Applied Econometrics
Second edition

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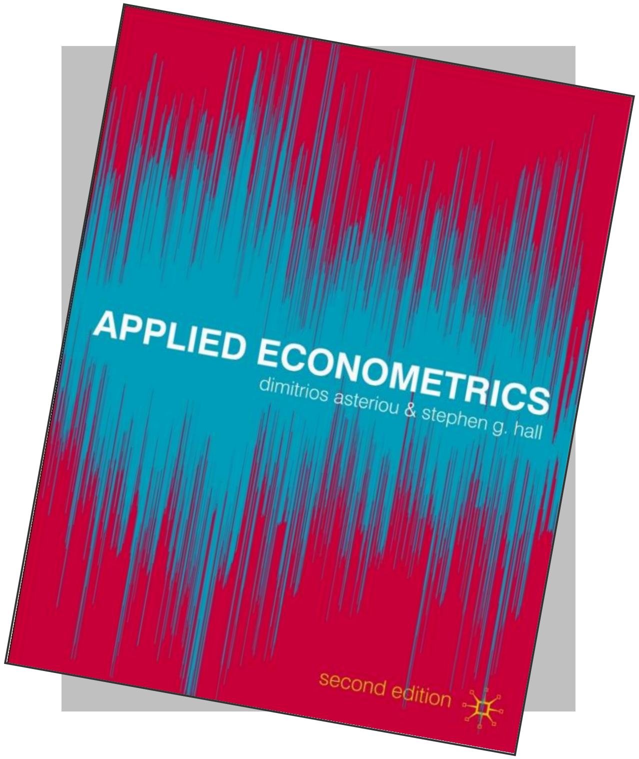
Chapter 7:
Autocorrelation



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Autocorrelation

1. What is autocorrelation
2. What causes autocorrelation
3. First and higher orders
4. Consequences of autocorrelation
5. Detecting autocorrelation
6. Resolving autocorrelation

Learning Objectives

1. Understand meaning of autocorrelation in the CLRM
2. What causes autocorrelation
3. Distinguish among first and higher orders of autocorrelation
4. Understand consequences of autocorrelation on OLS estimates
5. Detect autocorrelation through graph inspection
6. Detect autocorrelation through formal econometric tests
7. Distinguish among the wide range of available tests for detecting autocorrelation
8. Perform autocorrelation tests using econometric software
9. Resolve autocorrelation using econometric software

What is Autocorrelation

Assumption 6 of the CLRM states that the covariances and correlations between different disturbances are all zero:

$cov(u_t, u_s)=0$ for *all* $t \neq s$

This assumption states that the disturbances u_t and u_s are independently distributed, which is called serial independence

What is Autocorrelation (2)

If this assumption is no longer valid, then the disturbances are not pairwise independent, but pairwise autocorrelated (or serially correlated).

This means that an error occurring at period *t* may be carried over to the next period *t+1*.

Autocorrelation most likely to occur in time series data.

In cross-sectional we can change the arrangement of the data without altering the results.

What Causes Autocorrelation

One factor that can cause autocorrelation is **omitted variables.**

Suppose Y_t is related to X_{2t} and X_{3t}, but we wrongfully do not include X_{3t} in our model.

The effect of X_{3t} will be captured by the disturbances u_t.

If X_{3t} like many economic series exhibit a trend over time, then X_{3t} depends on X_{3t-1}, X_{3t-2} and so on.

Similarly then u_t depends on u_{t-1}, u_{t-2} and so on.

What Causes Autocorrelation (2)

Another possible reason is **misspecification**.

Suppose Y_t is related to X_{2t} with a quadratic relationship:

$$Y_t=\beta_1+\beta_2 X^2_{2t}+u_t$$

but we wrongfully assume and estimate a straight line:

$$Y_t=\beta_1+\beta_2 X_{2t}+u_t$$

Then the error term obtained from the straight line will depend on X^2_{2t}.

What Causes Autocorrelation (3)

A third reason is **systematic errors in measurement**.

Suppose a company updates its inventory at a given period in time.

If a systematic error occurred then the cumulative inventory stock will exhibit accumulated measurement errors.

These errors will show up as an autocorrelated procedure.

First-order Autocorrelation

The simplest and most commonly observed is the **first-order** autocorrelation.

Consider the multiple regression model:

$$Y_t=\beta_1+\beta_2 X_{2t}+\beta_3 X_{3t}+\beta_4 X_{4t}+\dots+\beta_k X_{kt}+u_t$$

in which the current observation of the error term u_t is a function of the previous (lagged) observation of the error term:

$$u_t=\rho u_{t-1}+e_t$$

First-order Autocorrelation (2)

The coefficient ρ is called the first-order autocorrelation coefficient and takes values from -1 to +1.

It is obvious that the size of ρ will determine the strength of serial correlation.

There can be three different cases.

First-order Autocorrelation (3)

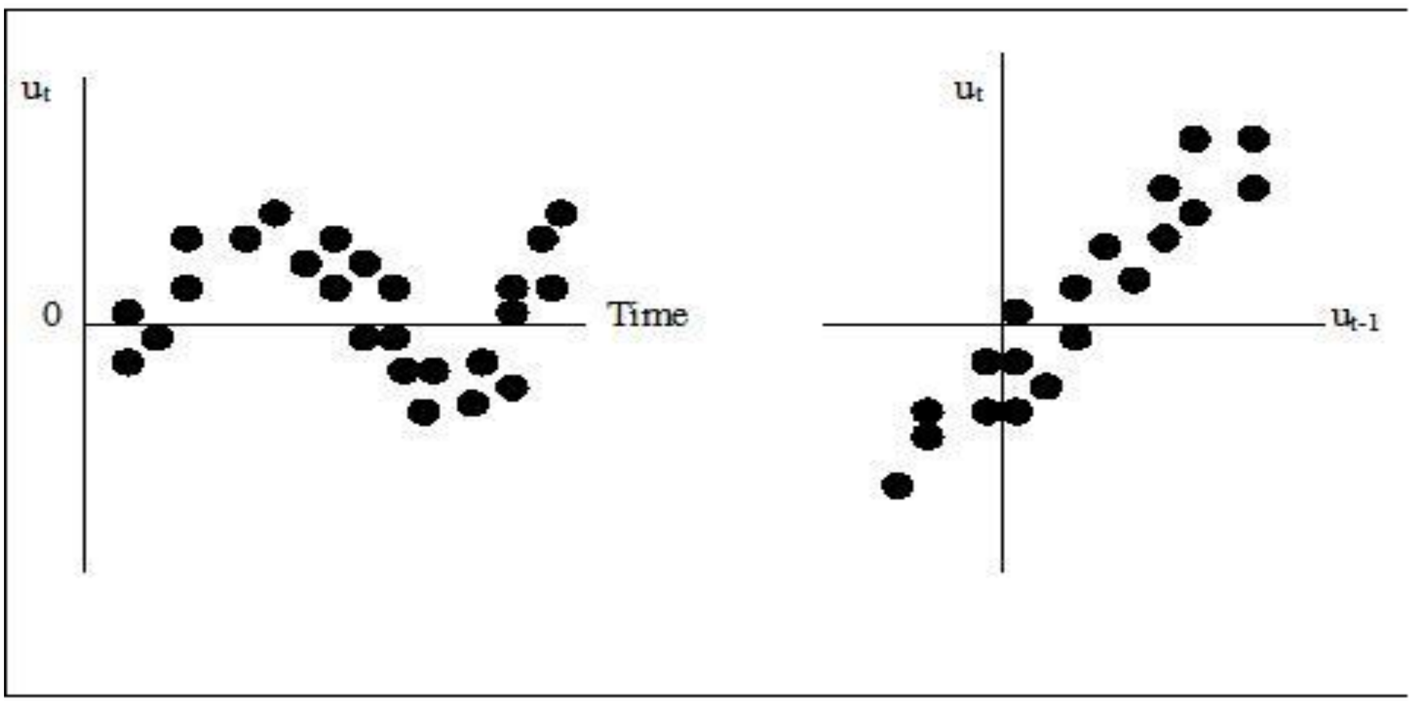
(a) If ρ is zero, then we have **no autocorrelation.**

(b) If ρ approaches unity, the value of the previous observation of the error becomes more important in determining the value of the current error and therefore high degree of autocorrelation exists. In this case we have **positive autocorrelation.**

(c) If ρ approaches -1, we have high degree of **negative autocorrelation.**

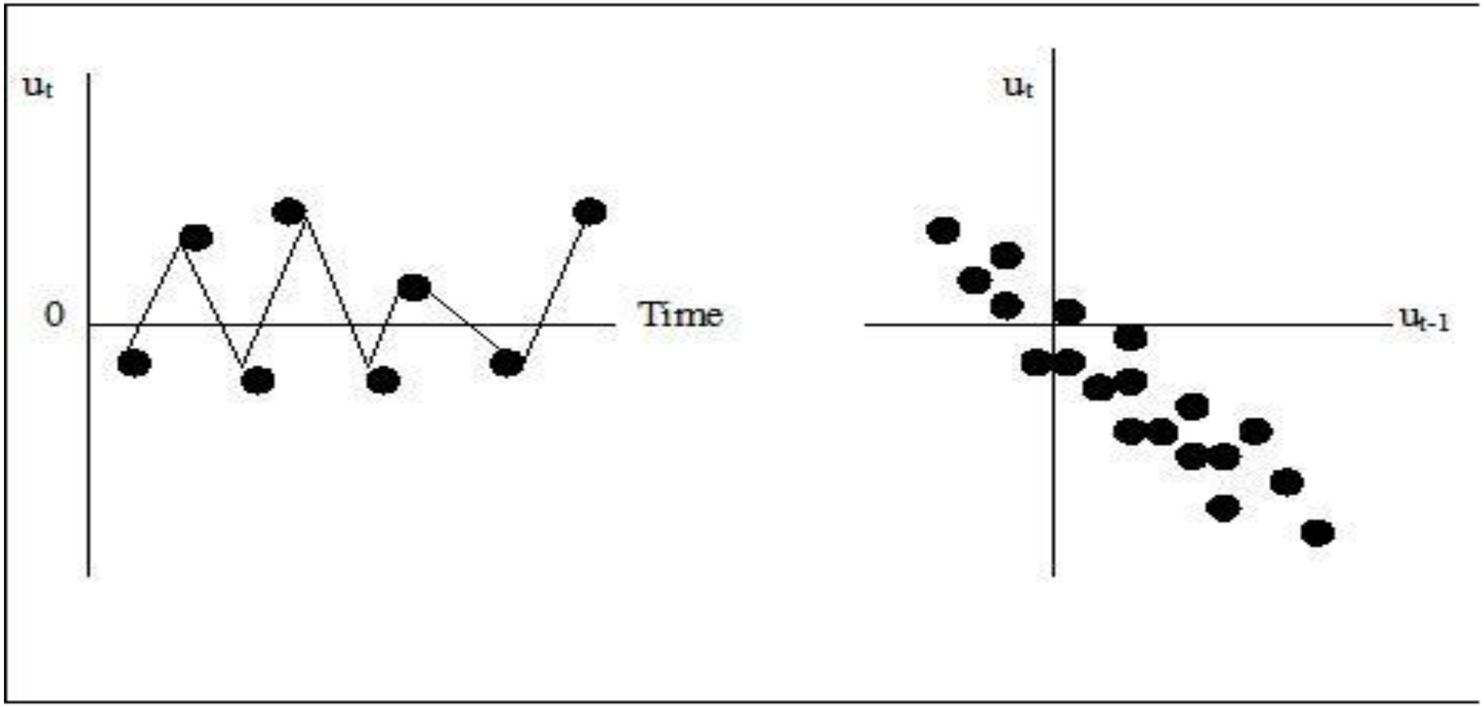
First-order Autocorrelation (4)

Figure 11.1: Positive Serial Correlation



First-order Autocorrelation (5)

Figure 11.2: Negative Serial Correlation



Higher-order Autocorrelation

Second order when:

$$u_t=\rho_1 u_{t-1}+ \rho_2 u_{t-2}+et$$

Third order when:

$$u_t=\rho_1 u_{t-1}+ \rho_2 u_{t-2}+\rho_3 u_{t-3} +e_t$$

p-th order when:

$$u_t=\rho_1 u_{t-1}+ \rho_2 u_{t-2}+\rho_3 u_{t-3} +\ldots+ \rho_p u_{t-p} +e_t$$

Consequences of Autocorrelation

1. OLS estimators still unbiased and consistent, because both unbiasedness and consistency do not depend on assumption 6, which in this case is violated.
2. OLS estimators will be inefficient and not BLUE.
3. Estimated variances of regression coefficients will be biased and inconsistent, so hypothesis testing no longer valid. Mostly, R^2 will be overestimated and t-statistics will tend to be higher.

Detecting Autocorrelation

Two ways in general.

First is informal, done through graphs, and therefore called **graphical method**.

Second is through **formal tests** for autocorrelation, such as:

1. Durbin Watson test
2. Breusch-Godfrey test
3. Durbin’s h test (for the presence of lagged dependent variables)
4. Engle’s ARCH test

Detecting Autocorrelation (2)

Take the following series (quarterly data from 1985q1 to 1994q2):

lcons = the consumer's expenditure on food

ldisp = disposable income

lprice = the relative price index of food

Typing the following command in Eviews :

ls lcons c ldisp lprice

gives the regression results

Detecting Autocorrelation (3)

Then we can store the residuals of this regression in a vector by typing the command:

genr res01=resid

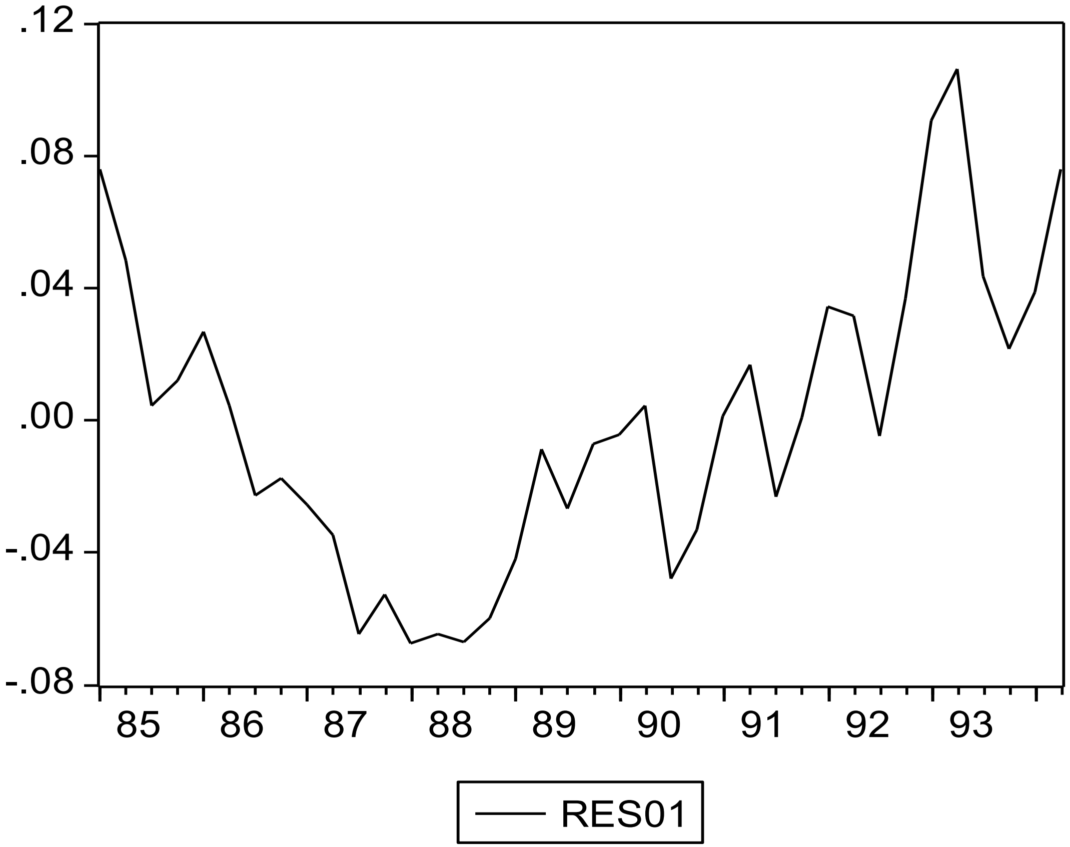
And a plot of the residuals can be obtained with:

plot res01

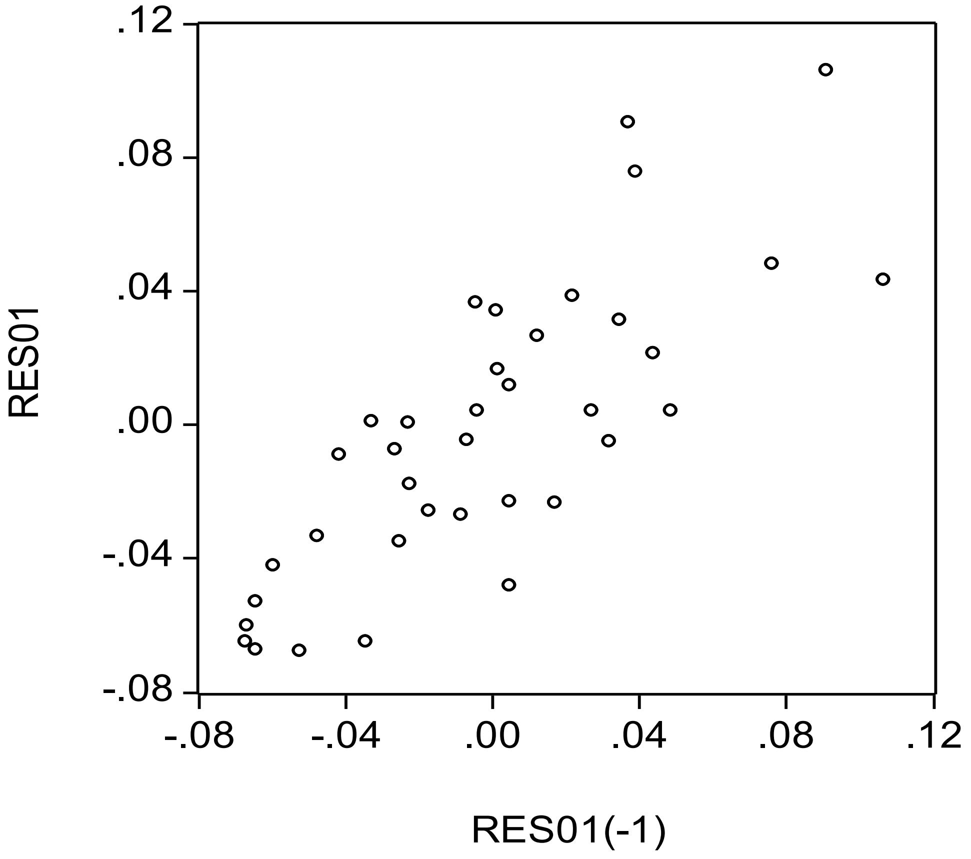
While a scatter of the residuals against their lagged terms can be obtained by:

scat res01(-1) res01

Detecting Autocorrelation (4)



Detecting Autocorrelation (5)



Durbin Watson Test

The following assumptions should be satisfied:

1. The regression model includes a constant
2. Autocorrelation is assumed to be of first-order only
3. The equation does not include a lagged dependent variable as explanatory variable

Durbin Watson Test (2)

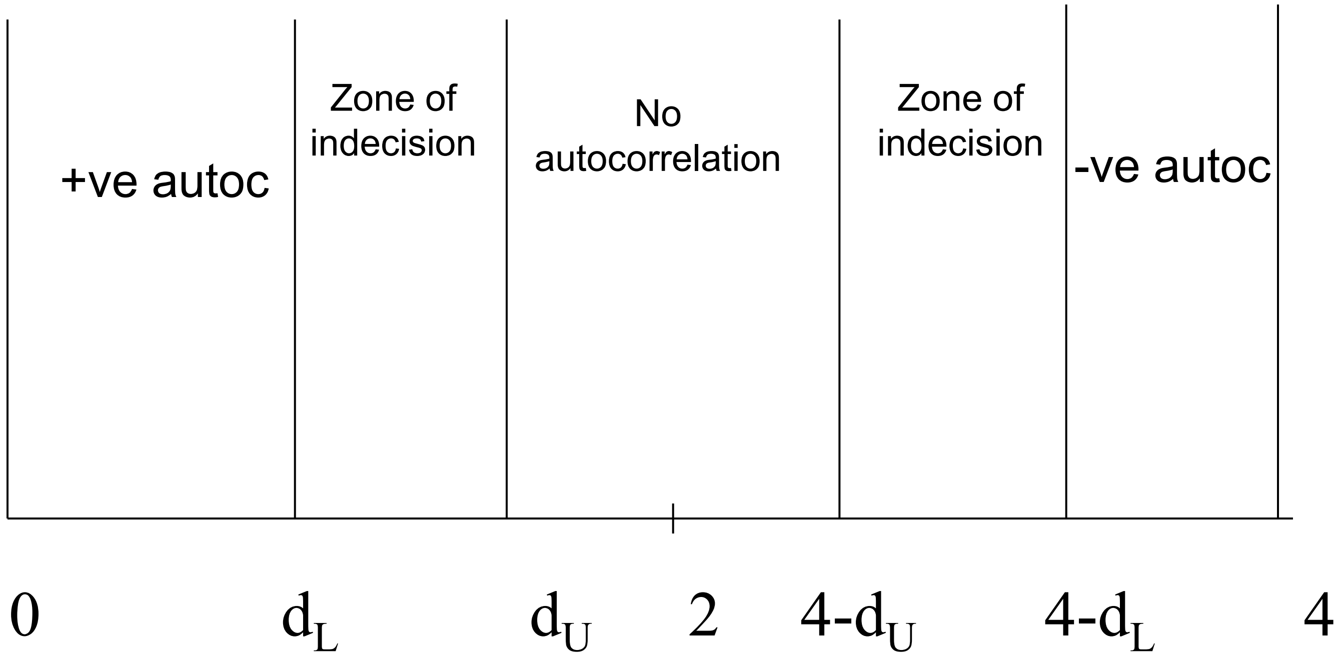
Step 1: Estimate model by OLS and obtain the residuals

Step 2: Calculate DW statistic

Step 3: Construct table with calculated DW statistic and the d_U, d_L, $4\text{-}d_U$ and $4\text{-}d_L$ critical values.

Step 4: Conclude

Durbin Watson Test (3)



Durbin Watson Test (4)

Drawbacks of the DW test:

1. May give inconclusive results
2. Not applicable when a lagged dependent variable is used
3. Can’t take into account higher order of autocorrelation

Breusch-Godfrey Test

A Lagrange Multiplier test that resolves the drawbacks of the DW test.

Consider the model:

$$Y_t=\beta_1+\beta_2 X_{2t}+\beta_3 X_{3t}+\beta_4 X_{4t}+\ldots+\beta_k X_{kt}+u_t$$

where:

$$u_t=\rho_1 u_{t-1}+ \rho_2 u_{t-2}+\rho_3 u_{t-3} +\ldots+ \rho_p u_{t-p} +e_t$$

Breusch-Godfrey Test (2)

Combining those two we get:

$$Y_t=\beta_1+\beta_2 X_{2t}+\beta_3 X_{3t}+\beta_4 X_{4t}+\ldots+\beta_k X_{kt}+$$

$$+\rho_1 u_{t-1}+\rho_2 u_{t-2}+\rho_3 u_{t-3}+\ldots+\rho_p u_{t-p}+e_t$$

The null and the alternative hypotheses are:

H_0: $\rho_1=\rho_2=\ldots=\rho_p=0$ *no autocorrelation*

H_a: *at least one of the ρ's is not zero, thus, autocorrelation*

Breusch-Godfrey Test (3)

Step 1: Estimate model and obtain the residuals

Step 2: Run full LM model with the number of lags used being determined by the assumed order of autocorrelation

Step 3: Compute *LM statistic* = $(n\text{-}\rho)R^2$ from the LM model and compare it with the chi-square critical value

Step 4: Conclude

Durbin’s h Test

When there are lagged dependent variables (i.e. Y_{t-1}) then the DW test is not applicable.

Durbin developed an alternative test statistic, named the h-statistic, which is calculated by:

$$h = \left(1 - \frac{DW}{2}\right)\sqrt{\frac{n}{1 - n\sigma_{\hat{\gamma}}^2}}$$

Where sigma of gamma hat square is the variance of the estimated coefficient of the lagged dependent variable.

Statistic is distributed following the normal distribution

Durbin's h Test (2)

Dependent Variable: LOG(CONS)

Included observations: **37** after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.834242	0.626564	1.331456	0.1922
LOG(INC)	0.227634	0.188911	1.204981	0.2368
LOG(CPI)	-0.259918	0.110072	-2.361344	0.0243
LOG(CONS(-1))	0.854041	**0.089494**	9.542982	0.0000

R-squared	0.940878	Mean dependent var	4.582683
Adjusted R-squared	0.935503	S.D. dependent var	0.110256
S.E. of regression	0.028001	Akaike info criterion	-4.211360
Sum squared resid	0.025874	Schwarz criterion	-4.037207
Log likelihood	81.91016	F-statistic	175.0558
Durbin-Watson stat	**1.658128**	Prob(F-statistic)	0.000000

Durbin's h Test (3)

$$h = \left(1 - \frac{DW}{2}\right)\sqrt{\frac{n}{1 - n\sigma_{\hat{\gamma}}^2}}$$

$$= \left(1 - \frac{1.658}{2}\right)\sqrt{\frac{37}{1 - 37 * 0.089^2}} = 1.2971$$

Resolving Autocorrelation

Two different cases:

(a) When ρ is known

(b) When ρ is unknown

Resolving Autocorrelation when ρ is known

Consider the model:

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + \beta_4 X_{4t} + \ldots + \beta_k X_{kt} + u_t$$

where:

$$u_t = \rho_1 u_{t-1} + e_t$$

Resolving Autocorrelation when ρ is known (2)

Write the model of *t-1*:

$$Y_{t-1}=\beta_1+\beta_2 X_{2t-1}+\beta_3 X_{3t-1}+\beta_4 X_{4t-1}+\ldots+\beta_k X_{kt-1}+u_{t-1}$$

Multiply both sides by ρ to get:

$$\rho Y_{t-1}=\rho\beta_1+\rho\beta_2 X_{2t-1}+\rho\beta_3 X_{3t-1}+\rho\beta_4 X_{4t-1}$$
$$+\ldots+\rho\,\beta_k X_{kt-1}+\rho u_{t-1}$$

Resolving Autocorrelation when ρ is known (3)

Subtract those two equations:

$$Y_t - \rho Y_{t-1} = (1-\rho)\beta_1 + \beta_2(X_{2t} - \rho X_{2t-1}) + \beta_3(X_{3t} - \rho X_{3t-1}) + \\ + \ldots + \beta_k(X_{kt} - \rho X_{kt-1}) + (u_t - \rho u_{t-1})$$

or

$$Y^*_t = \beta^*_1 + \beta^*_2 X^*_{2t} + \beta^*_3 X^*_{3t} + \ldots + \beta^*_k X^*_{kt} + e_t$$

Where the problem of autocorrelation is now resolved because e_t is no longer autocorrelated.

Resolving Autocorrelation when ρ is known (4)

Note that because from the transformation we lose one observation, in order to avoid that loss we generate Y1 and X_{i1} as follows:

$Y^*_1 = Y_1 \, sqrt(1 - \rho^2)$

$X^*_{i1} = X_{i1} \, sqrt(1-\rho^2)$

This transformation is called quasi-differencing or generalized differencing.

Resolving Autocorrelation (when ρ is unknown)

Cochrane-Orcutt iterative procedure

Step 1: Estimate regression and obtain residuals

Step 2: Estimate *ρ* from regressing the residuals to its lagged terms

Step 3: Transform the original variables as starred variables using those obtained from step 2

Step 4: Run the regression again with the transformed variables and obtain residuals

Step 5 and on: Continue repeating steps 2 to 4 for several rounds until (**stopping rule**) the estimates of from two successive iterations differ by no more than some preselected small value, such as 0.001