Formulário

Notação

$F_X(x) = P(X \leq x)$; $S(x) = P(X > x) = 1 - F_X(x)$; $M_X(t) = E(e^{tX})$.

Distribuição Gamma: Seja $X \sim Gamma(\alpha, \lambda)$

$$f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}, \quad x \geq 0, \quad E(X) = \frac{\alpha}{\lambda}, \quad \text{Var}(X) = \frac{\alpha}{\lambda^2}$$

$$M_X(t) = \left(\frac{\lambda}{\lambda - t}\right)^\alpha, \quad t < \lambda, \quad S(t) = e^{-\lambda t} \sum_{k=0}^{\alpha-1} \frac{(\lambda t)^k}{k!}, \quad \alpha \in \mathbb{N}$$

Com,

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x}\, dx, \quad \alpha > 0$$

$$\Gamma(\alpha+1) = \alpha\Gamma(\alpha), \quad \Gamma(n) = (n-1)!,\ n \in \mathbb{N}$$

$$\Gamma(\tfrac{1}{2}) = \sqrt{\pi}, \quad \Gamma(\tfrac{3}{2}) = \tfrac{1}{2}\sqrt{\pi}$$

$$\int_0^\infty x^k e^{-ax}\, dx = \frac{\Gamma(k+1)}{a^{k+1}}, \quad a > 0$$

Distribuição Beta: Seja $X \sim \text{Beta}(a, b)$

$$f(x) = \frac{1}{B(a,b)} x^{a-1}(1-x)^{b-1}, \quad 0 < x < 1, \quad E(X) = \frac{a}{a+b},$$

$$\text{Var}(X) = \frac{ab}{(a+b)^2(a+b+1)}$$

Com,

$$B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}\, dx, \quad a, b > 0$$

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

FGM

$$M_X(t) = E(e^{tX}), \quad M_X(0) = 1$$

$$E(X) = M_X'(0), \quad E(X^2) = M_X''(0)$$

$$\text{Var}(X) = M_X''(0) - [M_X'(0)]^2$$

$$M_{aX+b}(t) = e^{bt} M_X(at)$$

Distribuição Qui-quadrado: Seja $X \sim \chi^2_\nu$

$$f(x) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} x^{\nu/2-1} e^{-x/2}, \quad x \geq 0$$

$$E(X) = \nu, \quad \text{Var}(X) = 2\nu$$

$$M_X(t) = (1-2t)^{-\nu/2}, \quad t < \tfrac{1}{2}$$

$$X \sim \chi^2_\nu \iff X \sim \Gamma\left(\tfrac{\nu}{2}, \tfrac{1}{2}\right)$$

Se $Z_i \overset{iid}{\sim} N(0,1)$:

$$\sum_{i=1}^{\nu} Z_i^2 \sim \chi^2_\nu$$

Distribuição Weibull: Seja $X \sim Weibull(\alpha, \beta)$

$$f(x) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} e^{-(x/\alpha)^\beta}, \quad x \geq 0$$

$$F(x) = 1 - e^{-(x/\alpha)^\beta}, \quad S(x) = e^{-(x/\alpha)^\beta}$$

$$E(X) = \alpha\Gamma\left(1 + \frac{1}{\beta}\right)$$

$$\text{Var}(X) = \alpha^2\left[\Gamma\left(1 + \frac{2}{\beta}\right) - \Gamma^2\left(1 + \frac{1}{\beta}\right)\right]$$

Transformações (1D)

Monótona: $Y = g(X)$ inversível

$$f_Y(y) = f_X(g^{-1}(y)) \left|\frac{d}{dy} g^{-1}(y)\right|$$

Linear: $Y = aX + b$, $a \neq 0$

$$f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right)$$

$$E(Y) = aE(X) + b, \quad \text{Var}(Y) = a^2\text{Var}(X)$$

Série de Maclaurin de e^x

$$e^x = \sum_{k=0}^\infty \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$