Formulário

Notação

$F_X(x) = P(X \le x)$; $S(x) = P(X > x) = 1 - F_X(x)$; $M_X(t) = E(e^{tX})$.

Distribuição Gamma: Seja $X \sim Gamma(\alpha, \lambda)$

$$f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}, \quad x \ge 0, \quad E(X) = \frac{\alpha}{\lambda}, \quad \text{Var}(X) = \frac{\alpha}{\lambda^2}$$

$$M_X(t) = \left(\frac{\lambda}{\lambda - t}\right)^\alpha, \quad t < \lambda, \quad S(t) = e^{-\lambda t} \sum_{k=0}^{\alpha-1} \frac{(\lambda t)^k}{k!}, \quad \alpha \in \mathbb{N}$$

Com,

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x}\, dx, \quad \alpha > 0$$

$$\Gamma(\alpha+1) = \alpha\Gamma(\alpha), \quad \Gamma(n) = (n-1)!,\ n \in \mathbb{N}$$

$$\Gamma(\tfrac{1}{2}) = \sqrt{\pi}, \quad \Gamma(\tfrac{3}{2}) = \tfrac{1}{2}\sqrt{\pi}$$

$$\int_0^\infty x^k e^{-ax}\, dx = \frac{\Gamma(k+1)}{a^{k+1}}, \quad a > 0$$

Distribuição Beta: Seja $X \sim \text{Beta}(a, b)$

$$f(x) = \frac{1}{B(a,b)} x^{a-1}(1-x)^{b-1}, \quad 0 < x < 1, \quad E(X) = \frac{a}{a+b},$$

$$\text{Var}(X) = \frac{ab}{(a+b)^2(a+b+1)}$$

Com,

$$B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}\, dx, \quad a, b > 0$$

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

FGM

$$M_X(t) = E(e^{tX}), \quad M_X(0) = 1$$

$$E(X) = M'_X(0), \quad E(X^2) = M''_X(0)$$

$$\text{Var}(X) = M''_X(0) - [M'_X(0)]^2$$

$$M_{aX+b}(t) = e^{bt} M_X(at)$$

Distribuição Qui-quadrado: Seja $X \sim \chi^2_\nu$

$$f(x) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} x^{\nu/2-1} e^{-x/2}, \quad x \ge 0$$

$$E(X) = \nu, \quad \text{Var}(X) = 2\nu$$

$$M_X(t) = (1-2t)^{-\nu/2}, \quad t < \tfrac{1}{2}$$

$$X \sim \chi^2_\nu \iff X \sim \Gamma\left(\tfrac{\nu}{2}, \tfrac{1}{2}\right)$$

Se $Z_i \overset{iid}{\sim} N(0,1)$:

$$\sum_{i=1}^{\nu} Z_i^2 \sim \chi^2_\nu$$

Distribuição Weibull: Seja $X \sim Weibull(\alpha, \beta)$

$$f(x) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} e^{-(x/\alpha)^\beta}, \quad x \ge 0$$

$$F(x) = 1 - e^{-(x/\alpha)^\beta}, \quad S(x) = e^{-(x/\alpha)^\beta}$$

$$E(X) = \alpha\Gamma\left(1 + \frac{1}{\beta}\right)$$

$$\text{Var}(X) = \alpha^2\left[\Gamma\left(1 + \frac{2}{\beta}\right) - \Gamma^2\left(1 + \frac{1}{\beta}\right)\right]$$

Transformações (1D)

Monótona: $Y = g(X)$ inversível

$$f_Y(y) = f_X(g^{-1}(y))\left|\frac{d}{dy}g^{-1}(y)\right|$$

Linear: $Y = aX + b$, $a \ne 0$

$$f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right)$$

$$E(Y) = aE(X) + b, \quad \text{Var}(Y) = a^2\text{Var}(X)$$

Série de Maclaurin de e^x

$$e^x = \sum_{k=0}^\infty \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

Distribuições Conjuntas

$$p_{X,Y}(x,y) = P(X=x, Y=y)$$

$$p_{X,Y}(x,y) \geq 0, \qquad \sum_x \sum_y p_{X,Y}(x,y) = 1$$

$$f_{X,Y}(x,y) \geq 0, \qquad \int\int f_{X,Y}(x,y)\,dx\,dy = 1$$

$$P((X,Y) \in A) = \int\int_A f_{X,Y}(x,y)\,dx\,dy$$

Distribuições Marginais

$$p_X(x) = \sum_y p_{X,Y}(x,y), \qquad p_Y(y) = \sum_x p_{X,Y}(x,y)$$

$$f_X(x) = \int f_{X,Y}(x,y)\,dy, \qquad f_Y(y) = \int f_{X,Y}(x,y)\,dx$$

Distribuições Condicionais

$$p_{X|Y}(x|y) = \frac{p_{X,Y}(x,y)}{p_Y(y)}$$

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

Esperança Matemática

$$E[g(X,Y)] = \sum_x \sum_y g(x,y) p_{X,Y}(x,y)$$

$$E[g(X,Y)] = \int\int g(x,y) f_{X,Y}(x,y)\,dx\,dy$$

$$E(X) = \sum_x x\,p_X(x) \quad \text{ou} \quad E(X) = \int x f_X(x)dx$$

$$E(Y) = \sum_y y\,p_Y(y) \quad \text{ou} \quad E(Y) = \int y f_Y(y)dy$$

$$E(X) = E_Y[E_X(X|Y)]$$

$$E(Y) = E_X[E_Y(Y|X)]$$

$$E(aX+bY) = aE(X) + bE(Y)$$

Variância e Covariância

$$Var(X) = E(X^2) - [E(X)]^2$$

$$Cov(X,Y) = E(XY) - E(X)E(Y)$$

$$Cov(X,Y) = E[(X-\mu_X)(Y-\mu_Y)]$$

$$Var(aX+bY) = a^2Var(X) + b^2Var(Y) + 2abCov(X,Y)$$

$$Var(X+Y) = Var(X) + Var(Y)$$

$$\rho_{X,Y} = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$$

$$-1 \leq \rho_{X,Y} \leq 1$$

Independência

$$p_{X,Y}(x,y) = p_X(x)p_Y(y)$$

$$f_{X,Y}(x,y) = f_X(x)f_Y(y)$$

Transformações Bidimensionais

$$U = g(X,Y), \qquad V = h(X,Y)$$

$$f_{U,V}(u,v) = f_{X,Y}(x(u,v), y(u,v)) \cdot |J|$$

$$J = \begin{vmatrix} \partial x/\partial u & \partial x/\partial v \\ \partial y/\partial u & \partial y/\partial v \end{vmatrix}$$