

Formulário

Notação

$$F_X(x) = P(X \leq x); \quad S(x) = P(X > x) = 1 - F_X(x); \\ M_X(t) = E(e^{tX}).$$

Distribuição Gamma: Seja

$$X \sim \text{Gamma}(\alpha, \lambda)$$

$$f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}, \quad x \geq 0, \quad E(X) = \frac{\alpha}{\lambda}, \quad \text{Var}(X) = \frac{\alpha}{\lambda^2}$$

$$M_X(t) = \left(\frac{\lambda}{\lambda - t} \right)^\alpha, \quad t < \lambda, \quad S(t) = e^{-\lambda t} \sum_{k=0}^{\alpha-1} \frac{(\lambda t)^k}{k!}, \quad \alpha \in \mathbb{N}$$

Com,

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx, \quad \alpha > 0$$

$$\Gamma(\alpha + 1) = \alpha \Gamma(\alpha), \quad \Gamma(n) = (n-1)!, \quad n \in \mathbb{N}$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}, \quad \Gamma\left(\frac{3}{2}\right) = \frac{1}{2}\sqrt{\pi}$$

$$\int_0^\infty x^k e^{-ax} dx = \frac{\Gamma(k+1)}{a^{k+1}}, \quad a > 0$$

Distribuição Beta: Seja $X \sim \text{Beta}(a, b)$

$$f(x) = \frac{1}{B(a, b)} x^{a-1} (1-x)^{b-1}, \quad 0 < x < 1, \quad E(X) = \frac{a}{a+b},$$

$$\text{Var}(X) = \frac{ab}{(a+b)^2(a+b+1)}$$

Com,

$$B(a, b) = \int_0^1 x^{a-1} (1-x)^{b-1} dx, \quad a, b > 0$$

$$B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

FGM

$$M_X(t) = E(e^{tX}), \quad M_X(0) = 1$$

$$E(X) = M'_X(0), \quad E(X^2) = M''_X(0)$$

$$\text{Var}(X) = M''_X(0) - [M'_X(0)]^2$$

$$M_{aX+b}(t) = e^{bt} M_X(at)$$

Distribuição Qui-quadrado: Seja $X \sim \chi^2_\nu$

$$f(x) = \frac{1}{2^{\nu/2} \Gamma(\nu/2)} x^{\nu/2-1} e^{-x/2}, \quad x \geq 0$$

$$E(X) = \nu, \quad \text{Var}(X) = 2\nu$$

$$M_X(t) = (1 - 2t)^{-\nu/2}, \quad t < \frac{1}{2}$$

$$X \sim \chi^2_\nu \iff X \sim \Gamma\left(\frac{\nu}{2}, \frac{1}{2}\right)$$

Se $Z_i \stackrel{iid}{\sim} N(0, 1)$:

$$\sum_{i=1}^{\nu} Z_i^2 \sim \chi^2_\nu$$

Distribuição Weibull: Seja

$$X \sim \text{Weibull}(\alpha, \beta)$$

$$f(x) = \frac{\beta}{\alpha} \left(\frac{x}{\alpha} \right)^{\beta-1} e^{-(x/\alpha)^\beta}, \quad x \geq 0$$

$$F(x) = 1 - e^{-(x/\alpha)^\beta}, \quad S(x) = e^{-(x/\alpha)^\beta}$$

$$E(X) = \alpha \Gamma\left(1 + \frac{1}{\beta}\right)$$

$$\text{Var}(X) = \alpha^2 \left[\Gamma\left(1 + \frac{2}{\beta}\right) - \Gamma^2\left(1 + \frac{1}{\beta}\right) \right]$$

Transformações (1D)

Monótona: $Y = g(X)$ inversível

$$f_Y(y) = f_X(g^{-1}(y)) \left| \frac{d}{dy} g^{-1}(y) \right|$$

Linear: $Y = aX + b, a \neq 0$

$$f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right)$$

$$E(Y) = aE(X) + b, \quad \text{Var}(Y) = a^2 \text{Var}(X)$$

Série de Maclaurin de e^x

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$