pbj-bem114-hw4

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1 Imports

[145]:
```python
import pandas as pd
import statsmodels.api as sm
import numpy as np
import matplotlib.pyplot as plt
```

2 Problem 1

2.1 Part (a)

[ ]:
```python
file_parsing = {
    "F-F_Momentum_Factor_CSV.zip": {
        "name": "mom_monthly",
        "header": 11,
        "nrows": 1176,
        "names": ["Date", "Mom"]
    },
    "F-F_Momentum_Factor_daily_CSV.zip": {
        "name": "mom_daily",
        "header": 11,
        "nrows": 25800,
        "names": ["Date", "Mom"]
    },
    "F-F_Research_Data_5_Factors_2x3_CSV.zip": {
        "name": "ff5_monthly",
        "header": 2,
        "nrows": 738,
        "names": ["Date", "Mkt-RF", "SMB", "HML", "RMW", "CMA", "RF"]
    },
    "F-F_Research_Data_5_Factors_2x3_daily_CSV.zip": {
        "name": "ff5_daily",
        "header": 2,
        "nrows": None,
```

```python
        "names": ["Date", "Mkt-RF", "SMB", "HML", "RMW", "CMA", "RF"]
    }
}

datasets = {}

for zip_file, params in file_parsing.items():
    df = pd.read_csv(
        zip_file,
        header=params["header"],
        names=params["names"],
        nrows=params["nrows"]
    )

    # Convert Date column to datetime
    if df["Date"].iloc[0] < 999999:  # YYYYMM format
        df["Date"] = pd.to_datetime(df["Date"].astype(str),
                                    format="%Y%m").dt.to_period('M')
    else:  # For daily data in YYYYMMDD format
        df["Date"] = pd.to_datetime(df["Date"].astype(str), format="%Y%m%d")

    datasets[params["name"]] = df
```

[96]:
```python
datasets["mom_monthly"]
```

[96]:
```
         Date   Mom
0     1927-01  0.36
1     1927-02 -2.14
2     1927-03  3.61
3     1927-04  4.30
4     1927-05  3.00
…         …     …
1171  2024-08  4.79
1172  2024-09 -0.60
1173  2024-10  2.87
1174  2024-11  0.90
1175  2024-12  0.05

[1176 rows x 2 columns]
```

[97]:
```python
# Build combined daily & monthly factor tables
# daily: outer-join on Date so any missing factor shows up as NaN
df_daily = pd.merge(
    datasets["ff5_daily"],
    datasets["mom_daily"][["Date", "Mom"]],
    on="Date",
    how="outer"
```

```python
).sort_values("Date").set_index("Date")

# monthly: same as above
df_monthly = pd.merge(
    datasets["ff5_monthly"],
    datasets["mom_monthly"][["Date", "Mom"]],
    on="Date",
    how="outer"
).sort_values("Date").set_index("Date")
```

[98]:
```python
df_monthly
```

[98]:
```
         Mkt-RF   SMB   HML   RMW   CMA    RF   Mom
Date
1927-01     NaN   NaN   NaN   NaN   NaN   NaN  0.36
1927-02     NaN   NaN   NaN   NaN   NaN   NaN -2.14
1927-03     NaN   NaN   NaN   NaN   NaN   NaN  3.61
1927-04     NaN   NaN   NaN   NaN   NaN   NaN  4.30
1927-05     NaN   NaN   NaN   NaN   NaN   NaN  3.00
…           …     …     …     …     …     …     …
2024-08    1.61 -3.65 -1.13  0.85  0.86  0.48  4.79
2024-09    1.74 -1.02 -2.59  0.04 -0.26  0.40 -0.60
2024-10   -0.97 -0.88  0.89 -1.38  1.03  0.39  2.87
2024-11    6.51  4.78 -0.05 -2.62 -2.17  0.40  0.90
2024-12   -3.17 -3.87 -2.95  1.82 -1.10  0.37  0.05

[1176 rows x 7 columns]
```

[99]:
```python
df_daily
```

[99]:
```
            Mkt-RF   SMB   HML   RMW   CMA     RF   Mom
Date
1926-11-03     NaN   NaN   NaN   NaN   NaN    NaN  0.56
1926-11-04     NaN   NaN   NaN   NaN   NaN    NaN -0.50
1926-11-05     NaN   NaN   NaN   NaN   NaN    NaN  1.17
1926-11-06     NaN   NaN   NaN   NaN   NaN    NaN -0.03
1926-11-08     NaN   NaN   NaN   NaN   NaN    NaN -0.01
…              …     …     …     …     …      …     …
2024-12-24    1.11 -0.12 -0.05 -0.13 -0.37  0.017  0.67
2024-12-26    0.02  1.09 -0.19 -0.44  0.35  0.017  0.01
2024-12-27   -1.17 -0.44  0.56  0.41  0.03  0.017 -0.88
2024-12-30   -1.09  0.24  0.74  0.55  0.14  0.017  0.06
2024-12-31   -0.46  0.31  0.71  0.33  0.00  0.017 -1.07

[25800 rows x 7 columns]
```

2.2 Part (b)

[ ]:
```
# compute rolling variance (window=22 days, require full 22 observations)
df_daily_var22 = df_daily.rolling(window=22, min_periods=22).var()

# suffix column names
df_daily_var22.columns = [col + "_var22" for col in df_daily_var22.columns]

# concatenate
df_daily = pd.concat([df_daily, df_daily_var22], axis=1)
```

[101]:
```
df_daily
```

[101]:
```
            Mkt-RF   SMB   HML   RMW   CMA     RF   Mom  Mkt-RF_var22  \
Date
1926-11-03     NaN   NaN   NaN   NaN   NaN    NaN  0.56           NaN
1926-11-04     NaN   NaN   NaN   NaN   NaN    NaN -0.50           NaN
1926-11-05     NaN   NaN   NaN   NaN   NaN    NaN  1.17           NaN
1926-11-06     NaN   NaN   NaN   NaN   NaN    NaN -0.03           NaN
1926-11-08     NaN   NaN   NaN   NaN   NaN    NaN -0.01           NaN
…              …     …     …     …     …      …     …             …
2024-12-24    1.11 -0.12 -0.05 -0.13 -0.37  0.017  0.67      0.807481
2024-12-26    0.02  1.09 -0.19 -0.44  0.35  0.017  0.01      0.795482
2024-12-27   -1.17 -0.44  0.56  0.41  0.03  0.017 -0.88      0.851588
2024-12-30   -1.09  0.24  0.74  0.55  0.14  0.017  0.06      0.888292
2024-12-31   -0.46  0.31  0.71  0.33  0.00  0.017 -1.07      0.890354

            SMB_var22  HML_var22  RMW_var22  CMA_var22      RF_var22  \
Date
1926-11-03        NaN        NaN        NaN        NaN           NaN
1926-11-04        NaN        NaN        NaN        NaN           NaN
1926-11-05        NaN        NaN        NaN        NaN           NaN
1926-11-06        NaN        NaN        NaN        NaN           NaN
1926-11-08        NaN        NaN        NaN        NaN           NaN
…                 …          …          …          …             …
2024-12-24   0.491921   0.279187   0.494185   0.208387  1.655844e-06
2024-12-26   0.452180   0.258875   0.479230   0.202073  1.402597e-06
2024-12-27   0.349691   0.277311   0.486100   0.196355  1.110390e-06
2024-12-30   0.324535   0.308790   0.488469   0.194205  7.792208e-07
2024-12-31   0.316454   0.326266   0.470142   0.192986  4.090909e-07

            Mom_var22
Date
1926-11-03        NaN
1926-11-04        NaN
1926-11-05        NaN
1926-11-06        NaN
```

```
1926-11-08        NaN
…               …
2024-12-24   0.965847
2024-12-26   0.956825
2024-12-27   0.909293
2024-12-30   0.841711
2024-12-31   0.865381

[25800 rows x 14 columns]
```

2.3 Part (c)

```
[ ]: # Collapse df_daily var columns to month-end
df_monthly_var22 = (
    df_daily
      .filter(like="_var22") # pick out the var22 cols
      .resample("ME")        # calendar month-end
      .last()                # last trading-day var per month
      .to_period("M")        # keep month and year only
)

# Merge the two tables on month
df_monthly_all = df_monthly.join(df_monthly_var22)

# Get factors (excluding RF)
factors = ["Mkt-RF", "SMB", "HML", "RMW", "CMA", "Mom"]

# For each factor, regress var_t on lagged var (plus constant)
records = []
for factor in factors:
    factor_var = factor + "_var22"
    series = df_monthly_all[factor_var].dropna()
    # pull t and t-1
    df_reg = pd.DataFrame({
        "var_t": series,
        "var_t_1": series.shift(1)
    }).dropna() # drop NaN values

    X = sm.add_constant(df_reg["var_t_1"])
    model = sm.OLS(df_reg["var_t"], X).fit()

    records.append({
        "factor": factor_var.replace("_var22",""),
        "coef_lag": model.params["var_t_1"],
        "pvalue_lag": model.pvalues["var_t_1"],
        "signif_5pct": model.pvalues["var_t_1"] < 0.05
    })
```

```
summary_df = pd.DataFrame(records).set_index("factor")
```

[103]:
```
summary_df
```

[103]:
```
        coef_lag     pvalue_lag  signif_5pct
factor
Mkt-RF  0.455271   5.353625e-39         True
SMB     0.368813   3.564584e-25         True
HML     0.746163  4.775988e-132         True
RMW     0.722323  1.237260e-119         True
CMA     0.688149  1.548166e-104         True
Mom     0.635900  3.773912e-134         True
```

Yes. For all of the risky factors (Mkt–RF, SMB, HML, RMW, CMA and Mom) the lag-variance coefficient is positive and highly significant (p-values << 0.05), so current variance does predict next month's variance.

2.4 Part (d)

[ ]:
```
# For each factor, regress next month's return on this month's var22
records_partd = []
for factor in factors:
    factor_var = factor + "_var22"
    df_reg = pd.DataFrame({
        "ret_t1": df_monthly_all[factor],              # future return
        "var_t":  df_monthly_all[factor_var].shift(1) # current variance
    }).dropna()                                        # drop any NaNs

    X = sm.add_constant(df_reg["var_t"])
    model = sm.OLS(df_reg["ret_t1"], X).fit()

    records_partd.append({
        "factor": factor,
        "coef_var": model.params["var_t"],
        "pvalue_var": model.pvalues["var_t"],
        "signif_5pct": model.pvalues["var_t"] < 0.05
    })

summary_df_partd = pd.DataFrame(records_partd).set_index("factor")
```

[105]:
```
summary_df_partd
```

[105]:
```
        coef_var    pvalue_var  signif_5pct
factor
Mkt-RF -0.007016  9.262857e-01        False
SMB     0.105143  6.451133e-01        False
```

```
HML    -0.105553  5.291985e-01        False
RMW     0.991033  2.783912e-04         True
CMA     1.403083  6.202527e-05         True
Mom    -0.621533  1.585757e-07         True
```

Variance carries no significant predictive power for Mkt–RF, SMB, or HML (all p-values > 0.5). However, higher RMW and CMA volatility predicts greater next-month returns ($p < 0.001$), while higher momentum volatility predicts lower future momentum returns ($p < 0.001$).

2.5 Part (e)

[106]:
```python
# compute c_i = long-run (sample) mean variance for each factor
c = df_monthly_all.filter(f + "_var22" for f in factors).mean()

# build weight series w_{i,t} = c_i / sigma_{i,t}^2  (here sigma^2 = our var22)
weights = c / df_monthly_all.filter(f + "_var22" for f in factors)

# shift weights by one month: w_{i,t-1}
w_lag = weights.shift(1).dropna() # drop NaN weights

# rename weight columns so they match your factor names
w_lag.columns = [col.replace("_var22", "") for col in w_lag.columns]

# compute volatility-managed returns: w_{i,t-1} * R_{i,t}
# .multiply() aligns on index and columns
df_vm = w_lag.multiply(
    df_monthly_all.filter(items=[f for f in factors])).dropna()

# rename columns to indicate vol-managed
df_vm.columns = [f + "_vm" for f in df_vm.columns]
df_vm
```

[106]:

Date	Mkt-RF_vm	SMB_vm	HML_vm	RMW_vm	CMA_vm	Mom_vm
1963-08	23.494188	-11.360359	15.999648	2.563364	-1.568581	7.551417
1963-09	-18.719853	-4.594272	1.107102	-8.426587	1.778621	2.294551
1963-10	17.007489	-7.860621	-0.960593	15.372185	-10.152892	24.727927
1963-11	-5.142293	-3.377773	5.060059	-1.304268	3.706516	-4.515657
1963-12	1.269919	-3.006226	-0.084214	0.050958	-0.121279	1.575423
…	…	…	…	…	…	…
2024-08	1.994843	-0.637998	-0.378571	0.493635	0.369540	3.472945
2024-09	1.073044	-0.488459	-1.824588	0.023099	-0.112735	-0.761483
2024-10	-1.275691	-0.540291	0.653250	-1.383892	0.728873	3.599848
2024-11	14.223740	3.126467	-0.037845	-2.710448	-2.724036	1.839396
2024-12	-3.778558	-1.310505	-0.657790	1.198532	-0.704987	0.048999

[737 rows x 6 columns]

2.6 Part (f)

[ ]:
```python
# Define the three models by their predictor lists
models = {
    "CAPM": ["Mkt-RF"],
    "FF3": ["Mkt-RF", "SMB", "HML"],
    "FF5+Mom": ["Mkt-RF", "SMB", "HML", "RMW", "CMA", "Mom"]
}

records_partf = []
for col in df_vm.columns: # i.e. "Mkt-RF_vm", "SMB_vm", ...
    factor = col.replace("_vm", "")
    for name, preds in models.items():
        # build regression table: vol-managed return and its predictors
        df_reg = pd.concat(
            [df_vm[col]] +
            [df_monthly[p] for p in preds],
            axis=1
        ).dropna()

        X = sm.add_constant(df_reg[preds])
        y = df_reg[col]
        res = sm.OLS(y, X).fit()

        records_partf.append({
            "factor": factor,
            "model": name,
            "alpha": res.params["const"],
            "pvalue_alpha": res.pvalues["const"],
            "signif_5pct": res.pvalues["const"] < 0.05
        })

alpha_summary = pd.DataFrame(records_partf).set_index(["factor", "model"])
alpha_summary
```

[ ]:

factor	model	alpha	pvalue_alpha	signif_5pct
Mkt-RF	CAPM	0.489159	1.085271e-01	False
	FF3	0.462963	1.320307e-01	False
	FF5+Mom	0.075567	8.102905e-01	False
SMB	CAPM	0.131401	6.422386e-01	False
	FF3	-0.023801	9.100764e-01	False
	FF5+Mom	-0.199699	3.589766e-01	False
HML	CAPM	1.123394	1.489803e-03	True
	FF3	0.415665	1.403872e-01	False
	FF5+Mom	0.780529	5.560307e-03	True
RMW	CAPM	0.874257	7.236082e-06	True

```
      FF3      1.025432  5.825360e-08         True
      FF5+Mom  0.589008  2.220986e-04         True
CMA   CAPM     0.525881  4.789172e-04         True
      FF3      0.295945  2.710813e-02         True
      FF5+Mom  0.206165  6.610711e-02        False
Mom   CAPM     3.924924  3.347800e-14         True
      FF3      4.104515  2.875791e-15         True
      FF5+Mom  2.710298  1.717412e-09         True
```

Under all three models, the RMW and Mom volatility-managed strategies earn positive and highly significant alpha ($p < 0.001$), while CMA earns significant alpha under CAPM and FF3 (but just misses significance in FF5+Mom, p~0.066). HML also produces significant alpha in the CAPM and FF5+Mom regressions (but not in FF3). By contrast, Mkt-RF and SMB show no significant alpha in any specification.

Even though variance positively predicts future variance, scaling each factor by lagged volatility can still harvest time-varying risk premia. When variance is low, the strategy ramps up exposure to that factor's expected premium; when variance is high, it scales back. This volatility timing boosts risk-adjusted returns—in effect buying low when conditional variance (and hence risk) is low and selling high when risk is elevated, yielding net alpha even in factors whose variance itself is persistent.

3 Problem 2

3.1 Part (a)

[ ]:
```python
# build var & return panels
var_df = df_monthly_var22[[f + "_var22" for f in factors]]

# inverse-variance weights each month
inv_var = 1.0 / var_df # 1/sigma^2
w = inv_var.div(inv_var.sum(axis=1), axis=0).dropna() # normalize: sum of w = 1
w.columns = factors # drop the "_var22" suffix

# apply the hint from part (e): lag the weights by one month
w_lag = w.shift(1).dropna()
w_lag
```

[ ]:

	Mkt-RF	SMB	HML	RMW	CMA	Mom
Date						
1963-08	0.026554	0.283737	0.156820	0.268094	0.187691	0.077103
1963-09	0.055850	0.144301	0.122815	0.365272	0.209959	0.101801
1963-10	0.046921	0.137634	0.206431	0.251784	0.257674	0.099556
1963-11	0.088687	0.196203	0.130505	0.246332	0.177283	0.160990
1963-12	0.015738	0.113206	0.294017	0.253118	0.287177	0.036744
…	…	…	…	…	…	…
2024-08	0.111213	0.054706	0.092582	0.342501	0.281880	0.117118

```
2024-09  0.045005  0.121859  0.158287  0.276904  0.231263  0.166682
2024-10  0.066644  0.108484  0.114515  0.333895  0.262076  0.114387
2024-11  0.082618  0.086239  0.088119  0.257030  0.346915  0.139079
2024-12  0.086164  0.085353  0.049626  0.312779  0.338590  0.127487

[737 rows x 6 columns]
```

[150]:
```python
# build the matrix of w_{i,t-1} * R_{i,t} and drop any all-NA rows, then sum
# across factor-columns to get the single vol-managed portfolio return
vm_port = w_lag.multiply(df_monthly_all[factors]).dropna().sum(axis=1)
vm_port.name = "vm_ret"
vm_port
```

[150]:
```
Date
1963-08    0.298613
1963-09   -0.325869
1963-10    0.404444
1963-11    0.132693
1963-12   -0.163020
              …
2024-08    0.969295
2024-09   -0.605011
2024-10    0.079261
2024-11   -0.355390
2024-12   -0.546671
Freq: M, Name: vm_ret, Length: 737, dtype: float64
```

3.2 Part (b)

[144]:
```python
records_partb = []
for name, preds in models.items():
    # align VM returns and the predictors, drop any months with missing data
    df_reg = pd.concat(
        [vm_port] +
        [df_monthly[p] for p in preds],
        axis=1
    ).dropna()

    # regress VM returns on the chosen factors
    X = sm.add_constant(df_reg[preds])
    y = df_reg["vm_ret"]
    res = sm.OLS(y, X).fit()

    records_partb.append({
        "model": name,
        "alpha": res.params["const"],
        "pvalue_alpha": res.pvalues["const"],
```

```
        "signif_5pct": res.pvalues["const"] < 0.05
    })

alpha_vm = pd.DataFrame(records_partb).set_index("model")
alpha_vm
```

[144]:

model	alpha	pvalue_alpha	signif_5pct
CAPM	0.328980	8.750218e-17	True
FF3	0.249655	1.290598e-15	True
FF5+Mom	0.075478	5.743551e-04	True

The fully-aggregated VM portfolio delivers a positive, highly significant alpha in every specification: about 0.33% under CAPM (p~10^-16), 0.25% under FF3 (p~10^-15), and 0.08% under FF5+Mom (p~0.0005). Even after controlling for all five Fama-French factors plus momentum, the multi-factor strategy still earns a statistically meaningful excess return, outperforming each single-factor VM tilt.

3.3 Part (c)

[147]:

```
df_monthly_all
```

[147]:

Date	Mkt-RF	SMB	HML	RMW	CMA	RF	Mom	Mkt-RF_var22	SMB_var22	\
1927-01	NaN	NaN	NaN	NaN	NaN	NaN	0.36	NaN	NaN	
1927-02	NaN	NaN	NaN	NaN	NaN	NaN	-2.14	NaN	NaN	
1927-03	NaN	NaN	NaN	NaN	NaN	NaN	3.61	NaN	NaN	
1927-04	NaN	NaN	NaN	NaN	NaN	NaN	4.30	NaN	NaN	
1927-05	NaN	NaN	NaN	NaN	NaN	NaN	3.00	NaN	NaN	
…	…	…	…	…	…	…	…	…	…	
2024-08	1.61	-3.65	-1.13	0.85	0.86	0.48	4.79	1.681124	0.620879	
2024-09	1.74	-1.02	-2.59	0.04	-0.26	0.40	-0.60	0.788305	0.484273	
2024-10	-0.97	-0.88	0.89	-1.38	1.03	0.39	2.87	0.474499	0.454578	
2024-11	6.51	4.78	-0.05	-2.62	-2.17	0.40	0.90	0.869764	0.878025	
2024-12	-3.17	-3.87	-2.95	1.82	-1.10	0.37	0.05	0.890354	0.316454	

Date	HML_var22	RMW_var22	CMA_var22	RF_var22	Mom_var22
1927-01	NaN	NaN	NaN	NaN	0.153899
1927-02	NaN	NaN	NaN	NaN	0.126329
1927-03	NaN	NaN	NaN	NaN	0.205204
1927-04	NaN	NaN	NaN	NaN	0.211921
1927-05	NaN	NaN	NaN	NaN	0.239990
…	…	…	…	…	…
2024-08	0.477990	0.273233	0.327158	0.000000e+00	0.453913
2024-09	0.458769	0.157342	0.200460	3.463203e-07	0.459282
2024-10	0.444879	0.152521	0.113003	0.000000e+00	0.281870

```
2024-11   1.510145   0.239602   0.221337  7.792208e-07   0.587843
2024-12   0.326266   0.470142   0.192986  4.090909e-07   0.865381

[1176 rows x 14 columns]
```

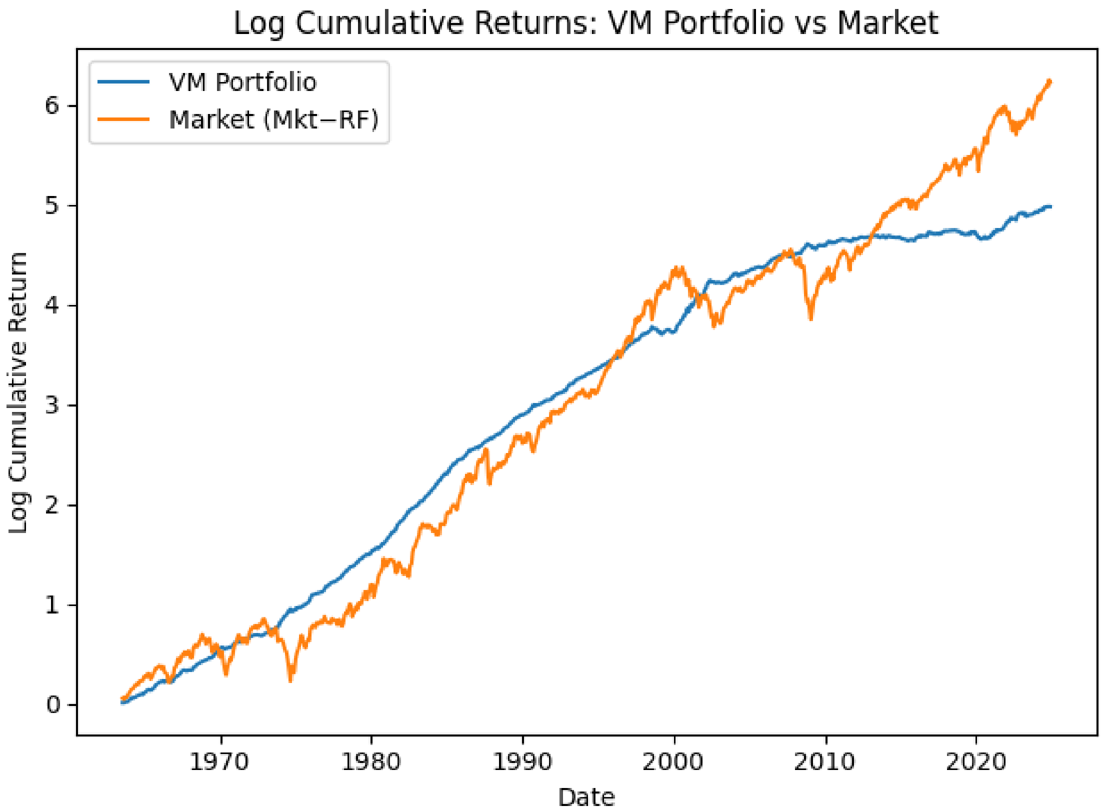
[151]:

```
# limit df_monthly_all to dates in vm_port
returns_aligned = df_monthly_all.loc[vm_port.index, ['RF', 'Mkt-RF']]

# Align and convert the PeriodIndex to Timestamps for matplotlib
dates = vm_port.index.to_timestamp()

# Compute log cumulative returns
cum_log_vm = np.log(((vm_port + returns_aligned['RF']) / 100 + 1.0).cumprod())
cum_log_mkt = np.log(((returns_aligned['Mkt-RF'] + returns_aligned['RF']) / 100␣
 ↪+ 1.0).cumprod())

# Plot
plt.figure()
plt.plot(dates, cum_log_vm, label='VM Portfolio')
plt.plot(dates, cum_log_mkt, label='Market (Mkt-RF)')
plt.legend()
plt.xlabel('Date')
plt.ylabel('Log Cumulative Return')
plt.title('Log Cumulative Returns: VM Portfolio vs Market')
plt.tight_layout()
plt.show()
```



Although the VM portfolio earned a smoother, more steadily rising growth path from the 1960s through the 1990s, it has flattened out relative to the market since 2000. In raw cumulative terms the market has gone on to outpace the volatility-managed strategy over the last two decades.

Yet the VM strategy still produces a significant positive alpha in a factor regression because it systematically tilts into each factor when its conditional variance is low and tilts out when variance is high. That timing harvests extra risk premia on average, even though the compounded dollar-growth of the VM portfolio lagged the simple market buy-and-hold after 2000, so that on a risk-adjusted (alpha) basis it still outperforms.