Formulário — Probabilidade II

Notação

$F_X(x) = P(X \leq x)$; $S(x) = P(X > x) = 1 - F_X(x)$; $M_X(t) = E(e^{tX})$.

Função Gamma

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}\,dx, \quad \alpha > 0$$
$$\Gamma(\alpha+1) = \alpha\Gamma(\alpha), \quad \Gamma(n) = (n-1)!,\ n \in \mathbb{N}$$
$$\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}, \quad \Gamma\left(\tfrac{3}{2}\right) = \tfrac{1}{2}\sqrt{\pi}$$
$$\int_0^\infty x^k e^{-ax}\,dx = \frac{\Gamma(k+1)}{a^{k+1}}, \quad a > 0$$

Função Beta

$$B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}\,dx, \quad a,b > 0$$
$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$
$$B(a+1,b) = \frac{a}{a+b}B(a,b), \quad B(a,b+1) = \frac{b}{a+b}B(a,b)$$

FGM

$$M_X(t) = E(e^{tX}), \quad M_X(0) = 1$$
$$E(X) = M_X'(0), \quad E(X^2) = M_X''(0)$$
$$\mathrm{Var}(X) = M_X''(0) - [M_X'(0)]^2$$
$$M_{aX+b}(t) = e^{bt}M_X(at)$$

Gamma $\Gamma(\alpha, \lambda)$ (taxa)

$$f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\lambda x}, \quad x \geq 0$$
$$E(X) = \frac{\alpha}{\lambda}, \quad \mathrm{Var}(X) = \frac{\alpha}{\lambda^2}$$
$$M_X(t) = \left(\frac{\lambda}{\lambda - t}\right)^\alpha, \quad t < \lambda$$
$$S(t) = e^{-\lambda t}\sum_{k=0}^{\alpha-1}\frac{(\lambda t)^k}{k!}, \quad \alpha \in \mathbb{N}$$

Se $X_i \overset{iid}{\sim} \Gamma(\alpha_i, \lambda)$:

$$\sum_i X_i \sim \Gamma\Big(\sum_i \alpha_i, \lambda\Big)$$

Qui-quadrado χ^2_ν

$$X \sim \chi^2_\nu \iff X \sim \Gamma\left(\tfrac{\nu}{2}, \tfrac{1}{2}\right)$$
$$f(x) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)}x^{\nu/2-1}e^{-x/2}, \quad x \geq 0$$
$$E(X) = \nu, \quad \mathrm{Var}(X) = 2\nu$$
$$M_X(t) = (1-2t)^{-\nu/2}, \quad t < \tfrac{1}{2}$$

Se $Z_i \overset{iid}{\sim} N(0,1)$:

$$\sum_{i=1}^{\nu} Z_i^2 \sim \chi^2_\nu$$

Se $X \sim \chi^2_{\nu_1}$, $Y \sim \chi^2_{\nu_2}$ indep.:

$$X + Y \sim \chi^2_{\nu_1+\nu_2}$$

Beta $\mathrm{Beta}(a,b)$

$$f(x) = \frac{1}{B(a,b)}x^{a-1}(1-x)^{b-1}, \quad 0 < x < 1$$
$$E(X) = \frac{a}{a+b}$$
$$\mathrm{Var}(X) = \frac{ab}{(a+b)^2(a+b+1)}$$
$$\mathrm{Beta}(1,1) = U(0,1)$$

Weibull (α, β)

$$f(x) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1}e^{-(x/\alpha)^\beta}, \quad x \geq 0$$
$$F(x) = 1 - e^{-(x/\alpha)^\beta}, \quad S(x) = e^{-(x/\alpha)^\beta}$$
$$E(X) = \alpha\Gamma\left(1 + \frac{1}{\beta}\right)$$
$$\mathrm{Var}(X) = \alpha^2\left[\Gamma\left(1+\frac{2}{\beta}\right) - \Gamma^2\left(1+\frac{1}{\beta}\right)\right]$$
$$t_p:\ F(t_p) = p \Rightarrow t_p = \alpha[-\ln(1-p)]^{1/\beta}$$

Se $Y = (X/\alpha)^\beta$:

$$Y \sim \mathrm{Exp}(1)$$

Transformações (1D)

Monótona: $Y = g(X)$ inversível

$$f_Y(y) = f_X(g^{-1}(y)) \left| \frac{d}{dy} g^{-1}(y) \right|$$

Linear: $Y = aX + b$, $a \neq 0$

$$f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a} \right)$$

$$E(Y) = aE(X) + b, \quad \mathrm{Var}(Y) = a^2 \mathrm{Var}(X)$$

Não-monótona:

$$f_Y(y) = \sum_{x_i:\ g(x_i)=y} f_X(x_i) \left| \frac{1}{g'(x_i)} \right|$$

Taylor

$$e^t = 1 + t + \frac{t^2}{2} + \frac{t^3}{6} + O(t^4)$$