



This session will  
be recorded

# **F**AST **R**EED-SOLOMON **I**NTERACTIVE ORACLE PROOFS of PROXIMITY

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# Three Sigma

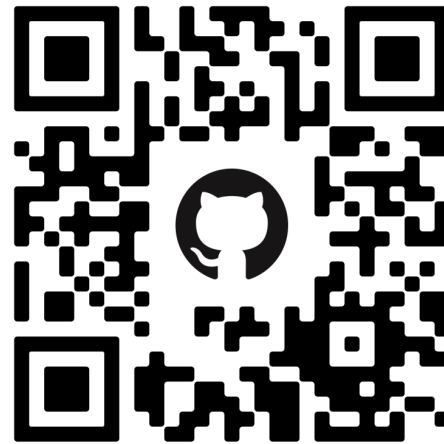
Venture builder firm focused on **blockchain engineering, research, and investment.**

## **Mission**

Advance the adoption of blockchain technology.  
Contribute for the healthy development of crypto/web3.

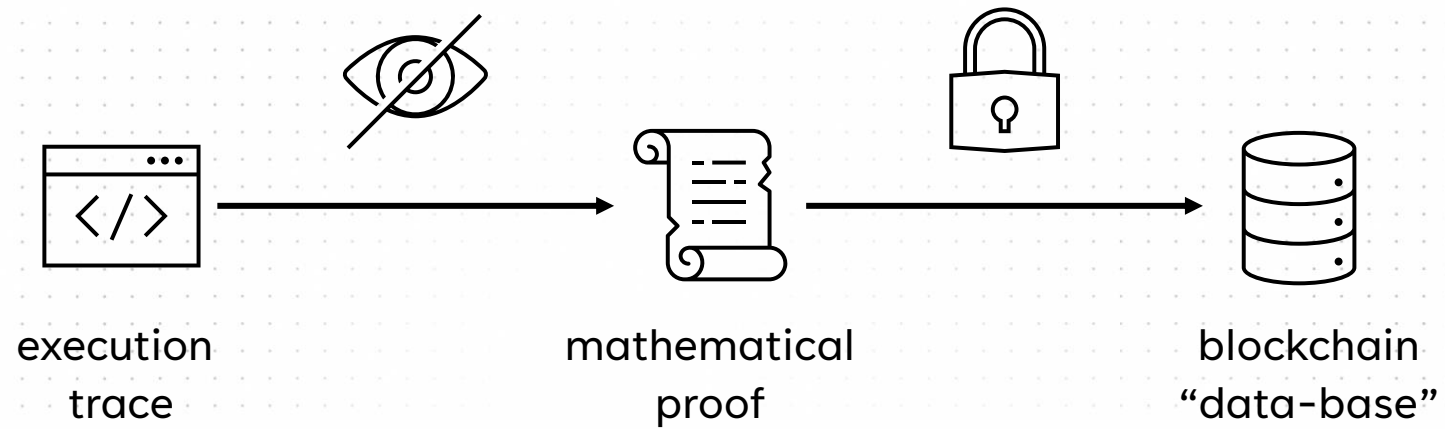
# LIBRARIES

If you haven't, please install Python  $\leq 3.10$  and the libraries **NumPy** and **Galois**. Check the GitHub repository for more:



<https://github.com/tiagomartins-threesigma/SINFO2023FRI4STARKs>

# ZERO KNOWLEDGE



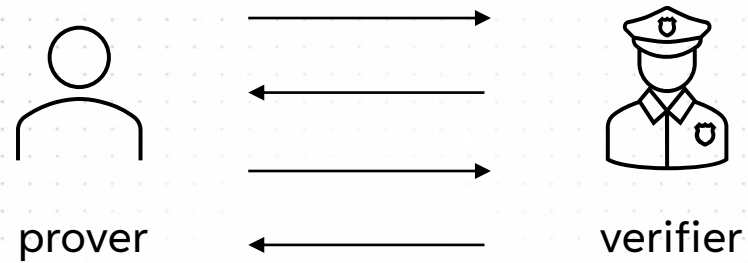
Can Vitalik prove that he  
knows a path from Lisbon to  
Berlin without disclosing it?



**Zero Knowledge Proofs**  
enable public consensus on  
private data!



# INTERACTIVE PROOF



Scalable Transparent **AR**guments of **K**nowledge

Vitalik must fulfil some challenges!



*Vitalik eating icecream  
at the wall of Berlin*



*Vitalik in  
Brandenburger Tor*





Section Break

# Diving into the Math

# BLOCKCHAIN SETTING



A non-necessarily honest prover wants to indirectly settle some **private** data in the blockchain with a ZK proof.

A verifier must check whether that data corresponds to honest computations and transactions.



# BLOCKCHAIN SETTING



STARKs operate over **low-degree polynomials**.

**How?**

Valid computations correspond to the evaluation table of low-degree polynomials.



Section Break

# Polynomials

# PROBLEM EXAMPLE

Trace index $k$	Group element $x$	Trace polynomial $f(x)$
0	$x_0 = g^0$	3
1	$x_1 = g^1$	9
2	$x_2 = g^2$	81
3	$x_3 = g^3$	6561
4	$x_4 = g^4$	43046721
5	$x_5 = g^5$	1853020188851841

$$f(\underbrace{x_{k+1}}_{g x_k}) = (f(x_k))^2 \quad \text{for } x_k \in \{x_0, x_1, x_2, x_3, x_4\}$$

$$\text{Example: } f(x_1) = 9 = (f(x_0))^2 = 3^2$$



# PROBLEM EXAMPLE

$$\underbrace{f(x_{k+1})}_{g x_k} = (f(x_k))^2 \quad \text{for} \quad x_k \in \{x_0, x_1, x_2, x_3, x_4\}$$

which implies that

$$f(g x) - (f(x))^2 = 0 \quad \text{for} \quad x \in \{x_0, x_1, x_2, x_3, x_4\}$$

which in turn implies that

$$p_0(x) = \frac{f(g x) - (f(x))^2}{(x - x_0)(x - x_1)(x - x_2)(x - x_3)(x - x_4)}$$

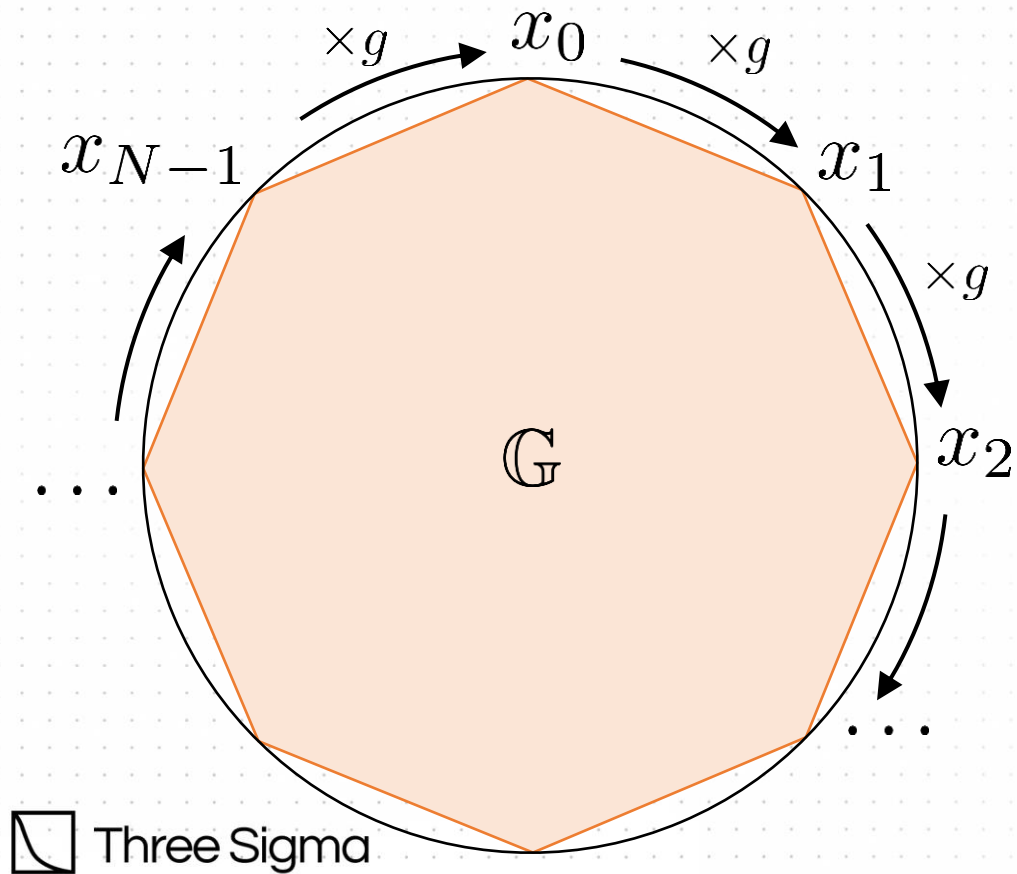
is a polynomial of degree  $\deg(p_0) = 2 \deg(f) - 5$ .

# POLYNOMIAL

$$p(x) = \sum_{n=0}^{N-1} c_n x^n$$

$$\underbrace{\begin{bmatrix} p(x_0) \\ p(x_1) \\ \vdots \\ p(x_{N-1}) \end{bmatrix}}_{\mathbf{p}} = \begin{bmatrix} 1 & x_0 & \dots & x_0^{N-1} \\ 1 & x_1 & \dots & x_1^{N-1} \\ \vdots & \vdots & & \vdots \\ 1 & x_{N-1} & \dots & x_{N-1}^{N-1} \end{bmatrix} \underbrace{\begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_{N-1} \end{bmatrix}}_{\mathbf{c}}$$

# FAST FOURIER TRANSFORM TO EVALUATE POLYNOMIALS

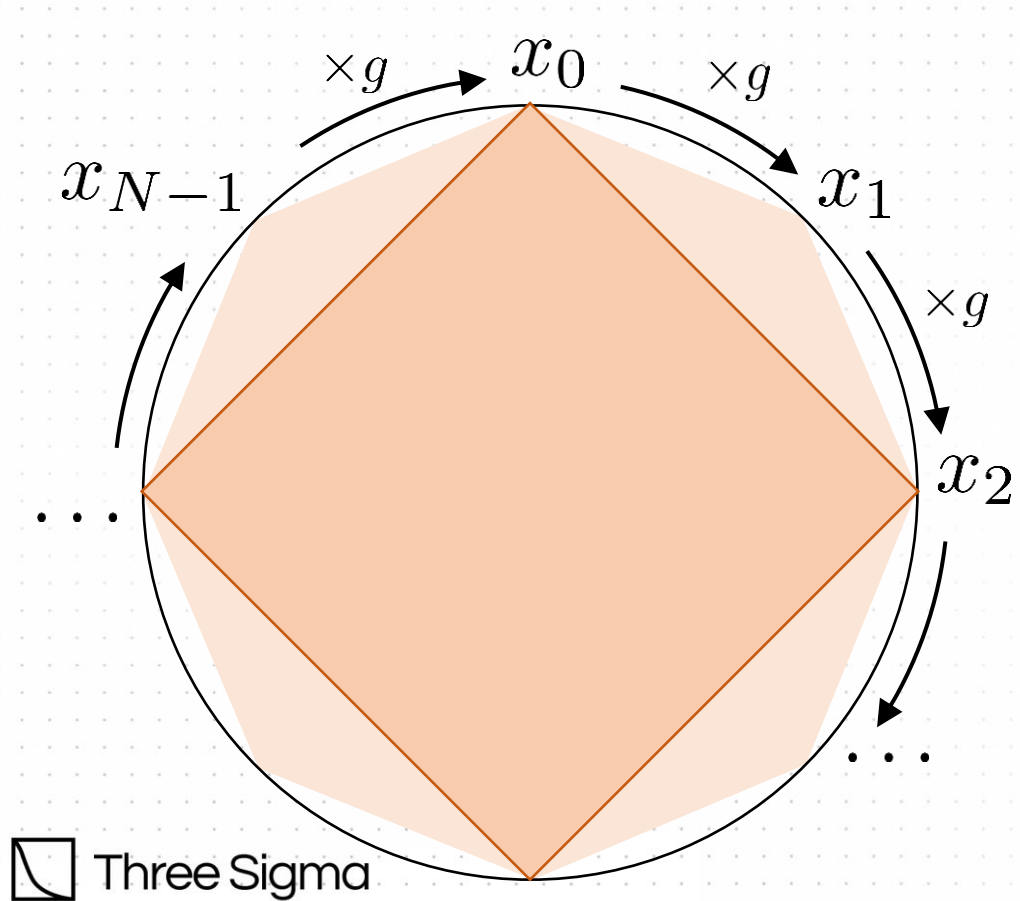


$$x \in \mathbb{G}$$

$$\mathbf{p} = \text{FFT}(\mathbf{c})$$

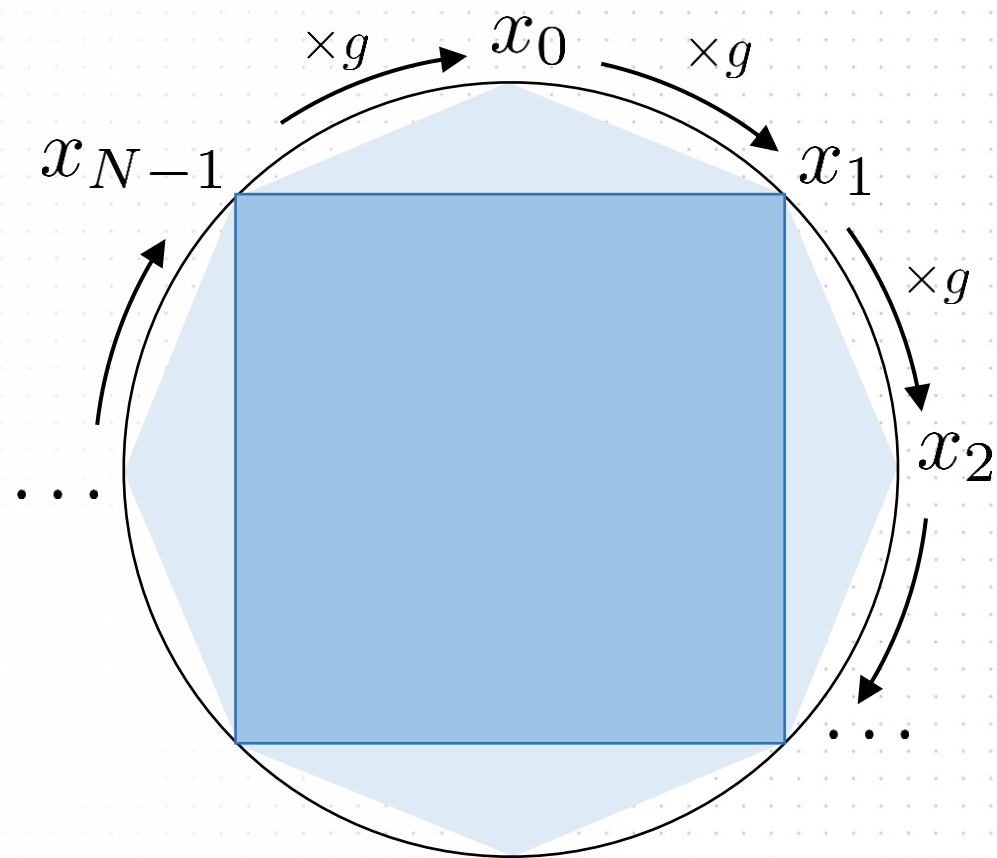
$$\mathbf{c} = \text{iFFT}(\mathbf{p})$$

# FAST FOURIER TRANSFORM to EVALUATE POLYNOMIALS



Three Sigma

$$x \in \mathbb{G}$$



# HANDS-ON!

Files:  
Base.ipynb  
FFT for function evaluation.ipynb



<https://github.com/tiagomartins-threesigma/SINFO2023FRI4STARKs>



Section Break

Let's FRI

# DEGREE RESPECTING PROJECTION

$$\begin{aligned}\mathbf{c} &= [c_0, & c_1, & c_2, & c_3, & \dots, & c_{N-2}, & c_{N-1}] \\ \mathbf{c}_{\text{even}} &= [c_0, & & c_2, & & \dots, & c_{N-2} & ] \\ \mathbf{c}_{\text{odd}} &= [ & c_1, & & c_3, & \dots, & & c_{N-1} ]\end{aligned}$$

$$\text{len}(\mathbf{c}) = N$$

$$\text{len}(\mathbf{c}_{\text{even}}) = N/2$$

$$\text{len}(\mathbf{c}_{\text{odd}}) = N/2$$

# DEGREE RESPECTING PROJECTION

$$\begin{aligned}\mathbf{c} &= [c_0, & c_1, & c_2, & c_3, & \dots, & c_{N-2}, & c_{N-1}] \\ \mathbf{c}_{\text{even}} &= [c_0, & & c_2, & & \dots, & c_{N-2}] \\ \mathbf{c}_{\text{odd}} &= [ & c_1, & & c_3, & \dots, & & c_{N-1}] \\ \mathbf{c}_{\text{new}} &= [c_0 z_{\text{even}} + & c_1 z_{\text{odd}}, & c_2 z_{\text{even}} + & c_3 z_{\text{odd}}, & \dots, & c_{N-2} z_{\text{even}} + & c_{N-1} z_{\text{odd}}]\end{aligned}$$

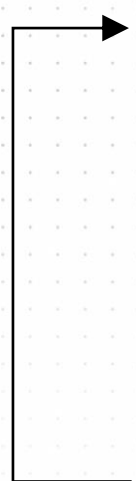
$$\text{len}(\mathbf{c}_{\text{new}}) = N/2$$

$$\mathbf{c}_{\text{new}} = \mathbf{c}_{\text{even}} z_{\text{even}} + \mathbf{c}_{\text{odd}} z_{\text{odd}}$$

```
c_new = c[::2]*z_even + c[1::2]*z_odd
```

# FRI COMMIT PHASE

For each FRI layer  $i \geq 0$  such that  $d_i > 1$ :

- 
1. the verifier selects random  $z_{i,\text{even}}$  and  $z_{i,\text{odd}}$ ,
  2. the prover defines  $p_{i+1}(x)$  through the degree respecting projection
$$\mathbf{c}_{i+1} = \mathbf{c}_{i,\text{even}} z_{i,\text{even}} + \mathbf{c}_{i,\text{odd}} z_{i,\text{odd}},$$
  3. the prover commits to the polynomial evaluation
$$\mathbf{p}_{i+1} = \text{FFT}(\mathbf{c}_{i+1}),$$
  4. the degree bound reduces as  $d_{i+1} = d_i/2$ .

At the end, the prover commits to a constant  $C$  allegedly equal to the last polynomial.

# HANDS-ON!

Files:  
FRI Commit.ipynb



<https://github.com/tiagomartins-threesigma/SINFO2023FRI4STARKs>



# DRP wo/ COEFFICIENTS

$$p_i(x) = \sum_{n=0}^{N-1} c_{i,n} x^n \quad p_i(-x) = \sum_{n=0}^{N-1} (-1)^n c_{i,n} x^n$$

$$p_{i+1}(x) = \sum_{n=0}^{N/2-1} c_{i+1,n} x^n \quad \text{where} \quad \mathbf{c}_{i+1} = \mathbf{c}_{i,\text{even}} z_{i,\text{even}} + \mathbf{c}_{i,\text{odd}} z_{i,\text{odd}}$$

$$p_{i+1}(x^2) = \underbrace{\frac{p_i(x) + p_i(-x)}{2}}_{g(x^2)} z_{i,\text{even}} + \underbrace{\frac{p_i(x) - p_i(-x)}{2x}}_{h(x^2)} z_{i,\text{odd}}$$

# FRI QUERY PHASE

The verifier samples a random  $x$ . The verifier queries for  $p_0(x)$ , and, subsequently, for each FRI layer  $i \geq 0$  such that  $d_i > 1$ :

- 1. the verifier queries for  $p_i(-x)$  and  $p_{i+1}(x^2)$ ,
- 2. the verifier computes  $g(x^2)$  and  $h(x^2)$  from  $p_i(x)$  and  $p_i(-x)$ ,
- 3. the verifier checks whether  $p_{i+1}(x^2) = g(x^2) z_{i+1,\text{even}} + h(x^2) z_{i+1,\text{odd}}$ ,
- 4. set  $x \leftarrow x^2$ .

At the end, the verifier checks whether the last value equals the constant  $C$ .

# HANDS-ON!

Files:  
FRI Query.ipynb



<https://github.com/tiagomartins-threesigma/SINFO2023FRI4STARKs>



# Three Sigma

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