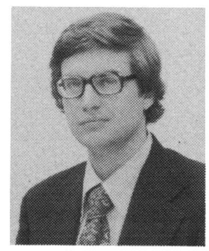
REFERENCES

[1] A. A. Sawchuk, "Space-variant image motion degradation and restoration," *Proc. IEEE,* vol. 60, pp. 854–861, 1972.
[2] ——, "Space-variant system analysis of image motion," *J. Opt. Soc. Amer.,* vol. 63, pp. 1052–1063, 1973.
[3] R. Nathan, "Digital video data handling," Jet Propulsion Lab., Pasadena, CA, Tech. Rep. 32-877, 1966.
[4] F. C. Billingsley, "Applications of digital image processing," *Appl. Optics,* vol. 9, pp. 289–299, 1970.
[5] W. B. Green and R. M. Ruiz, "Removal of photometric distortion from mariner-9 television images," *J. Opt. Soc. Amer.,* vol. 62, pp. 1351–1352, 1972.
[6] D. G. Luenberger, *Optimization by Vector Space Methods.* New York: Wiley, 1969, pp. 82–83.
[7] R. Deutsch, *Estimation Theory.* Englewood Cliffs, NJ: Prentice-Hall, 1965, ch. 4 and 5.
[8] G. H. Golub and C. Reinsch, "Singular-value decomposition and least squares solutions," *Numer. Math.,* vol. 14, pp. 403–420, 1970.
[9] H. C. Andrews and C. L. Patterson, "Outer product expansions and their uses in digital image processing," *IEEE Trans. Comput.,* vol. C-25, pp. 140–148, Feb. 1976.
[10] C. Fisher and C. P. Bond, "The quantimet 720 D for densitometry in the life sciences," *Microscope,* vol. 20, pp. 203–216, 1972.
[11] C. L. Bristor, W. M. Callicott, and R. E. Bradford, "Operational processing of satellite cloud pictures by computer," *Mon. Weather Rev.,* vol. 94, pp. 512–527, 1966.
[12] J. M. Wozencraft and I. M. Jacobs, *Principles of Communication Engineering.* New York: Wiley, 1965, pp. 148–153.
[13] A. A. Sawchuk, "Space-variant image restoration by coordinate transformations," *J. Opt. Soc. Amer.,* vol. 64, pp. 138–144, 1974.
[14] W. S. Dorn and D. D. McCracken, *Numerical Methods with Fortran IV Case Studies.* New York: Wiley, 1972, pp. 33–34.
[15] D. E. Knuth, "Evaluation of polynomials by computer," *Commun. Assoc. Comput. Mach.,* vol. 5, pp. 595–599, 1962.



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Aircraft Identification by Moment Invariants

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***Abstract*—Although many systems for optical reading of printed matter have been developed and are now in wide use, comparatively little success has been achieved in the automatic interpretation of optical images of three-dimensional scenes. This paper is addressed to the latter problem and is specifically concerned with automatic recognition of aircraft types from optical images. An experimental system is described in which certain features called *moment invariants* are extracted from binary television images and are then used for automatic classification. This experimental system has exhibited a significantly lower error rate than human observers in a limited laboratory test involving 132 images of six aircraft types. Preliminary indications are that this performance can be extended to a wider class of objects and that identification can be accomplished in one second or less with a small computer.**

***Index Terms*—Bayes decision rule, boundary, moment invariants, nearest neighbor rule, pattern recognition, recognition accuracy, silhouette, three-dimensional objects.**

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I. INTRODUCTION

THE possibility of duplicating in a machine the ability of animals and man to interpret visual information has intrigued many investigators. Although this problem has been treated with considerable success relative to automatic classification of two-dimensional objects, especially with regard to recognition of printed characters, comparatively little has been achieved in automatic identification of three-dimensional objects. This paper is concerned with recognition of complex man-made objects rather than natural objects or simple solids.

A literature survey has revealed that most investigators interested in complex three-dimensional object identification have been concerned primarily with the problem of aircraft identification [1]–[6]. Research on this particular problem has been motivated by a need for automatic methods for identifying aircraft types in an air traffic control situation. Moreover, a collection of different types of aircraft represents a group of three-dimensional objects with an interesting degree of complexity and therefore serves as an appropriate example for a general study of the identification problem. More recently, the importance of the three-dimensional object identification problem in industrial automation has been recognized [7]. A detailed survey of various techniques which have been investigated for recognition of complex three-dimensional objects can be found in [8]. In the interest of conciseness, this work will not be summarized here.

The problem under consideration in this investigation can be summarized as follows. An on-line recognition system for the purpose of identification of three-dimensional objects is to be developed. Using an ordinary black and white television camera and the necessary interface equipment, a two-dimensional image is to be furnished to a digital computer. After some preprocessing on the input picture to discard undesired image clutter, a clean silhouette and its boundary are to be obtained. Using this silhouette and boundary, certain features, depending on the moment calculations to be described later in this paper, are then to be extracted. Finally, on the basis of these features, a recognition algorithm is to be designed to identify the three-dimensional object and also to estimate its position and orientation in space.

In what follows, it is assumed that an object to be recognized belongs to one of a set of known classes, and that all elements of a given class are essentially "cast from the same mold." That is, each object is a more or less perfect replica of a given solid model for its class. It is also assumed that classification must be automatically accomplished from the information available in a single optical image obtained with the object in an unknown orientation relative to a camera.

II. The Input System

A block diagram of the image acquisition and processing system developed in this research is shown in Fig. 1. A white plastic model airplane is placed in a fixture in front of the television camera and viewed against a black background. The resulting video output from the camera is digitized into 180 binary resolution elements per horizontal scan line by a simple threshold circuit. The binary image formed in this way consists of 256 lines from one-half frame of the interlaced scan. The one-cells of the binary image correspond to video signals above the brightness threshold; the zero-cells, to those below the threshold. The resulting 256×180 binary image is next preprocessed to clean up small areas of noise present. Finally, the boundary of the resulting aircraft silhouette is extracted using techniques similar to those described in [9]. Some typical examples of the aircraft boundaries obtained by this processing are shown in Fig. 2.

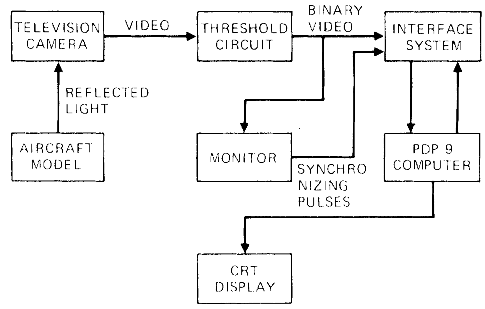

Fig. 1. Image acquisition and processing system for object identification experiments.

III. Feature Selection and Moment Invariants

One of the more difficult problems in the design of a pictorial pattern recognition system relates to the selection of a set of appropriate numerical attributes or *features* to be extracted from the object of interest for purposes of classification. The success of any practical system depends critically upon this decision. Although there is little in the way of a general theory to guide in the selection of features for an arbitrary problem [10], it is possible to state some desirable attributes of features for identification of solid objects. Among these are the following:

1) The features should be informative. That is, the dimensionality of a vector of measurements (feature vector) should be as low as possible, consistent with acceptable recognition accuracy.

2) The features should be invariant with translation of the object normal to the camera optical axis and with rotation about this axis.

3) The features should either be invariant or depend in a known way upon the distance of the object from the camera.

In the remainder of this section, it will be shown that certain functions of the central moments of a binary image satisfy the above criteria, and in addition are a class of features which can be rather readily computed.

A. Moment Transformations

Consider either a solid silhouette or its boundary which is represented by a matrix of ones and zeros in a discretized two-dimensional image plane. For such an image, the central moments are given by

$$\mu_{pq} = \frac{1}{N} \sum_{i=1}^{N} (u_i - \bar{u})^p \, (v_i - \bar{v})^q \tag{1}$$

where $\bar{u}$ and $\bar{v}$ are the mean values of the image coordinates u and v, respectively, and the summation is over all image points. Hu [11] derived a set of moment functions which have the desired property of invariance under image translation and rotation. Similar invariants have been proposed for use in automatic ship identification [12]. A

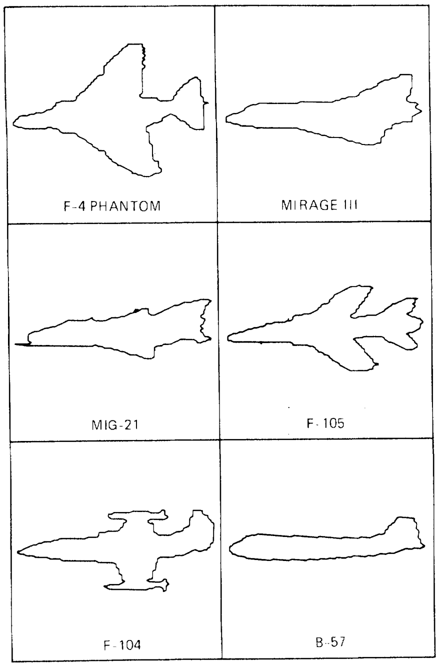

Fig. 2. Typical images obtained with the experimental image acquisition system.

set of moment invariant functions based only on the second- and third-order moments and which appear to be suitable for the present problem are given below [11].

$$M_1 = (\mu_{20} + \mu_{02}) \tag{2}$$

$$M_2 = (\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2 \tag{3}$$

$$M_3 = (\mu_{30} - 3\mu_{12})^2 + (3\mu_{21} - \mu_{03})^2 \tag{4}$$

$$M_4 = (\mu_{30} + \mu_{12})^2 + (\mu_{21} + \mu_{03})^2 \tag{5}$$

$$\begin{aligned} M_5 = {} & (\mu_{30} - 3\mu_{12})(\mu_{30} + \mu_{12}) \\ & \cdot [(\mu_{30} + \mu_{12})^2 - 3(\mu_{21} + \mu_{03})^2] \\ & + (3\mu_{21} - \mu_{03})(\mu_{21} + \mu_{03}) \\ & \cdot [3(\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2] \end{aligned} \tag{6}$$

$$\begin{aligned} M_6 = {} & (\mu_{20} - \mu_{02})[(\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2] \\ & + 4\mu_{11} (\mu_{30} + \mu_{12})(\mu_{21} + \mu_{03}) \end{aligned} \tag{7}$$

$$\begin{aligned} M_7 = {} & (3\mu_{21} - \mu_{03})(\mu_{30} + \mu_{12}) \\ & \cdot [(\mu_{30} + \mu_{12})^2 - 3(\mu_{21} + \mu_{03})^2] \\ & - (\mu_{30} - 3\mu_{12})(\mu_{21} + \mu_{03}) \\ & \cdot [3(\mu_{30} + \mu_{12})^2 - (\mu_{21} + \mu_{03})^2]. \end{aligned} \tag{8}$$

The functions M_1 through M_6 are invariant under rotation, reflection, or a combination of rotation and reflection. However, the function M_7 is invariant only in its absolute magnitude under a reflection as it changes its sign for a given reflection.

B. Moment Invariance with Angle of Elevation

Although the concept of Euler angles is used in many fields, there is no general agreement on its definition. There are, in fact, a large number of choices for the three angles required to define a Euler set of angles [13]. For the purpose of this research, a particular set of angles, called *camera orientation angles,* was selected with the aim of obtaining moment invariance with one of the angles of the set. Let xyz be a fixed coordinate system attached to a particular aircraft model with the origin at the center of gravity, positive x-axis toward the nose, positive y-axis along the right wing, and positive z-axis toward the bottom surface; let XYZ be another reference system such that XZ is the image plane and the Y-axis is the same as the optical axis of the camera. Now starting with the xyz and the XYZ coordinate systems initially aligned, an aircraft or other solid object is oriented by first rotating it about the camera Y-axis (elevation), then rotating it about the rotated aircraft z-axis (azimuth), and finally rotating it about the aircraft x-axis (roll).

When the elevation angle of a certain object is changed, the locus of the projection of each point of the object on the image plane is a circle with its center at the origin. Consequently, it follows that the result on the image of varying the elevation angle of a certain object is to produce merely a rotation in the image plane, with no change in size and shape. The moment functions M_1 through M_7 mentioned earlier are therefore invariant with the elevation angle θ.

C. Moment Invariance with Distance Along Optical Axis

It is clear that as an object is moved along the optical axis of the camera, the first-order effect on the image is just a change in size. The second-order effect is that a few small portions of the image may appear or disappear when the object is moved in this direction. This second-order effect diminishes as the distance of the object from the camera increases.

The radius of gyration r of a planar pattern is defined as follows:

$$r = (\mu_{20} + \mu_{02})^{1/2}. \tag{9}$$

The radius of gyration is directly proportional to the size of the image or inversely proportional to the distance of the object along the optical axis. Thus, the product of the radius of gyration of the image and the distance B along the optical axis of the object is a constant:

$$(\mu_{20} + \mu_{02})^{1/2} \cdot B = \text{constant}. \tag{10}$$

Therefore, the radius of gyration r should be used to normalize the moment functions M_2 through M_7 to obtain the desired size invariance. Thus, the following moment functions M_1 through M_7 are invariant with respect to elevation angle θ and the distance B along the optical axis:

$$M_1' = (\mu_{20} + \mu_{02})^{1/2} \cdot B = r \cdot B \tag{11}$$

$$M_2' = M_2/r^4 \tag{12}$$

$$M_3' = M_3/r^6 \tag{13}$$

$$M_4' = M_4/r^6 \tag{14}$$

$$M_5' = M_5/r^{12} \tag{15}$$

$$M_6' = M_6/r^8 \tag{16}$$

$$M_7' = M_7/r^{12}. \tag{17}$$

D. Feature Vector Construction

It has been shown that the moment functions M_1' through M_7' are invariant with elevation angle and distance of an object from the camera. Therefore, these moment features can be used to represent, or characterize, the image for a given *object viewing aspect* [2], which is defined uniquely by a pair of values for the azimuth angle ψ and roll angle ϕ.

The central moments of an image defined earlier in (1) can be computed either from the image boundary or the solid silhouette. Each of the two sets of moments—the set computed from the image boundary and the set from the silhouette—contains some information not carried by the other set. The moments derived from the boundary contain more information about the high-frequency portions of the optical image than those derived from the silhouette. Therefore, the minute details of the optical image, such as the shape of the nose or the tail of an aircraft, are better characterized by the moments computed from the boundary. On the other hand, the gross structural features of the aircraft, such as the fuselage and the wings, can be better characterized by those moments derived from the silhouette; also, these moments are less susceptible to noise. Hence, in order to take advantage of the information content of both the boundary and the silhouette, two sets of seven moment invariant functions (M_1', M_2', M_3', M_4', M_5', M_6', M_7') one set derived from the boundary and the other from the silhouette are computed. The feature vector $\vec{\rho} = (\rho_1, \rho_2, \rho_3, \rho_4, \cdots, \rho_{14})$ can therefore be defined as

$$\rho_i = M_i', \; i = 1,7 \tag{18}$$

where the previously defined invariant functions M_i', $i = 1,7$, are computed using the boundary of the optical image, and

$$\rho_{i+7} = M_i', \; i = 1,7 \tag{19}$$

where the previously defined invariant functions, M_i', $i = 1,7$, are here computed using the silhouette of the optical image.

It should be noted that the components ρ_1 and ρ_8 of the feature vector $\vec{\rho}$ require information about the distance B of the object along the optical axis. For situations where this distance B is not available, these two components should not be used in constructing the feature vector. Hence, the feature vector in those cases will have only twelve components.

IV. Training the Recognizer

In the last section, certain moment features were introduced to provide a parametric characterization in a fourteen-dimensional vector space for the image of an object for any given values of azimuth and roll angles. In order to make use of this feature vector for recognition purposes, it is necessary to construct a training sample set using the perspective projections (optical images) of the given object for various values of azimuth ψ and roll ϕ angles selected from the *significant range*. The significant range of values of azimuth and roll is the one which at least covers all *distinct views* of the object. Two views of an object are called distinct if and only if they produce images which do not merely differ from each other by a rotation, reflection, or a combination of rotation and reflection. It is shown in [6] that for three-dimensional objects which possess symmetry about a plane, such as an aircraft, the significant range for azimuth and roll angles is as follows:

$$-\frac{\pi}{2} \le \psi \le \frac{\pi}{2} \tag{20}$$

$$0 \le \phi \le \frac{\pi}{2}. \tag{21}$$

In practice, the significant range of values of azimuth and roll angles is discretized; then, for all the possible combinations of these discretized values of azimuth and roll angles, the images for the given object are obtained, and from each of these images a set of fourteen features is computed. The collection of these feature vectors, computed from the distinct views of the object, forms the training sample set for that object. Such a training sample must be generated for every object in the given class of objects to be recognized.

For the system under consideration, a lengthy experiment was carried out to construct the training sample set. Six different aircraft types were chosen to be included in the recognition class: namely, an F-4 Phantom, a Mirage IIIC, a MIG 21, an F-105, an F-104, and a B-57. The significant ranges of azimuth and roll angles were discretized at intervals of 5° each. This interval was selected so that as much information as possible would be included in the training set. Intervals smaller than 5° could not accurately be measured with the equipment at hand. Next, for each of the different combinations of these discretized values of the two angles, live images were obtained by holding the aircraft model in the proper orientation with the help of a fixture which has provisions for independent adjustment of each of the three orientation angles. For each of the images obtained, the corresponding vector $\vec{\rho}$ was computed. The complete training sample set, thus constructed, was based on over 3000 live images for the six different types of aircraft. To compensate for scaling effects, all of the training samples were subjected to a linear transformation to obtain zero mean and unit variance for each component when averaged over the entire set. The complete tables for the normalized training sets of the six aircraft can be found in [14].

For each of the images in the training set, 14 different feature components were computed and stored. In order to cut down on the amount of core storage for the training set, a feature ordering technique was employed. A transformation is made from the original feature space to a new space defined by the set of eigenvectors corresponding to the training sample covariance matrix. The new set of feature components obtained by the above transformation can be ordered in terms of their information content [15]. For our problem, a subset of only the five most informative feature components were used to form the training set and later used for classification purposes. The aggregate information content of these five new feature components amounted to 95 percent of the total information available from the original training set [6].

V. Image Classification

In this research, two distinct decision rules were used in classification experiments: a Bayes decision rule and a distance-weighted k-nearest-neighbor rule. The details of these two methods are presented below.

A. Bayes Decision Rule

Let $\omega = (i,\psi,\phi)$ represent a random variable where i is the type of aircraft, ψ is the azimuth angle, and ϕ is the roll angle. Let the space covered by the random variable, ω be denoted by Ω, where

$$\Omega = (i \in 1,2,3, \cdots ,6) \times \left(-\frac{\pi}{2} \leq \psi \leq \frac{\pi}{2}\right) \times \left(0 \leq \phi \leq \frac{\pi}{2}\right). \tag{22}$$

The problem of designing an optimal classifier can be stated as a problem of nonlinear programming: given a measurement $\vec{\rho}\,'$ and some *a priori* knowledge of ω, it is required to find $\omega \in \Omega$ which maximizes the *a posteriori* probability $p(\omega/\vec{\rho}\,')$. By Bayes rule [16],

$$p(\omega/\vec{\rho}\,') = \frac{p(\vec{\rho}\,'/\omega)p(\omega)}{p(\vec{\rho}\,')} \tag{23}$$

where $p(\omega)$ is the *a priori* probability of ω, $p(\vec{\rho}\,'/\omega)$ is the probability of $\vec{\rho}\,'$ given the random variable ω, and $p(\vec{\rho}\,')$ is the probability of occurrence of $\vec{\rho}\,'$ irrespective of ω.

For the experimental system under consideration, the following composite form [6] for $p(\omega)$ was assumed:

$$p(\omega) = P(i)p(\psi/\psi_0)p(\phi/\phi_0) \tag{24}$$

where ψ_0 and ϕ_0 are azimuth and roll estimates obtained from some other sources such as radar, flight plans, etc. For computation of $p(\vec{\rho}\,'/\omega)$, it was assumed that $\vec{\rho}\,'$ is normally distributed with a mean $\vec{\rho}_0$ depending on ω and a covariance matrix Σ_n independent of ω, i.e.,

$$p(\vec{\rho}\,'/\omega) = N(\vec{\rho}_0(\omega), \Sigma_n) \tag{25}$$

where $\vec{\rho}_0(\omega)$ is obtained from the training set, and the noise covariance matrix Σ_n is estimated experimentally by taking several images of a certain view of an aircraft repetitively one after another. The noise associated with an image is chiefly due to three sources—variation in the sensitivity of the vidicon in the television camera, variations in the lighting conditions on the object, and an error inherent to quantization electronics.

The classification can now be achieved by obtaining the value of ω which maximizes the probability $p(\omega/\vec{\rho}\,')$ of (23). To the extent that the assumed models for the underlying probability density functions are correct, Bayes estimates are, by definition, optimal with respect to average loss or risk. However, if such models are erroneous, simpler procedures may give better results. In the next subsection, one such classification procedure is described.

B. Distance-Weighted k-Nearest Neighbor Rule

Among the simplest and most intuitively appealing classes of nonprobabilistic classification procedures are those that weigh the evidence of nearby sample observations most heavily [16]. It is reasonable to assume that observations which are close together (according to some appropriate metric) will have the same classification. Furthermore, it is also reasonable to say that one might wish to weigh the evidence of a neighbor close to an unclassified observation more heavily than the evidence of another neighbor which is at a greater distance from the unclassified observation. Therefore, it would be desirable to have a weighting function which varies with the distance between the sample and the considered neighbor in a manner such that the value decreases with increasing sample-to-neighbor distance. One such weighting function is employed below in defining a distance-weighted k-nearest-neighbor rule.

When an unknown observation $\vec{\rho}\,'$ is to be classified, the k nearest neighbors of $\vec{\rho}\,'$ (according to a suitable metric) are found among the given samples constituting the training set. Let the corresponding distances of these neighbors from the unknown observation ρ' be given by d_i, $i = 1, \cdots ,k$. Also, let these distances be arranged in an ascending order of magnitude. Then, the weight attributed to the jth nearest neighbor is defined as follows [17]:

$$w_j = \frac{d_k - d_j}{d_k - d_1}, \qquad d_k \neq d_1 \tag{26}$$

$$= 1, \qquad d_k = d_1. \tag{27}$$

Having computed the weights w_j, the distance-weighted k-nearest-neighbor rule then assigns the unknown observation $\vec{\rho}\,'$ to that class for which the weights of representatives among the k-nearest neighbors sum to the greatest value. The weight function defined above assigns weights ranging from one to zero and is of a form which permits a few very close neighbors to overcome the effects of a large number of more distant neighbors. This tends to make the choice of k less critical than in an unweighted rule. Moreover, the use of such a distance-weighted k-nearest-neighbor rule with training sample sets of small or moderate size will yield smaller probabilities of error, at least for certain classes of distributions. The admissibility of such a rule together with certain other properties is treated further by Dudani [6], [17].

For our aircraft identification problem, a plot was obtained for the probability of error against the number of nearest neighbors k. It was seen from the plot that the probability of error did not decrease appreciably for values of k greater than 10. Thus, the value of k was chosen to be 10 for the classifier based on this rule.

VI. Estimation of Position and Orientation

In the last section, two different classification procedures were described. The estimation of ω by Bayes rule provides the values of azimuth and roll angles along with the type of aircraft identified. Use of the distance-weighted k-nearest-neighbor rule gives the type of aircraft identified. The corresponding values of azimuth and roll angles for the given image observation can be taken as the values associated with that of the nearest neighbor of the identified type found from the training set.

The inclination θ_m of the major axis of the ellipse fitted to the given image of an object is found by using the following relation [11]:

$$\tan 2\theta_m = -2\mu_{11}/(\mu_{20} - \mu_{02}). \tag{28}$$

The estimation for the elevation angle θ defined earlier is then given by [5], [6]

$$\theta = \theta_m + \theta_e(\omega) \tag{29}$$

where $\theta_e(\omega)$ is called the *elevation bias error,* and is a function of the variable ω. In the experiment reported in this paper the values for $\theta_e(\omega)$ were experimentally determined and stored at the time of training set construction.

The estimate for the distance B of the object along the optical axis is based on the radius of gyration r of its image. The feature component ρ_1 was defined earlier as the product of r based on boundary and the distance B. Therefore, the estimate for B is found by

$$B = \frac{\rho_1(\omega)}{r} \tag{30}$$

where $\rho_1(\omega)$ can be obtained from the training set.

Let A and C represent the Euclidian distances from the optical axis Y to the center of gravity of the aircraft along the X and Z axes, respectively. Then A and C are given by the following relations [6]:

$$A = \bar{u}(B + f)/f \tag{31}$$

$$C = \bar{v}(B + f)/f \tag{32}$$

where $(\bar{u}, \bar{v})$ is the centroid of the image based on the boundary, and f is the focal length of the television camera used.

VII. Experimental Results

Having settled on a Bayes rule and the particular distance-weighted k-nearest-neighbor rule given by (26) with k set to 10, a test sample of 132 new images consisting of 22 images of each of the 6 classes in the recognition group was prepared. These images were obtained at random viewing aspects and were stored in digital form for subsequent processing. In an attempt to provide a more meaningful evaluation of the classification techniques used, four human observers were asked to view the corresponding binary images and to decide upon a classification for each. The observers were given as much time as they liked to view each image and were also provided with all six aircraft models if they wished. The same images were also classified automatically by the two selected decision rules. The results are summarized in Table I. The detailed results may be found in [6]. Each computer decision required 30 s, while human decisions were generally made in less time, about 10–15 s. As can be seen, the computer outperformed all four observers to an impressive degree. This was an unexpected result since it had been thought that, in this problem, human performance represented an implicit upper bound on the classification accuracy attainable in an automatic system. The significance of these results is increased if it is taken into account that one might reasonably expect the gap between the classifier performance and that of humans to increase for instances where the recognition process is to be continuously performed over an extended period.

TABLE I
Results of the Six Class Aircraft Identification Experiment with 132 Test Images

Observer	Number of Misclassifications						
	Phantom	Mirage	MIG	F105	F104	B57	Total
Bayes Rule	2	0	2	2	2	1	9
Distance-Weighted Rule	1	0	0	2	3	0	6
Technical, A	2	1	3	2	2	0	10
Technical, B	1	1	5	2	2	1	12
Technical, C	3	2	4	0	3	2	14
Non-Technical, D	5	2	10	2	7	2	28

The errors in estimation of values of the three orientation angles, namely, azimuth, roll, and elevation, on an average were on the order of 5 to 10 degrees. The distance of the object from the camera along the optical axis was estimated with an average error of only about 10 percent of its value. These results, therefore, show that the accuracy achieved by the recognition system developed in this research is sufficient for many of the applications that it may be used for.

VIII. Conclusions

In this investigation, a recognition class consisting of only six aircraft types was used. It was difficult to arrive at any meaningful results regarding the relationship of recognition accuracy to the number of aircraft in the given class because of the fact that similarity or dissimilarity in shapes of aircraft under consideration greatly affect the recognition accuracy. However, for the aircraft used in the recognition class, the accuracy of correct classification did not increase significantly when lowering the number of aircraft in the recognition class was lowered to three. Hence, it is reasonable to hope that an increase in the number of types to be identified beyond the present six classes might be accomplished without a serious degradation in recognition accuracy. Further research will be needed, however, before specific limits can be established on the number of aircraft types which can be considered simultaneously.

The recognition time of 30 s per image associated with the system is not felt to represent any sort of limit on system speed since all programs were written in Fortran and were executed without the benefit of floating point hardware. Simulation studies on a PDP-10 computer have shown that processing time can be reduced to one second or less by the addition of such hardware. Straightforward improvements to the interface system combined with the use of floating point hardware can be expected to reduce identification time to the order of one second. With special purpose image processing hardware and parallel table search, it should eventually be possible to obtain identification in one television frame time, thereby realizing true real-time operation.

It is generally difficult to make a comparison of different recognition systems, even for the same problem, since different test sets are used for evaluating performance. If a set of data for a particular problem is processed by several recognition systems, however, some comparisons become possible. The test set presented in this research can be obtained in computer-compatible form by writing the authors. It is hoped that the availability of these data along with publication of the results included in this paper will encourage others to undertake research in this important area of automatic data processing.

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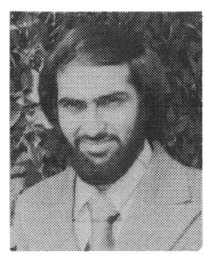
References

[1] R. L. Cosgriff, "Identification of shape," Ohio State Univ. Res. Foundation, Columbus, OH, Rep. 820-11, ASTIA AD-254 792, Dec. 1960.
[2] J. Sklansky and G. A. Davidson, "Recognizing three-dimensional objects by their silhouettes," *J. Soc. Photo-Opt. Instrum. Eng.*, vol. 10, pp. 10–17, Oct. 1971.
[3] C. W. Richard and H. Hemami, "Identification of three-dimensional objects using Fourier descriptors of the boundary curve," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-4, pp. 371–378, July 1974.
[4] J. G. Advani, "Computer recognition of three-dimensional objects from optical images," Ph.D. dissertation, Ohio State Univ., Columbus, OH, Aug. 1971.
[5] S. A. Dudani, "Moment methods for the identification of three-dimensional objects from optical images," M.S. thesis, Ohio State Univ., Columbus, OH, Aug. 1971.
[6] ——, "An experimental study of moment methods for automatic identification of three-dimensional objects from television images," Ph.D. dissertation, Ohio State Univ., Columbus, OH, Aug. 1973.
[7] A. Pugh, W. B. Heginbotham, and P. W. Kitchin, "Visual feedback applied to programmable assembly machines," in *Proc. 2nd Int. Symp. Industrial Robots,* May 1972, pp. 77–88.
[8] R. B. McGhee, "Automatic recognition of complex three-dimensional objects from optical images," in *Proc. U.S.–Japan Seminar on Learning Control and Intelligent Control,* Oct. 1973.
[9] A. Rosenfeld, *Picture Processing by Computer.* New York: Academic, 1969.
[10] M. D. Levine, "Feature extraction: A survey," in *Proc. IEEE,* vol. 50, pp. 1391–1407, Aug. 1969.
[11] M. K. Hu, "Visual pattern recognition by moment invariants," *IRE Trans. Inform. Theory,* vol. IT-8, pp. 179–187, Feb. 1962.
[12] F. W. Smith, and M. H. Wright, "Automatic ship photo interpretation by the method of moments," *IEEE Trans. Comput.*, vol. C-20, pp. 1089–1094, Sept. 1971.
[13] R. L. Pio, "Euler angle transformations," *IEEE Trans. Automat. Contr.*, vol. AC-11, pp. 707–715, Oct. 1966.
[14] S. A. Dudani, *User's Manual and Tables of Moment Invariants for an On-Line Automatic Aircraft Identification System,* Commun. Contr. Syst. Lab., Ohio State Univ., Columbus, OH, Tech. Note 15, Jan. 1974.
[15] S. Watanabe, "A method of self-feature information compression in pattern recognition," in *Proc. 1966 Bionics Meeting,* 1968, pp. 697–707.
[16] R. O. Duda and P. E. Hart, *Pattern Classification and Scene Analysis.* New York: Wiley, 1973.
[17] S. A. Dudani, "The distance-weighted k-nearest-neighbor rule," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-6, pp. 325–327, Apr. 1976.



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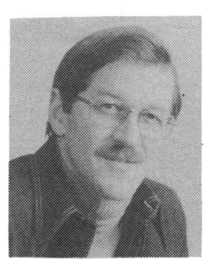

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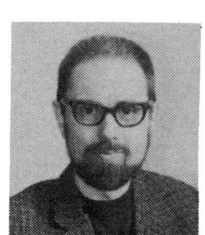

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Nonparametric Estimation of the Bayes Error of Feature Extractors Using Ordered Nearest Neighbor Sets

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***Abstract*—Since the Bayes classifier is the optimum classifier in the sense of having minimum probability of misclassification among all the classifiers using the same set of pattern features, the error rate of the Bayes classifier using the set of features provided by a feature extractor, called the *Bayes error* of the feature extractor, is the smallest possible for the feature extractor. Consequently, the Bayes error can be used to evaluate the effectiveness of the feature extractors in a pattern recognition system. In this paper, a nonparametric technique for estimating the Bayes error for any two-category feature extractor is presented. This technique uses the nearest neighbor sample sets and is based on an infinite series expansion of the general form of the Bayes error. It is shown that this technique is better than the existing methods, and the estimates obtained by this technique are more meaningful in evaluating the quality of feature extractors. Computer simulation as well as application to electrocardiogram analysis are used to demonstrate this technique.**

***Index Terms*—Bayes error, comparison, feature extractors, nearest neighbor sets, nonparametric estimation, pattern recognition systems, series expansion.**

I. INTRODUCTION

ELECTROCARDIOGRAM analysis, X-ray film analysis, speech recognition, weather prediction, and reading of printed and written material are examples of pattern recognition problems. Pattern recognition may be considered as a two-stage process, extracting features from the input pattern and then classifying the input pattern into one of the possible classes based on the extracted features. A pattern recognition system may be considered as consisting of two parts, feature extractor and classifier, for implementing the two-stage process. It is desirable to have some evaluation of the quality of the information provided by the feature extractor before designing the classifier. It is well known that Bayes classifier is the optimum classifier in the sense of having minimum probability of error in classifying input patterns among all the classifiers for the same feature extractor. Hence, the error rate of the Bayes classifier is the smallest possible for the given feature extractor, and is called the *Bayes error* of the feature extractor [1]. If the tolerable error rate of a pattern recognition system is smaller than the Bayes error of the feature extractor, it is necessary to improve the feature extractor. Otherwise, no classifier can be found to satisfy the tolerable error rate requirement. Consequently, the Bayes error of a feature extractor plays an important role in the design of a pattern recognition system.

If the *a priori* probabilities and probability density functions of all pattern classes are known or can easily be estimated, the Bayes error can simply be determined by means of Bayes theorem. However, in most cases it is necessary to estimate the Bayes error directly from the sample patterns without the knowledge of the *a priori* probabilities and probability density functions of pattern classes; that is, nonparametric estimation of Bayes error. To accomplish this, Cover [2], Hart [3], and Whitney [4], [5] independently suggested the use of the error rate of the K-nearest neighbor classifier as an estimate of the Bayes error for its feature extractor. Later, Fukunaga and Kessell [6] investigated another method based on K-nearest neighbor sets. In this paper, we will present a nonparametric technique for estimating the Bayes error for any two-category feature extractor. Our approach is based on an infinite series expression of the general form of the Bayes error. It will be shown that our technique is better than the existing methods, and that our estimates are more

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