Cracking KD-Tree: The First Multidimensional Adaptive Indexing (Position Paper)

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Abstract: Workload-aware physical data access structures are crucial to achieve short response time with (exploratory) data analysis tasks as commonly required for Big Data and Data Science applications. Recently proposed techniques such as automatic index advisers (for a priori known static workloads) and query-driven adaptive incremental indexing (for a priori unknown dynamic workloads) form the state-of-the-art to build single-dimensional indexes for single-attribute query predicates. However, similar techniques for more demanding multi-attribute query predicates, which are vital for any data analysis task, have not been proposed, yet. In this paper, we present our on-going work on a new set of workload-adaptive indexing techniques that focus on creating multidimensional indexes. We present our proof-of-concept, the *Cracking KD-Tree*, an adaptive indexing approach that generates a KD-Tree based on multidimensional range query predicates. It works by incrementally creating partial multidimensional indexes as a by-product of query processing. The indexes are produced only on those parts of the data that are accessed, and their creation cost is effectively distributed across a stream of queries. Experimental results show that the *Cracking KD-Tree* is three times faster than creating a full KD-Tree, one order of magnitude faster than executing full scans and two orders of magnitude faster than using uni-dimensional full or adaptive indexes on multiple columns.

1 Introduction

Multidimensional range queries (MDRQ) are queries that select intervals in two or more dimensions of a multidimensional search space (e.g., a query that searches every person between thirty and fifty years old and that earns between 100 and 200 thousand dollars per year). They are very common in OLAP environments [Ho et al., 1997] and have many exploratory applications, like: sensor data [Li et al., 2003], geographic information systems [Alvanaki et al., 2015] and genomics [Li, 2011]. Many benchmarks are composed of at least one MDRQ. For instance, out of the 22 TPC-H [Poess and Stephens, 2004] benchmark queries, 6 are MDRQ.

In order to boost MDRQ, many multidimensional index (MDI) structures have been proposed. For instance: the KD-Tree [Bentley, 1975], R-Tree [Guttman, 1984] and the vector approximation file [Weber et al., 1998]. They index multiple dimensions in a single data structure, avoiding the need of scanning the whole searched dimensional space. However, these structures have a high up-front creation and maintenance cost, in terms of both computation time and storage space. Selecting which MDI to create is one difficult decision that a database administrator (DBA) must take since the trade-off between the speed-up of subsequent queries and creation/maintenance costs must be carefully analyzed [Comer, 1978].

Self-tuning tools [Bruno, 2011] try to alleviate this problem by automatically selecting the indexes through a what-if architecture that tricks the query optimizer into guessing the indexes costs. However, they only consider uni-dimensional indexes in order to prune the subset of indexes to be created. They are also not a good fit for exploratory data analysis where the workload is unpredictable and where there is no idle time to invest in a priori index creation.

Adaptive indexing techniques, such as database cracking [Idreos et al., 2007], attempt to solve the index selection problem for exploratory data analysis workloads by presenting an adaptive partial indexing approach for relational databases. It works by building a partial index as a co-product of query process-

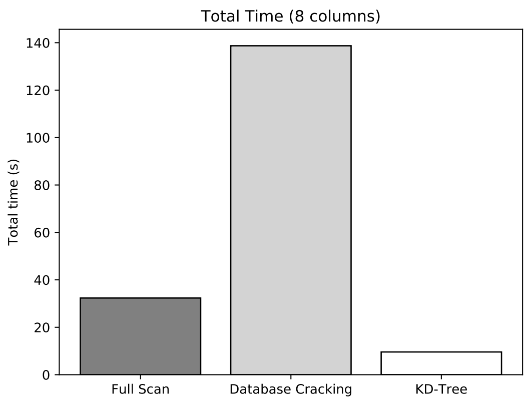


Figure 1: Total Time for multidimensional range queries in 8 Columns.

ing. An index is initiated the first time an attribute is queried, and then continuously refined as subsequent queries are executed. In this way, the cost of creating an index is distributed over a stream of queries. However, database cracking and its variations are designed to generate uni-dimensional indexes, only. Since each attribute is indexed separately from the others, to execute an MDRQ it is necessary to look up the individual indexes for each dimension and intersect their results. The latter can be an expensive task, given that the individual intermediate results can be much larger than the final result, and because index lookups yield the matching tuple IDs (or bit-vectors) in different order.

Figure 1 depicts the total cost for a query stream of 1000 MDRQs, with 20% selectivity per attribute, over an 8-dimensional data set of 10^7 elements. We use eight instances of database cracking, one per each column. The high costs of intersecting the individual intermediate results of different columns turn the unidimensional indexing solutions unfit for querying multidimensional data. Database cracking costs' surpass the cost of a full scan and is one order of magnitude slower than a full multidimensional index.

Covering indexes [Zhang, 2009] or side-ways cracking [Idreos et al., 2009] avoid the intersection of per-column intermediate results by keeping all remaining dimensions aligned with the leading index dimension. However, this comes at the expense of having index-support only for one leading dimension, while the remaining dimensions need to be scanned. In case the selectivity per dimension varies strongly across queries, multiple "wide" indexes would need to be built and maintained to suit all queries optimally.

To address these needs we propose a novel approach for indexing multidimensional data: *Multidimensional Adaptive Indexing (MDAI)*. It works by extending adaptive indexing in order to produce an MDI as a side-effect of query processing. In this paper, we describe our ongoing work on MDAI and present the *Cracking KD-Tree* as the first MDAI that generates a KD-Tree in an adaptive fashion.

Paper Structure. The rest of this paper is structured as follows. Section 2 provides an overview of related work. Then, Section 3, describes multidimensional adaptive indexing. Section 4 presents a brief proof of concept and experimental analysis. Finally, in Section 5, we present our conclusions and discuss future steps.

2 Related Work

In this section, we present the state of the art on automatic physical database design and MDI structures.

2.1 Automatic Physical Tuning

Self-Tuning Tools [Chaudhuri and Narasayya, 1997, Agrawal et al., 2000, Valentin et al., 2000] attempt to solve the index selection problem by automatically recommending a set of indexes to optimize a known workload of the system. They work by selecting a relevant workload, generating a set of indexes that might be beneficial for it and running them through the *What-If* architecture [Chaudhuri and Narasayya, 1998] in order to check if the indexes should be created. However, these systems depend on previous workload knowledge, are only able to create full indexes and only consider uni-dimensional indexes in order to prune the index search space. Therefore, they are not suitable for exploratory systems with MDRQ.

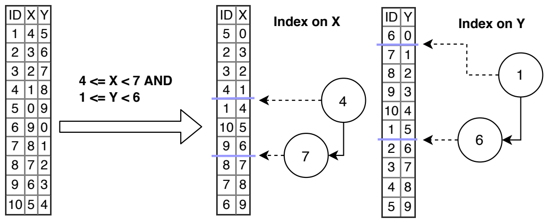


Figure 2: Database cracking with two dimensional range query.

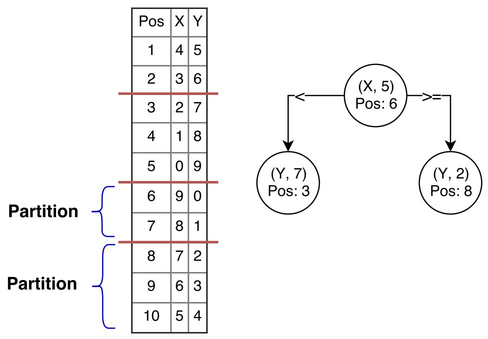
Adaptive Indexing [Schuhknecht et al., 2013] is an alternative to the self-tuning tools. It is especially useful in scenarios where the workload is unpredictable and there is no idle time to invest in index creation. It tackles these problems by creating indexes that are workload dependent in an incremental fashion. Figure 2 depicts an example of database cracking [Idreos et al., 2007] answering a multidimensional range query. The query starts by triggering the creation of the cracker column (i.e., initially a copy of column X) where the tuples are clustered in three pieces reflecting the range predicate on column X. The result is then retrieved as a view on the piece between 4 and 7. Column Y is treated in a similar way, using predicate boundaries 1 and 6. Finally, both views need to be intersected by their id column in order to retrieve the overall result.

Multiple issues with database cracking have been identified and resulted in different research paths, such as poor convergence towards a full index [Graefe and Kuno, 2010, Idreos et al., 2011], inefficient tuple reconstruction [Idreos et al., 2009], unpredictable performance [Halim et al., 2012], inefficient updates [Holanda and de Almeida, 2017]. More recently a generic algorithm for adaptive indexing, the adaptive adaptive indexing [Schuhknecht et al., 2018] was proposed to unify all algorithms in one, where previous data access paths can be mimicked by setting different properties.

To the best of our knowledge, all existing adaptive indexing techniques focus on creating only single-dimensional indexes, and are thus not suited to efficiently and effectively deal with MDRQs.

2.2 Multidimensional Index Structures

MDI structures can be exploited to accelerate MDRQs by avoiding the intersection cost necessary with uni-dimensional indexes and scans. Sprenger et al. [Sprenger et al., 2018] select three different multidimensional index structures considered to be the state-of-the-art for querying multidimensional data. In the following, we present a brief description of these three techniques.



Pos	X	Y
1	4	5
2	3	6
3	2	7
4	1	8
5	0	9
6	9	0
7	8	1
8	7	2
9	6	3
10	5	4

Figure 3: KD-Tree indexing two dimensions.

KD-Tree [Bentley, 1975]. It is a generalization of a binary search tree to multidimensional data. Every node of a KD-Tree holds a key, a discriminator column and, at most, two pointers for its children. The traditional method is using the median of each column to split the data horizontally. Every level of the tree is focused in one specific dimension chosen in a round-robin fashion. Figure 3 depicts a KD-Tree that indexes the dimensions X and Y. The root indexes the dimension X in its median value 5. The next level indexes the next dimension Y in the medians of the new pieces defined by X. 7 when $X < 5$ and 2 otherwise. A query that requests the ranges $X > 5$ and $2 < Y < 10$ would transverse the tree until reaching node $(Y, 2)$ and would scan the column partition from position 8 until its end.

R-Tree [Guttman, 1984]. Similar to B+-Trees, they store data in the leaves. However, they use the inner nodes to hold information in minimum bounding rectangles. Lookup starts at the root and traverses the tree to the leaves intersecting the query with the minimum bounding rectangles to determine which subtrees may have the searched data, and pruning the remaining subtrees.

VA File [Weber et al., 1998]. It partitions the data space into rectangular cells that generate a bit-encoded approximation of points. Dividing a k dimensional space into 2^b rectangular cells. Where b is the number of bits used for approximation.

3 Multidimensional Adaptive Indexing

The previous section gave us the necessary motivation for MDAI. (1) Selecting which MDI to create in an unpredictable environment is a hard task, (2) a priori index creation requires idle time that is not available in exploratory scenarios, (3) although adaptive indexing techniques aim to alleviate problems (1) and (2) they only produce uni-dimensional indexes. Hence, they are not suited to boost MDRQ.

We propose MDAI as a new technique that brings adaptivity to multidimensional indexes. MDAI is designed to produce an MDI while taking advantage of the lightweight adaptive properties from adaptive indexing. We believe that the following modifications from adaptive indexing must be taken: (1) We group all columns that are queried with range predicates together, maintaining the tuple alignment, and copy them to a table, that we call *cracker table*, (2) when swapping elements based on a pivot, we swap the entire row of our cracker table, instead of column elements and (3) we use an MDI structure to keep track of the cracker table pieces.

Figure 4 depicts how MDAI works. Consider an

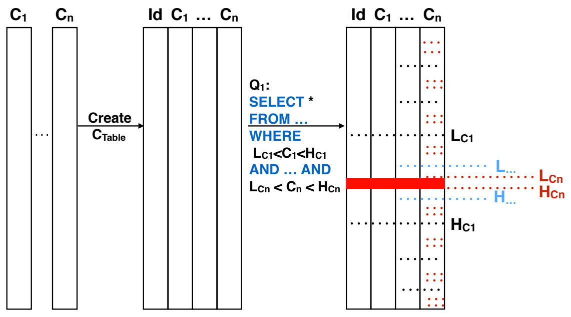


Figure 4: Multidimensional Adaptive Indexing

unindexed table, a query Q_1 that queries ranges from columns C_1 to C_n, where $1 < n \leq d$, d being the maximum number of dimensions of the multidimensional space. Q_1 triggers the first phase of MDAI, by creating a cracker table with all the columns aligned by an explicit id. After the cracker table is created the cracking phase starts. Each range predicate will be executed sequentially from their order in the query (i.e., in this case from C_1 to C_n). At the end of the cracking phase, we simply need to do a lookup in our cracking index and retrieve a view, marked in red, that answers our query.

3.1 Cracking KD-Tree

In this section we describe the *Cracking KD-Tree*, the first MDAI algorithm. It implements all the modifications of an MDAI and produces a KD-Tree as index.

Cracking. The major difference between the Cracking KD-Tree and a regular KD-Tree is how they are built. The regular KD-Tree is constructed based on the medians of each column, whereas the Cracking KD-Tree is constructed based on incoming MDRQs. Given an MDRQ, for example $x_1 \leq X < x_2$ *AND* $y_1 \leq Y < y_2$, we iterate over the (column,key) pairs that represent the individual predicate terms, e.g. $(X, x_1), (X, x_2), (Y, y_1), (Y, y_2)$, and use them to successively grow and refine the index. It is important to notice that one pair can be inserted in multiple locations and that the levels in the Cracking KD-Tree are not dimension specific as opposed to the regular KD-Tree. We also implemented a minimal partition size (i.e., L2 cache size) in order to avoid non-proportionally increased index maintenance overhead and unnecessary random access that would result from too many too small partitions.

Lookup. A lookup in the Cracking KD-Tree consists of comparing the current node key with the given range. For example, given a key x and a range $x_1 \leq X < x_2$, there are three possible outcomes:

1. $x \leq x_1$: the result of the query is on the right of the key.
2. $x_2 \leq x$: the result of the query is on the left of the key.
3. $x \in [x_1, x_2]$:the result of the query is on both sides.

Figure 5 depicts a bi-dimensional example of the Cracking KD-Tree. In this example, Figure 5(a) represents the uncracked cracker table. Query Q_1 starts by triggering the first cracking iteration on column X using its lower predicate boundary 4. After swapping the elements of X around 4 the Cracking KD-Tree root node is inserted. It holds the information regarding the cracked column, the pivot value and the table position, depicted in Figure 5(b). The second cracking iteration is then started and we continue to crack the column X. However, we now crack it using its upper predicate boundary 7, creating a child node to the root, depicted in Figure 2(c). Finally, the last cracking iteration starts, now on column Y. Since the root node and its child do not give us any information about Y we must follow all possible paths, and crack Y on 1 in all existing pieces, resulting in Figure 5(d). After the cracking phase is finished, we perform a lookup operation at the index. Starting from the root of the tree, we can see that its key 4 is equal to the inclusive lower predicate boundary of $4 \leq R.X < 7$. This means that need to descend to the right child of the root node. There, we see that its key 7 is equal to the exclusive upper predicate boundary of $4 \leq R.X < 6$, which leads us to its left child. Finally, the leaf with key 1 is equal to the inclusive lower predicate boundary of $1 \leq R.Y$. Hence, we can return the tuples in positions 6 and 7 of the cracker table, shaded in red in Figure 5(d), as the query answer.

4 Experiments

In this section, we present a brief experimental analysis to demonstrate the strong potential benefits of MDAI.

Setup. We implemented the *Cracking KD-Tree* in a single-threaded stand-alone program written in C++ and compiled with GNU g++ version 7.3.1 using optimization level `-O3`. All experiments were conducted on a machine equipped with 256 GB of main memory and two 2.6 GHz Intel Xeon E5-2650 v2 CPUs, each with 20 MB L3 cache, 8 cores and hyper-threading enabled, running Fedora 26.

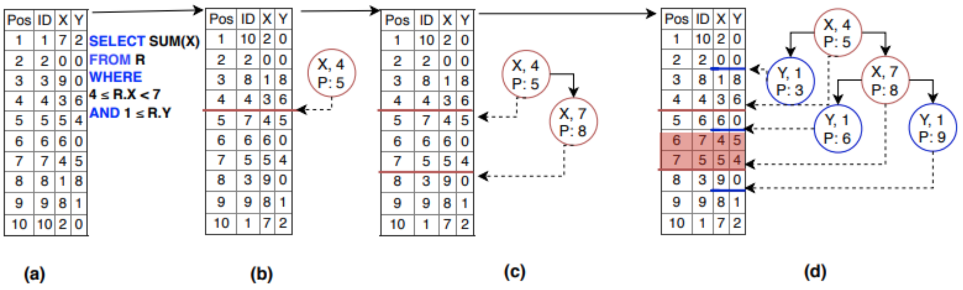


Figure 5: Cracking KD-Tree.

```
SELECT COUNT(R.C1)
FROM R
WHERE  LowC1 < R.C1 < HighC1 AND ...
AND LowCn < R.Cn < HighCn
```

Listing 1: Query form used on experiments

Our data set consists of a table with 8 8-byte integer attributes holding 10^7 tuples. The values per attribute are independently uniformly distributed.

All queries are of the form depicted in Listing 1. Where *n* is the number of dimensions queried. All the queries have selectivity equal to 0.2 per column, one might notice that the total selectivity of the queries in the query stream will vary since the query predicates are selected in a random pattern. We repeat the entire workload 10 times and take the average runtime of each query as the reported time.

We implemented four different algorithms to compare with our Cracking KD-Tree.

Full Scan. We use a vectorized, predicated scan approach [Boncz et al., 2005] that produces a candidate list per scanned vector of a column. The vector size is in accordance to the L2 cache size.

Standard Cracking AVL. Each column goes through the process of database cracking separately. Afterwards, the results are intersected by the creation of bit-vectors[1].

Full Index B+ Tree. Each column is indexed using a B+ Tree created before running the workload. To answer the queries, we do a lookup in each column and intersect the results with bit-vectors.

Full Index KD-Tree. All columns are indexed using a KD-Tree pivoting by median values and choosing the dimensions in a round robin fashion. In order to find the medians, we use a quick-sort variant that instead of ordering a column, stops execution when finding its median value. The query result is then given by a lookup in the KD-Tree.

[1]We use the standard c++ library: *boost::dynamic_bitset* to generate the bit-vectors

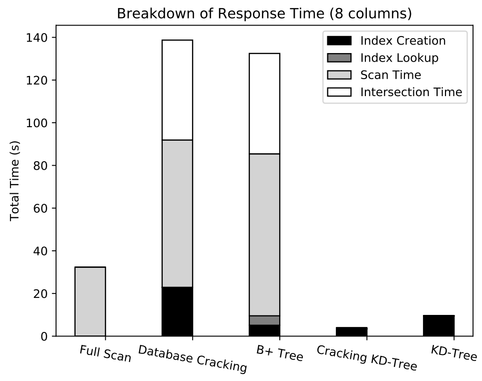


Figure 6: Total response time breakdown of workload with 8 columns.

Figure 6 depict the breakdown of accumulated response time for all algorithms. We can see that both uni-dimensional indexes have a similar cost, around 140 seconds in total, mainly due to the time spent in scanning, creating and intersecting the intermediate results with the bit-vectors. The full scan is four times cheaper than building, traversing and intersecting multiple uni-dimensional indexes to answer the query. The KD-Tree achieves a three times better response time than the full scan, with index creation being its highest cost. This issue is mitigated by the Cracking KD-Tree that efficiently spreads it throughout the query stream. However, the index creation cost still takes a considerable chunk of time on the Cracking KD-Tree mainly due to the relaxation of the one dimension per level restriction of the KD-Tree.

Figure 7 depict the cumulative response time for the full scan and both KD-Trees. The full scan has a linear cost, presenting about the same cost for every scan. The Full KD-Tree has the highest initial cost due to its a priori creation, costing two orders of magnitude more than a full scan, but all subse-

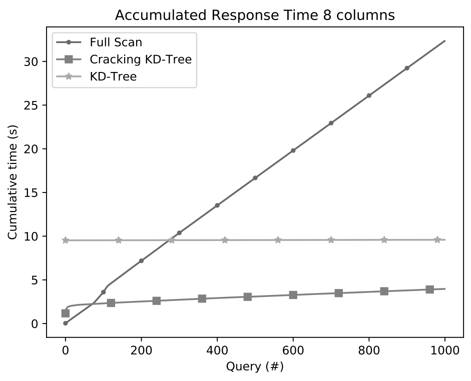


Figure 7: Cumulative response time for 8 columns.

quent queries have low additional costs, taking around 280 queries for this initial investment to pay off. The Cracking KD-Tree's first query cost is one order of magnitude higher than a full scan, and it quickly converges towards a full index speed, presenting a lower response time than a full scan around query 7, and only needing 70 queries for its creation investment to pay off.

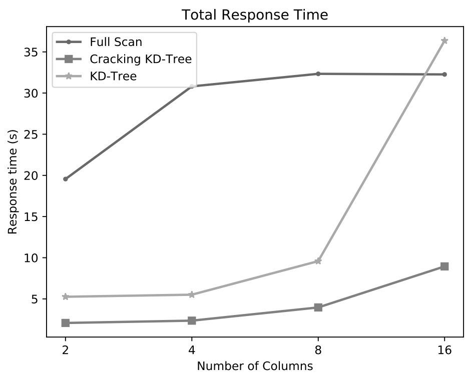


Figure 8: Total time for multiple columns.

Figure 8 depicts the total time for running our query stream under a different number of dimensions. Since we maintain the same selectivity per column (i.e., 0,2%), for 16 columns our query results are all empty. We can see that the full scan increases its total time dependent on the intermediate result size due to the use of candidate lists. Since the overall selectivity of the queries decreases when we increase the number of dimensions the full scan can maintain good scalability after four dimensions. The Full KD-Tree depends mostly on time spent creating the index, so its cost grows exponentially when we increase the number of columns. The Cracking KD-Tree presents better scalability when compared to a Full KD-tree due to its lazy nature.

5 Conclusion & Future Work

MDAI introduces new aspects that were unexplored by adaptive indexing and that require further investigation. We describe the following as the aspects that shall be explored as our next steps in this research:

- **Related work optimizations:** In our proof-of-concept, we always use one uni-dimensional index per column and later intersect their intermediate results using bit-vectors. However, we can also use a covering index which might be more competitive than the current approach. Another possibility would be to map the n-dimensions to one dimension by using Z-Ordering [Ramsak et al., 2000] and indexing it.
- **Cracking KD-Tree:** The Cracking KD-Tree presented in this work produces multiple dimensions in the same level by constructing it using the order of range predicates presented in the query. Other design choices can be made, as ignoring parts of the predicates to preserve the dimensions per level, or completely ignoring the query predicates and select the pivot points by calculating the medians, one for each predicate.
- **Adapting other MDI:** Other data structures are also good candidates to MDAI when increasing the number of dimensions. For instance, Vantage-point Tree, Ball-trees and M-Tree [Liu et al., 2006], and Locality Sensitive Hashing [Andoni, 2009] present similar searching properties to KD-Trees, although their structure demand heavier storage space compared to the KD-Tree increasing the runtime for maintenance.
- **Benchmarks:** Our experimental evaluation is limited by only using uniformly random distributions for the data and a fixed selectivity for the queries. Other distributions and selectivities must be explored. Real-world multidimensional data and workloads should also be tested (e.g., the genomic multidimensional range query benchmark [Sprenger et al., 2018]).
- **Machine Learning:** In-database Machine Learning is a trend and KD-Trees are broadly used for approximate nearest neighbor (k-NN) search: given a labeled object, find the most similar labeled object. Applications of the k-NN search, include, text categorization, searching image databases, finding duplicate records. Our agenda includes studying the impact of our Cracking KD-Tree to save search time at little cost in quality of the nearest neighbor in some of these applications.

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REFERENCES

[Agrawal et al., 2000] Agrawal, S., Chaudhuri, S., and Narasayya, V. R. (2000). Automated Selection of Materialized Views and Indexes in SQL Databases. In *VLDB*, pages 496–505.

[Alvanaki et al., 2015] Alvanaki, F., Goncalves, R., Ivanova, M., Kersten, M., and Kyzirakos, K. (2015). Gis navigation boosted by column stores. *PVLDB*, 8(12):1956–1959.

[Andoni, 2009] Andoni, A. (2009). *NN search : the old, the new, and the impossible*. PhD thesis, Massachusetts Institute of Technology, Cambridge, MA, USA.

[Bentley, 1975] Bentley, J. L. (1975). Multidimensional binary search trees used for associative searching. *Communications of the ACM*, 18(9):509–517.

[Boncz et al., 2005] Boncz, P. A., Zukowski, M., and Nes, N. (2005). Monetdb/x100: Hyper-pipelining query execution. In *CIDR*, volume 5, pages 225–237.

[Bruno, 2011] Bruno, N. (2011). *Automated Physical Database Design and Tunning*. CRC-Press.

[Chaudhuri and Narasayya, 1998] Chaudhuri, S. and Narasayya, V. (1998). AutoAdmin "What-if" Index Analysis Utility. *SIGMOD Record*, 27(2):367–378.

[Chaudhuri and Narasayya, 1997] Chaudhuri, S. and Narasayya, V. R. (1997). An Efficient, Cost-Driven Index Selection Tool for Microsoft SQL Server. In *VLDB*, volume 97, pages 146–155.

[Comer, 1978] Comer, D. (1978). The Difficulty of Optimum Index Selection. *TODS*, 3(4):440–445.

[Graefe and Kuno, 2010] Graefe, G. and Kuno, H. (2010). Self-selecting, self-tuning, incrementally optimized indexes. In *EDBT*, pages 371–381. ACM.

[Guttman, 1984] Guttman, A. (1984). R-trees: A dynamic index structure for spatial searching. In *SIGMOD*, pages 47–57.

[Halim et al., 2012] Halim, F., Idreos, S., Karras, P., and Yap, R. H. (2012). Stochastic Database Cracking: Towards Robust Adaptive Indexing in Main-Memory Column-Stores. *VLDB*, 5(6):502–513.

[Ho et al., 1997] Ho, C.-T., Agrawal, R., Megiddo, N., and Srikant, R. (1997). Range queries in OLAP data cubes. In *SIGMOD*, volume 26, pages 73–88.

[Holanda and de Almeida, 2017] Holanda, P. and de Almeida, E. C. (2017). SPST-Index: A Self-Pruning Splay Tree Index for Caching Database Cracking. In *EDBT*, pages 458–461.

[Idreos et al., 2009] Idreos, S., Kersten, M. L., and Manegold, S. (2009). Self-organizing Tuple Reconstruction in Column-stores. *SIGMOD*, pages 297–308.

[Idreos et al., 2007] Idreos, S., Kersten, M. L., Manegold, S., et al. (2007). Database Cracking. In *CIDR*, volume 3, pages 1–8.

[Idreos et al., 2011] Idreos, S., Manegold, S., Kuno, H., and Graefe, G. (2011). Merging What's Cracked, Cracking What's Merged: Adaptive Indexing in Main-Memory Column-Stores. *VLDB*, 4(9):586–597.

[Li, 2011] Li, H. (2011). Tabix: fast retrieval of sequence features from generic tab-delimited files. *Bioinformatics*, 27(5):718–719.

[Li et al., 2003] Li, X., Kim, Y. J., Govindan, R., and Hong, W. (2003). Multi-dimensional range queries in sensor networks. In *SenSys*, pages 63–75.

[Liu et al., 2006] Liu, T., Moore, A. W., and Gray, A. G. (2006). New algorithms for efficient high-dimensional nonparametric classification. *Journal of Machine Learning Research*, 7:1135–1158.

[Poess and Stephens, 2004] Poess, M. and Stephens, Jr., J. M. (2004). Generating thousand benchmark queries in seconds. In *VLDB*, pages 1045–1053.

[Ramsak et al., 2000] Ramsak, F., Markl, V., Fenk, R., Zirkel, M., Elhardt, K., and Bayer, R. (2000). Integrating the ub-tree into a database system kernel. In *VLDB*, volume 2000, pages 263–272.

[Schuhknecht et al., 2018] Schuhknecht, F. M., Dittrich, J., and Linden, L. (2018). Adaptive adaptive indexing. *ICDE*.

[Schuhknecht et al., 2013] Schuhknecht, F. M., Jindal, A., and Dittrich, J. (2013). The Uncracked Pieces in Database Cracking. *PVLDB*, 7(2):97–108.

[Sprenger et al., 2018] Sprenger, S., Schäfer, P., and Leser, U. (2018). Multidimensional range queries on modern hardware. *arXiv preprint arXiv:1801.03644*.

[Valentin et al., 2000] Valentin, G., Zuliani, M., Zilio, D. C., Lohman, G., and Skelley, A. (2000). DB2 Advisor: An Optimizer Smart Enough to Recommend Its Own Indexes. In *ICDE*, pages 101–110.

[Weber et al., 1998] Weber, R., Schek, H.-J., and Blott, S. (1998). A quantitative analysis and performance study for similarity-search methods in high-dimensional spaces. In *VLDB*, volume 98, pages 194–205.

[Zhang, 2009] Zhang, D. (2009). Covering index. In Liu, L. and Özsu, M. T., editors, *Encyclopedia of Database Systems*, pages 516–517. Springer US.