarded. Observe that, in expression (2), if the terms of the interaction effects, $\beta_{11}x_1x_2, \ldots, \beta_n x_{n-1}x_n$ are significant, it is recommended to keep the terms from the linear effects, $\beta_1 x_1, \beta_2 x_2, \ldots, \beta_n x_n$, even if they are not significant.

The justification for the choice of these confidence values for the CI that 95% is the standard used in experimental problems addressed by RSM and 99% represents the greatest range of variation of the coefficients of the empirical function in relation to the value estimated by the regression analysis.

Applying OLS algorithm, with the data from Table 3, we find the empirical quadratic function (5)–(6) and the constraints (7)–(9)

$$\hat{y} = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_2^2 + \beta_7 x_4^2 + \beta_8 x_5^2 + \beta_9 x_1 x_4 + \beta_{10} x_2 x_4 \tag{5}$$

$$\hat{y} = -0.01049784x_1 + -0.012858642x_2 - 0.002599383x_3 + 0.192685194 + 0.174519113x_5 + 0.000268939x_2^2 - 0.002288908x_4^2 - 0.006298995x_5^2 + 0.000170417x_1x_4 + 0.000253935x_2x_4 \tag{6}$$

subject to:

$$-20 \le x_i \le 20, i \in = \{1, 2, 3\}. \tag{7}$$

$$30 \le x_4 \le 50 \tag{8}$$

$$10 \le x_5 \le 20 \tag{9}$$

5.3. Step 3 - optimizing the empirical function without uncertainty

Upon applying the GRG algorithm to the problem Eqs. (6)–(9), we obtained the following optimal solution with the encoded values, according to the Box-Benken design used: thickness of material when molded in stamping process $= \hat{y} = 5.85$ mm, $x_1 = -20$, $x_2 = -20$, $x_3 = -20$, $x_4 = 41.35$ and $x_5 = 13.85$.

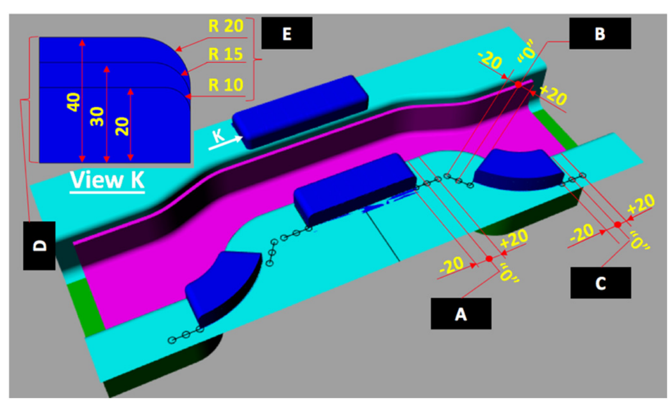

Fig. 9. Positioning the inserts for the RSM and Box - Behnken design.

Table 2
Experimental matrix of the stamping process example.

Factors	Description		Levels		
			−1	0	−1
A	Positioning of the insert at the entry of the traction region	x_1	−20	0	20
B	Positioning of the insert in the end of the traction region	x_2	−20	0	20
C	Positioning of the insert in the end of the compression region	x_3	−20	0	20
D	Heights of the compensating inserts	x_4	30	40	50
E	Radius of entry of compensating inserts	x_5	10	15	20

Table 3
Natural factor values for each experiment and the associated response values obtained by finite element simulation.

Trial	x_1	x_2	x_3	x_4	x_5	y
1	−20	−20	−20	30	10	5.465
2	−20	−20	−20	30	15	5.271
…		…				
…			…			
…				…		
243	20	20	20	50	20	5.039

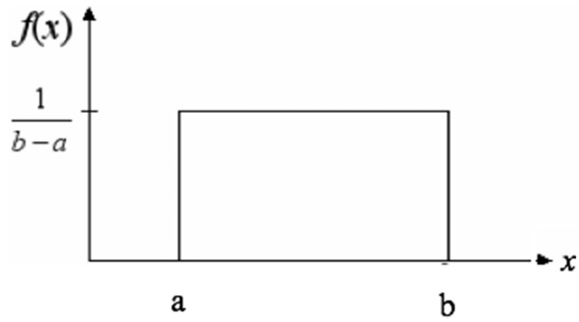

Fig. 10. Continuous uniform distribution.
Source: [14].

5.4. Step 4 - insert the occurrence of uncertainty in the coefficients of the empirical function, and Step 5 - optimize the empirical function with uncertainty

Enter the occurrence of uncertainties in the coefficients of the empirical function, obtained in Step 2. For this purpose, based on [14], the continuous uniform distribution in the [a, b] interval was chosen, which represents the situation in which any value, among the limits considered, has the same probability of occurring. The values of the lower (a) and upper (b) limits should be tested, associated with the CI of 95% and 99%, for each coefficient of the empirical function, as specified in Fig. 10.

$$f(x) = \left\{ \frac{1}{b-a} \rightarrow a \le x \le b 0 \rightarrow \text{Otherwise} \right. \quad (10)$$

In this context, [18] commented that based on the quantitative approach, the evaluation measures are calculated as the probabilities of occurrence of the main events and the reliability or insecurity of the main events.

Eq. (11) presents the new coefficients of the empirical function, with the insertion of uncertainty, as is in Table 4, for CI - 95% and CI - 99%:

$$\hat{y} = \widetilde{\beta_1}x_1 + \widetilde{\beta_2}x_2 + \widetilde{\beta_3}x_3 + \widetilde{\beta_4}x_4 + \widetilde{\beta_5}x_5 + \widetilde{\beta_6}x_2^2 + \widetilde{\beta_7}x_4^2 + \widetilde{\beta_8}x_5^2 + \widetilde{\beta_9}x_1x_4 + \tilde{\beta}_{10}x_2x_4 \quad (11)$$

To solve the problem with uncertainty, the value of the best feasible solution is obtained by the Crystal Ball™ software calibrated with number of trials = 500 and total number of simulations = 20,000. In Figs. 11 and 12 report all mean feasible solutions found out among the 20,000 simulations, respectively for OvMCS - CI - 95% and OvMCS - CI - 99%. Moreover, these figures present the best mean feasible solution found out (see Table 5): for OvMCS - CI - 95% we have $\hat{y} = 5.89$ mm and for OvMCS - CI - 99% we have $\hat{y} = 5.87$ mm.

5.5. Step 6 - compare optimization results with and without uncertainty

It is possible to verify, in Table 5, that:

- The optimized values of the response variable $\hat{y}$ calculated by the OvMCS algorithm were higher (therefore, better) in relation to what was obtained with the GRG algorithm ($\hat{y} = 5.85$ mm), for CI – 95% and $\hat{y} = 5.89$ mm, and 5.87 mm for CI – 99%.
- The optimized values of the independent variables x_3 and x_4, calculated by the OvMCS algorithm, for the CI - 95% ($x_4 = 41.55$ and $x_5 = 13.84$), as well as for the CI - 99% ($x_4 = 41.67$ and $x_5 = 13.64$), were different from those generated by the optimization with the

Table 4
Coefficients in the empirical function with occurrence of uncertainty.

Coefficients	Continuous uniform distribution (CI – 95%)	Continuous uniform distribution (CI – 99%)
$\beta_1 = -0.01049784$	$\tilde{\beta}_1$ ~ U[−0.01735401, −0.003641669	$\tilde{\beta}_1$ ~ U[−0.019535573, −0.001460106]
$\beta_2 = -0.012858642$	$\tilde{\beta}_2$ ~ U[−0.019714813, −0.006002471]	$\tilde{\beta}_2$ ~ U[−0.021896375, −0.003820909]
$\beta_3 = -0.002599383$	$\tilde{\beta}_3$ ~ U[−0.003970617, −0.001228149]	$\tilde{\beta}_3$ ~ U[−0.004406929, −0.000791836]
$\beta_4 = 0.192685194$	$\tilde{\beta}_4$ ~ U[0.174107666, 0.211262723]	$\tilde{\beta}_4$ ~ U[0.168196489, 0.2171739]
$\beta_5 = 0.174519113$	$\tilde{\beta}_5$ ~ U[0.124214602, 0.224823625]	$\tilde{\beta}_5$ ~ U[0.108208227, 0.24083]
$\beta_6 = 0.000268939$	$\tilde{\beta}_6$ ~ U[0.000150271, 0.000387608]	$\tilde{\beta}_6$ ~ U[0.000112512, 0.000425367]
$\beta_7 = -0.002288908$	$\tilde{\beta}_7$ ~ U[−0.002526497, −0.002051319]	$\tilde{\beta}_7$ ~ U[−0.002602095, −0.001975721]
$\beta_8 = -0.006298995$	$\tilde{\beta}_8$ ~ U[−0.007975476, −0.004622513]	$\tilde{\beta}_8$ ~ U[−0.0085089155, −0.004089074]
$\beta_9 = 0.000170417$	$\tilde{\beta}_9$ ~ U[2.47547E-06, 0.000338358]	$\tilde{\beta}_9$ ~ U[−5.09617E-055, 0.000391795]
$\beta_{10} = 0.000253935$	$\tilde{\beta}_{10}$ ~ U[8.5994E-05, 0.000421876]	$\tilde{\beta}_{10}$ ~ U[3.25568E-05, 0.000475314]

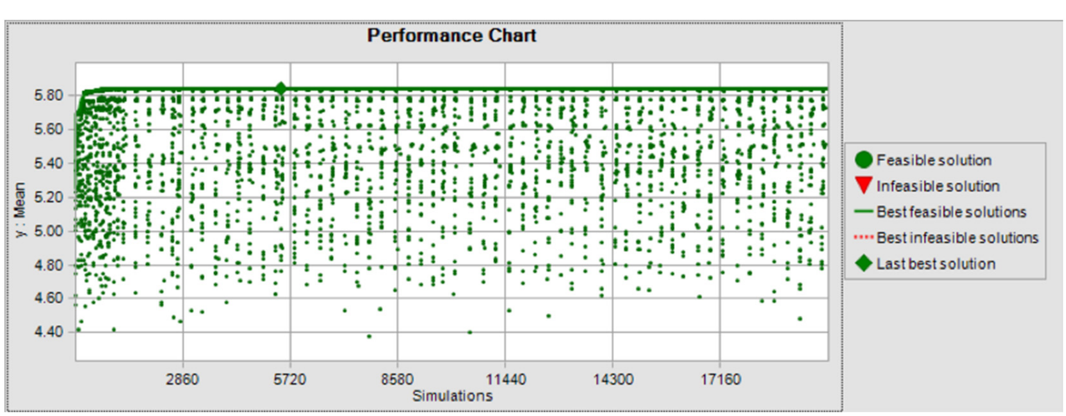

Fig. 11. Mean feasible solutions for OvMCS - CI - 95%.
Source: Crystal Ball™

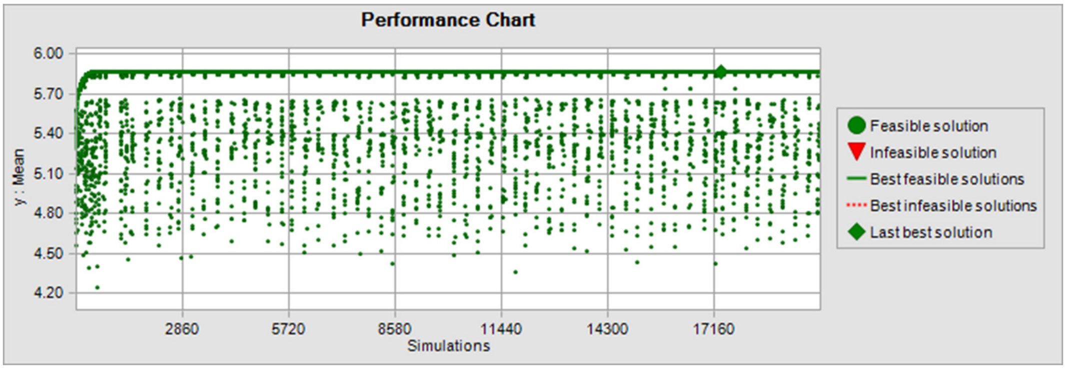

Fig. 12. Mean feasible solutions for OvMCS - CI - 99%.
Source: Crystal Ball™

GRG algorithm ($x_4 = 41.35$ and $x_5 = 13.85$). Also, observe that the optimized value of the independent variable $x_1, x_2, x_3 = -20$ was the same for all tested algorithms.

As illustrated in Table 5, it is possible to verify that, in OvMCS - CI - 95%, with the factors $x_1 = -20$, $x_2 = -20$, $x_3 = -20$, $x_4 = 41.55$ and $x_5 = 13.84$, $\hat{y}_{mean} = 5.89 \in [4.18, 7.59]$, which is the certainty range, as illustrated in Fig. 13.

As illustrated in Table 5, it is possible to verify that, in OvMCS - CI - 99%, with the factors $x_1 = -20$, $x_2 = -20$, $x_3 = -20$, $x_4 = 41.67$ and $x_5 = 13.64$, $\hat{y}_{mean} = 5.87 \in [4.13, 7.48]$, which is the certainty range, as illustrated in Fig. 14.

These results can be interpreted as evidence of a better quality of the optimization results when using OvMCS, since it enabled a better understanding of the effect of uncertainty in experimental problems modeled with RSM. In fact, with this proposed procedure, the analyst can perform a sensitivity analysis of the optimized values for each independent variable (x_i), and also, identify the range of variation for the

Table 5
Results of the optimizations for the real stamping process.
Source: Crystal Ball™

Factors/Approach	Encoded value	Natural value	Mean $\hat{y}$ [%]
x_1/OvMCS - CI – 95%	−1	−20	
x_2/OvMCS - CI – 95%	−1	−20	
x_3/OvMCS - CI – 95%	−1	−20	5.89
x_4/OvMCS - CI – 95%	0.155	41.55	
x_5/OvMCS - CI – 95%	−0.232	13.84	
x_1/OvMCS - CI – 99%	−1	−20	
x_2/OvMCS - CI – 99%	−1	−20	
x_3/OvMCS - CI – 99%	−1	−20	5.87
x_4/OvMCS - CI – 99%	0.167	41.67	
x_5/OvMCS - CI – 99%	−0.272	13.64	

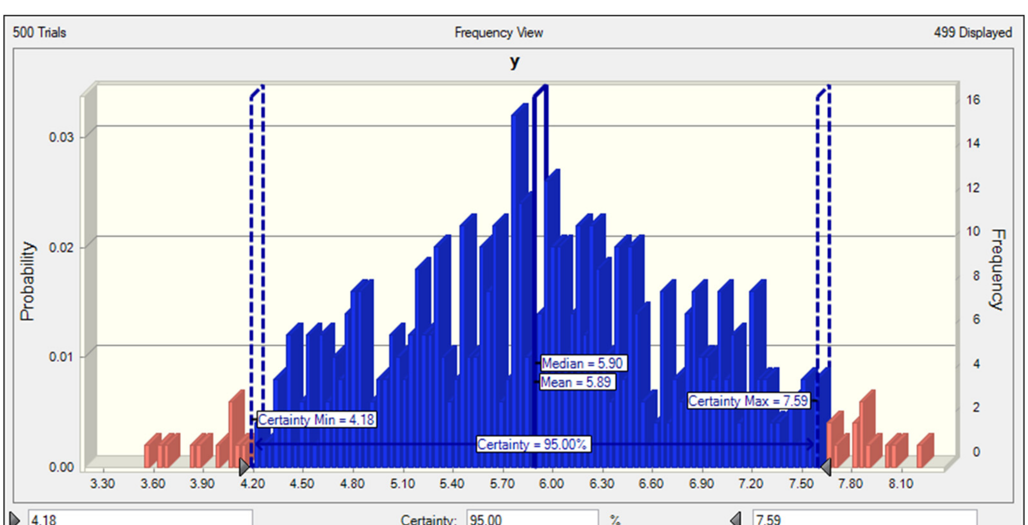

Fig. 13. Frequency chart for $\hat{\boldsymbol{y}}$ in OvMCS - CI - 95%.
Source: Crystal Ball™

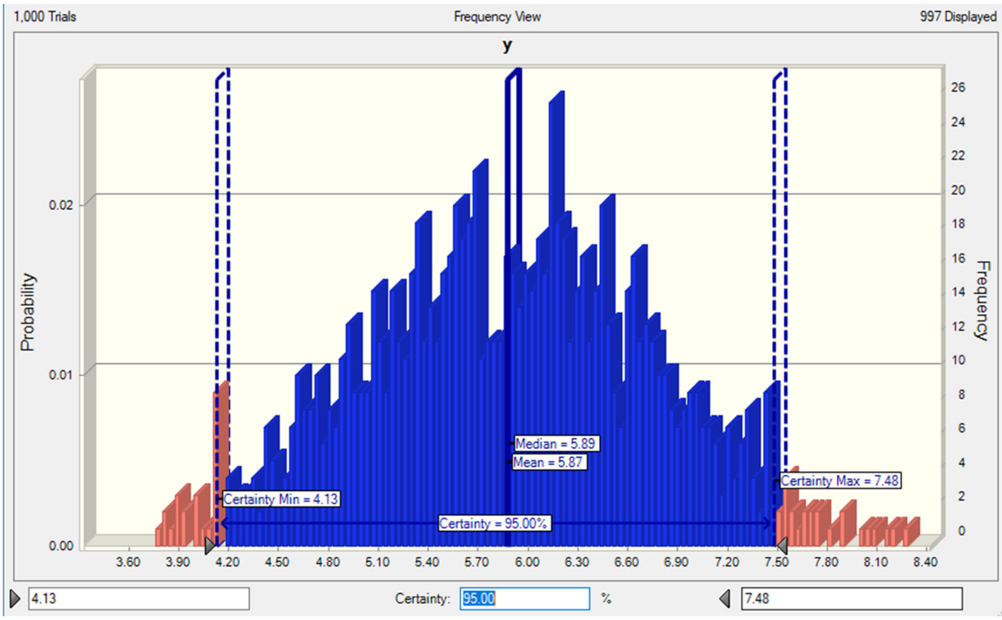

Fig. 14. Frequency chart for $\hat{\boldsymbol{y}}$ in OvMCS - CI - 99%.
Source: Crystal Ball™.

optimized dependent variable $\hat{y}$, which can help in the execution of more accurate and reliable experiments.

5.6. Step 7 - analyze the behavior of the response variable from the experimental problem optimized by OvMC, and validate the results, and Step 8 - recommendations

Fig. 6 shows the stamped product obtained from the original mold, as it was made by the company. There was a reduction of thickness in the critical region from 6.8 mm to 4.857 mm (a loss of 28.574%), and there were cracks.

Fig. 15 shows the stamped product obtained by the mold configuration with the inserts, but without the occurrence of uncertainty, utilizing the solution generated by the GRG algorithm. There were no cracks, and there was a reduction of thickness in the critical region from 6.8 mm to 5.423 mm (a loss of 20.25%).

Fig. 16 shows the stamped product obtained by the mold configuration with the inserts, but with the occurrence of uncertainty, utilizing the solution generated by the OvMCS - CI - 95% approach. There were no cracks, and there was a reduction of thickness in the critical region from 6.8 mm to 5.430 mm (a loss of 20.14%).

Fig. 17 shows the stamped product obtained by the mold configuration with the inserts, but with the occurrence of uncertainty, utilizing the solution generated by the OvMCS - CI - 99% approach. There were no cracks, and there was a lower reduction of thickness in the critical region from 6.8 mm to 5.439 mm (a loss of 20.00%).

Thus, the best solution for sizing the critical region was generated by the OvMCS - CI - 99% approach, as is evident from the comparison described below:

– The gain in relation to the current values practiced used by company was $\approx 11.99\% = \left(\frac{5.439}{4.857} - 1\right) \times 100\%$.

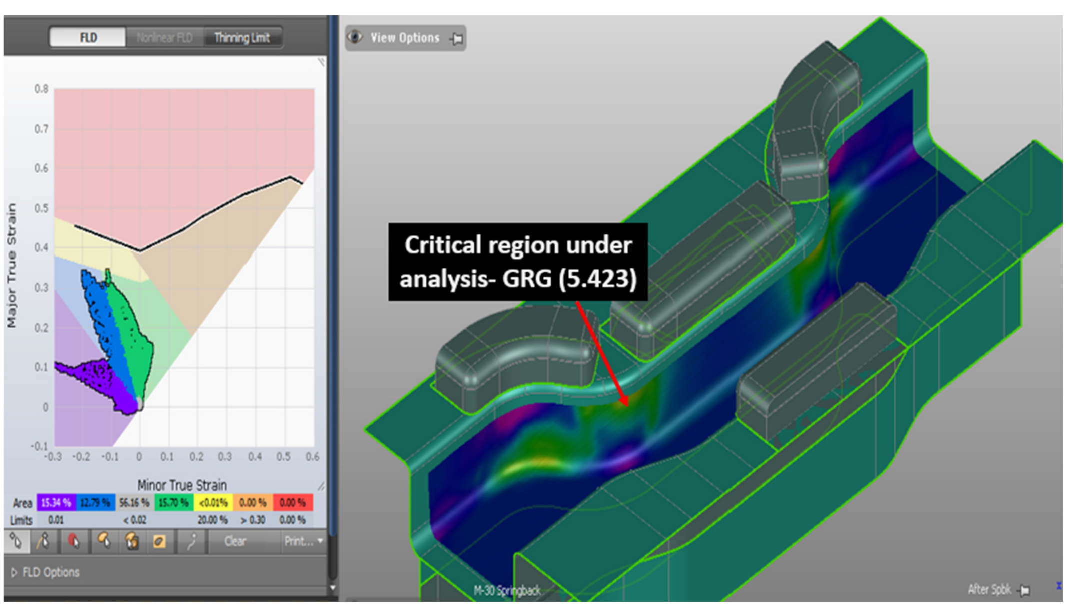

Fig. 15. Critical region under analysis - GRG algorithm.

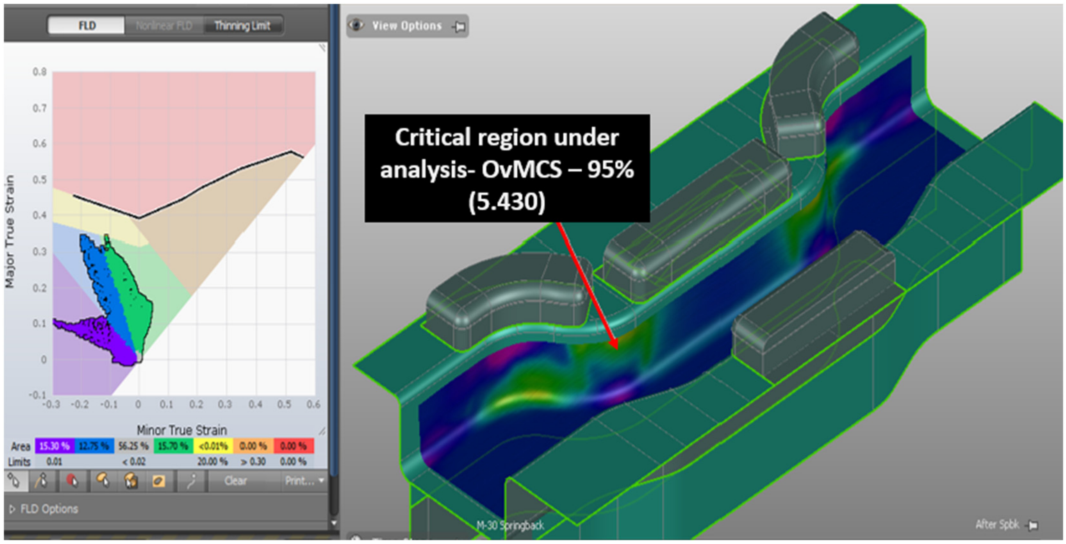

Fig. 16. Critical region under analysis - OvMCS - CI - 95% approach.

- The gain in relation to the solution generated by the use of the GRG algorithm was $\approx 0.3\% = \left(\frac{5.439}{5.423} - 1\right) \times 100\%$.
- The gain with respect to the solution generated by the OvMCS-95% approach was $\approx 0.17\% = \left(\frac{5.439}{5.430} - 1\right) \times 100\%$.

For the purpose of validation of the proposed procedure, with the optimized values suggested by the OvMCS - CI - 95% and by OvMCS - CI - 99% approaches, we obtained experimental values of the y response variable, using the finite element simulation in the Autoform™ software, as presented in Figs. 16 and 17. For $\hat{y}$ variable generated by the OvMCS - CI - 95% and OvMCS - CI - 99% approaches, using the Crystal Ball™ software, we constructed the associated confidence intervals, as shown in Figs. 13 and 14, respectively. It can be verified that experimental values of y variable are contained in these intervals, therefore statistically validating the proposed procedure.

6. Conclusions and recommendations for future research

Upon completing this study, we were able to answer all the research questions and fully meet all the proposed objectives. The proposed procedure of combining RSM with OvMCS, based on the tests completed, showed there are advantages in introducing the occurrence of uncertainties in the coefficients of the objective function in the experimental problems. We can also conclude that the continuous uniform distribution was shown to be appropriate to represent the occurrence of uncertainties.

The new procedure was applied to a real problem of stamping processes, and its results were compared to the results obtained by the classic RSM. The advantages offered by this innovative procedure were presented and discussed, including the statistical validation of its results.

It was possible to verify that RSM combined with OvMCS outperforms the use of (deterministic) optimization, using the GRG algorithm, which is traditionally used in RSM applications, providing engineers with useful information that will make their work easier

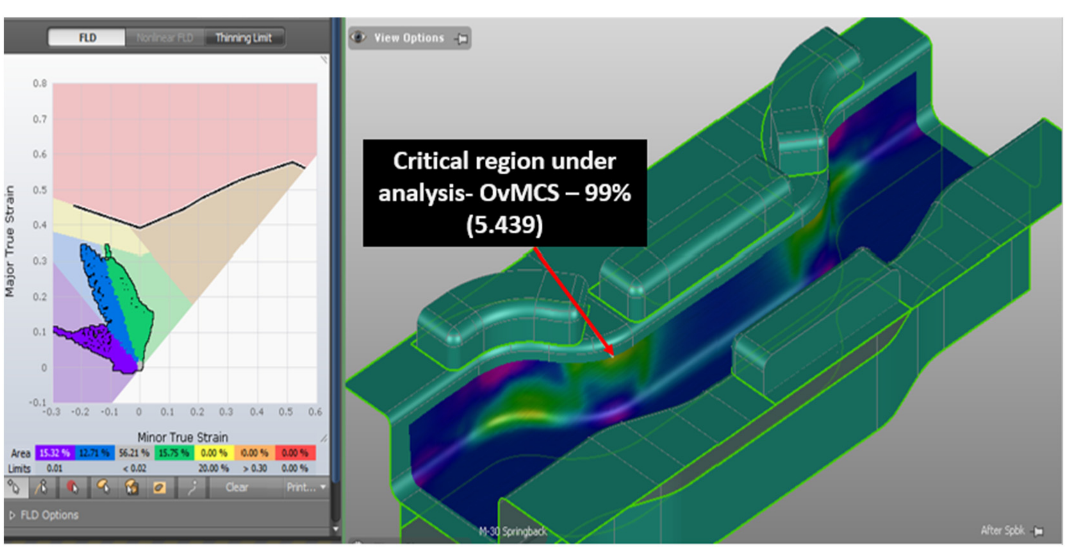

Fig. 17. Validate the results: Critical region under analysis - OvMCS - CI - 99%.

in searching for improvements to the process in question. In fact, the OvMCS - CI - 99%, with the continuous uniform distribution used to enter the occurrence of uncertainties in the coefficients of the objective function of RSM, presented interesting options to managers, and proposed a higher quality solution than that generated by the GRG algorithm.

We intend to test the proposed procedure in this article on experimental problems with multiple responses, such as in the case of various practical situations, entering the occurrence of uncertainties, which constitutes an important research gap, from the academic as well as the practical point of view.

Acknowledgements

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Appendix A

Table A1
- Influence Factors - Plackett Burman N24 Experiment.
Source: The authors (2018).

		Influence factors	Levels	
			−1	1
Raw material	**A**	Direction of the lamination process	0°	90°
Tool	**B**	Height of the central insert in the larger side	10	30
	C	Height of the insert in the straight central region in the larger side	10	30
	D	Height of the insert in the beginning of the traction region in the larger side	10	50
	E	Height of the 1st central insert in the traction region in the larger side	10	30
	F	Height of the 2nd central insert in the traction region in the larger side	10	30
	G	Height of the insert in the ending of the traction region in the larger side	10	50
	H	Height of the insert in the beginning of the compression region in the larger side	10	30
	I	Height of the 1st central insert in the compression region in the larger side	10	30
	J	Height of the 2nd central insert in the compression region in the larger side	10	30
	K	Height of the insert in the ending of the compression region in the larger side	10	30
	L	Height of the insert in the straight region of the end in the larger side	10	30
	M	Height of the central insert in the smaller side	10	30
	N	Height of 1st insert in the straight region of the end in the smaller side	10	30
	O	Height of 2nd insert in the straight region of the end in the smaller side	10	50
	P	Height of the insert in the beginning of the compression region in the smaller side	10	30
	Q	Height of the central insert in the compression region in the smaller side	10	30
	R	Height of the insert in the ending of the compression region in the smaller side	10	30
	S	Height of the insert between the compression and traction regions in the smaller side	10	30
	T	Height of the insert in the beginning of the traction region in the smaller side	10	30
	U	Height of the central insert in the traction region in the smaller side	10	30
	V	Height of the insert in the ending of the compression region in the smaller side	10	30
	W	Height of the insert in the straight region of the end in the smaller side	10	30

Table A2
– Response variable (Y) values - Plackett Burman N24 Experiment.
Source: The authors (2018).

Experiments	Influence factors																							
	Inputs variables																							Response variable
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	Y
1	**1**	**30**	**30**	**50**	**30**	**10**	**50**	**10**	**30**	**30**	**10**	**10**	**30**	**30**	**10**	**10**	**30**	**10**	**30**	**10**	**10**	**10**	**10**	**5.195**
2	**1**	**30**	**30**	**50**	**10**	**30**	**10**	**30**	**30**	**10**	**10**	**30**	**30**	**10**	**10**	**30**	**10**	**30**	**10**	**10**	**10**	**10**	**30**	**4.565**
3	**1**	**30**	**30**	**10**	**30**	**10**	**50**	**30**	**10**	**10**	**30**	**30**	**10**	**10**	**50**	**10**	**30**	**10**	**10**	**10**	**10**	**30**	**30**	**4.620**
4	**1**	**30**	**10**	**50**	**10**	**30**	**50**	**10**	**10**	**30**	**30**	**10**	**10**	**30**	**10**	**30**	**10**	**10**	**10**	**10**	**30**	**30**	**30**	**5.115**
5	**1**	**10**	**30**	**10**	**30**	**30**	**10**	**10**	**30**	**30**	**10**	**10**	**30**	**10**	**50**	**10**	**10**	**10**	**10**	**30**	**30**	**30**	**30**	**4.725**
6	**0**	**30**	**10**	**50**	**30**	**10**	**10**	**30**	**30**	**10**	**10**	**30**	**10**	**30**	**10**	**10**	**10**	**10**	**30**	**30**	**30**	**30**	**30**	**4.845**
7	**1**	**10**	**30**	**50**	**10**	**10**	**50**	**30**	**10**	**10**	**30**	**10**	**30**	**10**	**10**	**10**	**10**	**30**	**30**	**30**	**30**	**30**	**10**	**5.400**
8	**0**	**30**	**30**	**10**	**10**	**30**	**50**	**10**	**10**	**30**	**10**	**30**	**10**	**10**	**10**	**10**	**30**	**30**	**30**	**30**	**30**	**10**	**30**	**4.770**
9	**1**	**30**	**10**	**10**	**30**	**30**	**10**	**10**	**30**	**10**	**30**	**10**	**10**	**10**	**10**	**30**	**30**	**30**	**30**	**30**	**10**	**30**	**10**	**4.740**
10	**1**	**10**	**10**	**50**	**30**	**10**	**10**	**30**	**10**	**30**	**10**	**10**	**10**	**10**	**50**	**30**	**30**	**30**	**30**	**10**	**30**	**10**	**30**	**5.020**
11	**0**	**10**	**30**	**50**	**10**	**10**	**50**	**10**	**30**	**10**	**10**	**10**	**10**	**30**	**50**	**30**	**30**	**30**	**10**	**30**	**10**	**30**	**30**	**5.345**
12	**0**	**30**	**30**	**10**	**10**	**30**	**10**	**30**	**10**	**10**	**10**	**10**	**30**	**30**	**50**	**30**	**30**	**10**	**30**	**10**	**30**	**30**	**10**	**4.065**
13	**1**	**30**	**10**	**10**	**30**	**10**	**50**	**10**	**10**	**10**	**10**	**30**	**30**	**30**	**50**	**30**	**10**	**30**	**10**	**30**	**30**	**10**	**10**	**4.730**
14	**1**	**10**	**10**	**50**	**10**	**30**	**10**	**10**	**10**	**10**	**30**	**30**	**30**	**30**	**50**	**10**	**30**	**10**	**30**	**30**	**10**	**10**	**30**	**4.425**
15	**0**	**10**	**30**	**10**	**30**	**10**	**10**	**10**	**10**	**30**	**30**	**30**	**30**	**30**	**10**	**30**	**10**	**30**	**30**	**10**	**10**	**30**	**30**	**4.580**
16	**0**	**30**	**10**	**50**	**10**	**10**	**10**	**10**	**30**	**30**	**30**	**30**	**30**	**10**	**50**	**10**	**30**	**30**	**10**	**10**	**30**	**30**	**10**	**5.255**
17	**1**	**10**	**30**	**10**	**10**	**10**	**10**	**30**	**30**	**30**	**30**	**30**	**10**	**30**	**10**	**30**	**30**	**10**	**10**	**30**	**30**	**10**	**10**	**4.940**
18	**0**	**30**	**10**	**10**	**10**	**10**	**50**	**30**	**30**	**30**	**30**	**10**	**30**	**10**	**50**	**30**	**10**	**10**	**30**	**30**	**10**	**10**	**30**	**5.360**
19	**1**	**10**	**10**	**10**	**10**	**30**	**50**	**30**	**30**	**30**	**10**	**30**	**10**	**30**	**50**	**10**	**10**	**30**	**30**	**10**	**10**	**30**	**10**	**4.925**
20	**0**	**10**	**10**	**10**	**30**	**30**	**50**	**30**	**30**	**10**	**30**	**10**	**30**	**30**	**10**	**10**	**30**	**30**	**10**	**10**	**30**	**10**	**30**	**5.035**
21	**0**	**10**	**10**	**50**	**30**	**30**	**50**	**30**	**10**	**30**	**10**	**30**	**30**	**10**	**10**	**30**	**30**	**10**	**10**	**30**	**10**	**30**	**10**	**5.060**
22	**0**	**10**	**30**	**50**	**30**	**30**	**50**	**10**	**30**	**10**	**30**	**30**	**10**	**10**	**50**	**30**	**10**	**10**	**30**	**10**	**30**	**10**	**10**	**5.110**
23	**0**	**30**	**30**	**50**	**30**	**30**	**10**	**30**	**10**	**30**	**30**	**10**	**10**	**30**	**50**	**10**	**10**	**30**	**10**	**30**	**10**	**10**	**10**	**4.835**
24	**0**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**4.810**

Table A3
- Influence factors - Plackett Burman N16 Experiment.
Source: The authors (2018)

		Influence factors	Levels	
			−1	1
Tool	**A**	Height of the central insert in the larger side	10	15
	B	Height of the insert in the straight central region in the larger side	30	50
	C	Height of the insert in the beginning of the traction region in the larger side	10	15
	D	Height of the insertion in the traction region in the larger side	30	50
	E	Height of the insert in the compression region in the larger side	20	30
	F	Height of 1st insert in the straight region of the end in the larger side	10	15
	G	Height of 2nd insert in the straight region of the end in the larger side	10	15
	H	Height of 3rd insert in the straight region of the end in the larger side	10	15
	I	Height of the central insert in the smaller side	10	15
	J	Height of 1st insert in the straight region of the end in the smaller side	10	15
	K	Height of 2nd insert in the straight region of the end in the smaller side	10	15
	L	Height of 3rd insert in the straight region of the end in the smaller side	20	30
	M	Height of the insert in the compression region in the smaller side	10	15
	N	Height of the insertion in the traction region in the smaller side	10	15
	O	Height of the insert in the straight region of the end in the smaller side	20	30

Table A4
- Response variable (Y) values - Plackett Burman N16 Experiment.
Source: The authors (2018).

Experiments	Influence factors															
	Inputs variables															Response variable
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	Y
1	15	50	15	50	**20**	15	**10**	15	15	**10**	**10**	30	**10**	**10**	**20**	**5.206**
2	15	50	15	**30**	30	**10**	15	15	**10**	**10**	15	**20**	**10**	**10**	30	**5.203**
3	15	50	**10**	50	**20**	15	15	**10**	**10**	15	**10**	**20**	**10**	15	30	**5.167**
4	15	**30**	15	**30**	30	15	**10**	**10**	15	**10**	**10**	**20**	15	15	30	**4.855**
5	**10**	50	**10**	50	30	**10**	**10**	15	**10**	**10**	**10**	30	15	15	30	**5.148**
6	15	**30**	15	50	**20**	**10**	15	**10**	**10**	**10**	15	30	15	15	**20**	**4.936**
7	**10**	50	15	**30**	**20**	15	**10**	**10**	**10**	15	15	30	15	**10**	30	**5.199**
8	15	50	**10**	**30**	30	**10**	**10**	**10**	15	15	15	30	**10**	15	**20**	**5.242**
9	15	**30**	**10**	50	**20**	**10**	**10**	15	15	15	15	**20**	15	**10**	30	**5.132**
10	**10**	**30**	15	**30**	**20**	**10**	15	15	15	15	**10**	30	**10**	15	30	**4.754**
11	**10**	50	**10**	**30**	**20**	15	15	15	15	**10**	15	**20**	15	15	**20**	**5.151**
12	15	**30**	**10**	**30**	30	15	15	15	**10**	15	**10**	30	15	**10**	**20**	**4.885**
13	**10**	**30**	**10**	50	30	15	15	**10**	15	**10**	15	30	**10**	**10**	30	**4.995**
14	**10**	**30**	15	50	30	15	**10**	15	**10**	15	15	**20**	**10**	15	**20**	**4.991**
15	**10**	50	15	50	30	**10**	15	**10**	15	15	**10**	**20**	15	**10**	**20**	**5.034**
16	**10**	**30**	**10**	**30**	**20**	**10**	**10**	**10**	**10**	**10**	**10**	**20**	**10**	**10**	**20**	**4.811**

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