This session will be recorded

FAST REED-SOLOMON INTERACTIVE ORACLE PROOFS of PROXIMITY

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Venture builder firm focused on **blockchain engineering, research, and investment**.

Mission

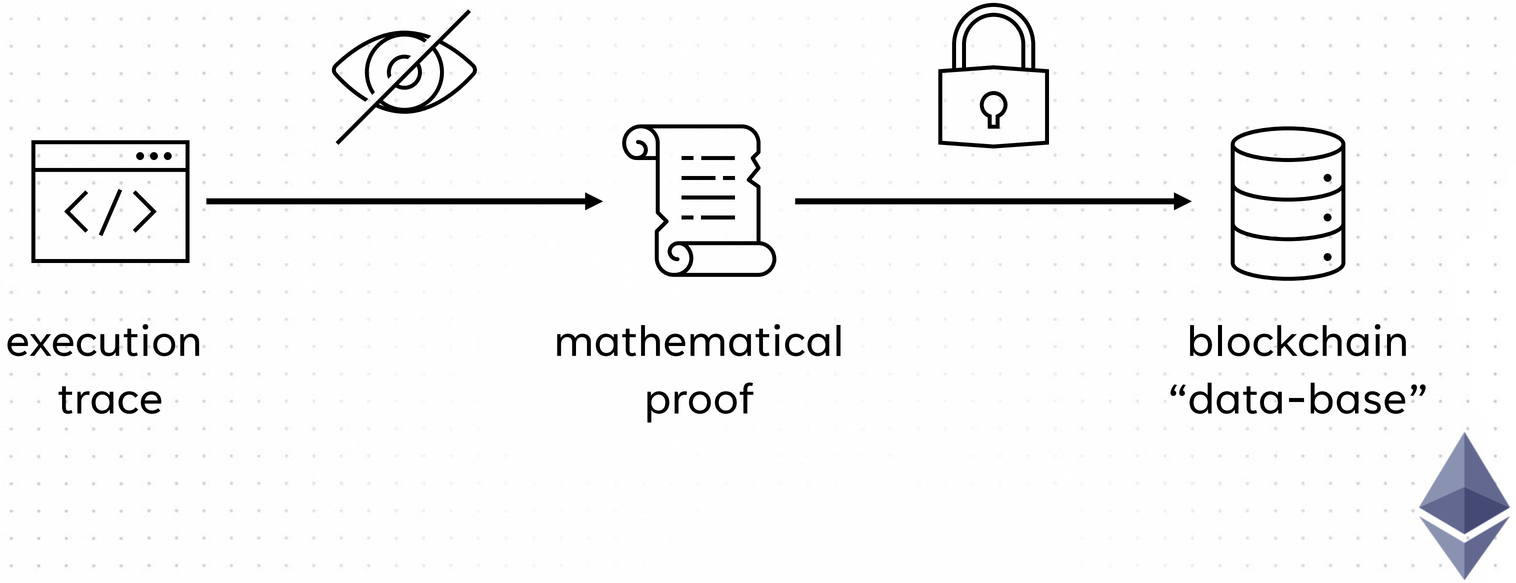
Advance the adoption of blockchain technology. Contribute for the healthy development of crypto/web3.

LIBRARIES

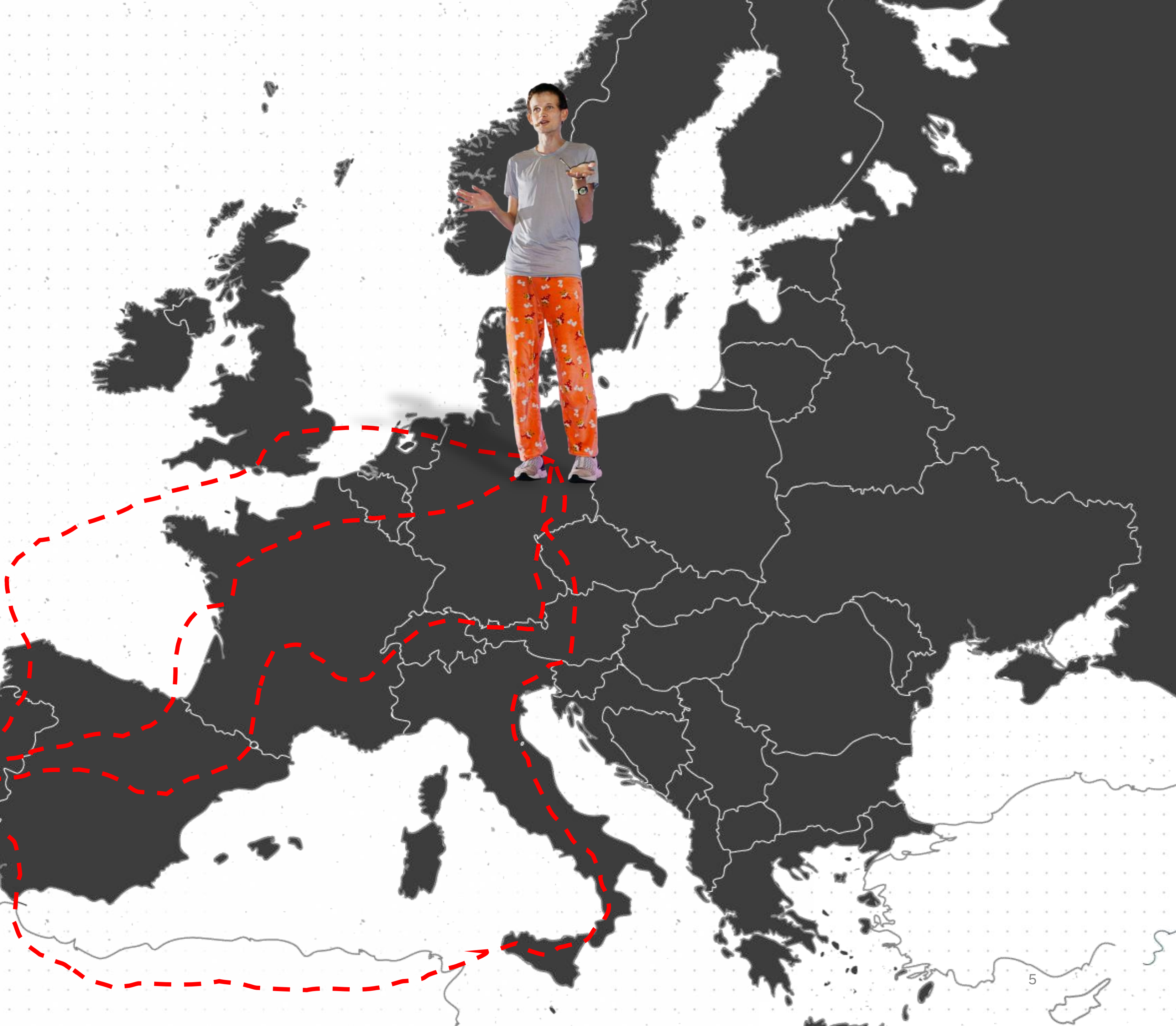
If you haven't, please install Python ≤ 3.10 and the libraries **NumPy** and **Galois**. Check the GitHub repository for more:

https://github.com/tiagomartins-threesigma/SINFO2023FRI4STARKs

ZERO KNOWLEDGE



Can Vitalik prove that he knows a path from Lisbon to Berlin without disclosing it?



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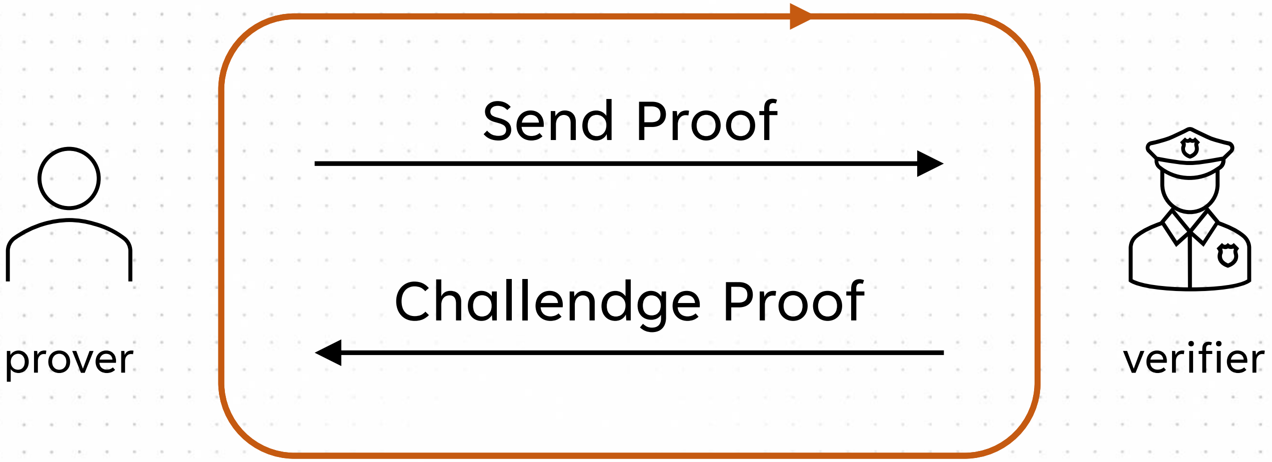
Zero **K**nowledge Proofs

enable **public consensus**

on **private data**!

INTERACTIVE PROOF



Until the Verifier is Satisfied

Scalable Transparent ARguments of Knowledge

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Vitalik must fulfil some challenges!

Vitalik eating icecream at the wall of Berlin

Vitalik in Brandenburger Tor

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Section Break

Diving into the Math

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BLOCKCHAIN SETTING

A non-necessarily honest prover wants to indirectly settle some **private** data in the blockchain with a ZK proof.

A verifier must check whether that data corresponds to honest computations and transactions.

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BLOCKCHAIN SETTING

STARKs operate over **low-degree polynomials**.

How?

Valid computations correspond to the evaluation table of low-degree polynomials.

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Section Break

Polynomials

PROBLEM EXAMPLE

Trace index k	Group element x	Trace polynomial $f(x)$
0	$x_0 = g^0$	3
1	$x_1 = g^1$	9
2	$x_2 = g^2$	81
3	$x_3 = g^3$	6561
4	$x_4 = g^4$	43046721
5	$x_5 = g^5$	1853020188851841

$$f(\underbrace{x_{k+1}}_{g\,x_k}) = (f(x_k))^2 \quad \text{for} \quad x_k \in \{x_0, x_1, x_2, x_3, x_4\}$$

$$\text{Example: } f(x_1) = 9 = (f(x_0))^2 = 3^2$$

PROBLEM EXAMPLE

$$f(\underbrace{x_{k+1}}_{g\,x_k}) = (f(x_k))^2 \quad \text{for} \quad x_k \in \{x_0, x_1, x_2, x_3, x_4\}$$

which implies that

$$f(g\,x) - (f(x))^2 = 0 \quad \text{for} \quad x \in \{x_0, x_1, x_2, x_3, x_4\}$$

which in turn implies that

$$p_0(x) = \frac{f(g\,x) - (f(x))^2}{(x - x_0)(x - x_1)(x - x_2)(x - x_3)(x - x_4)}$$

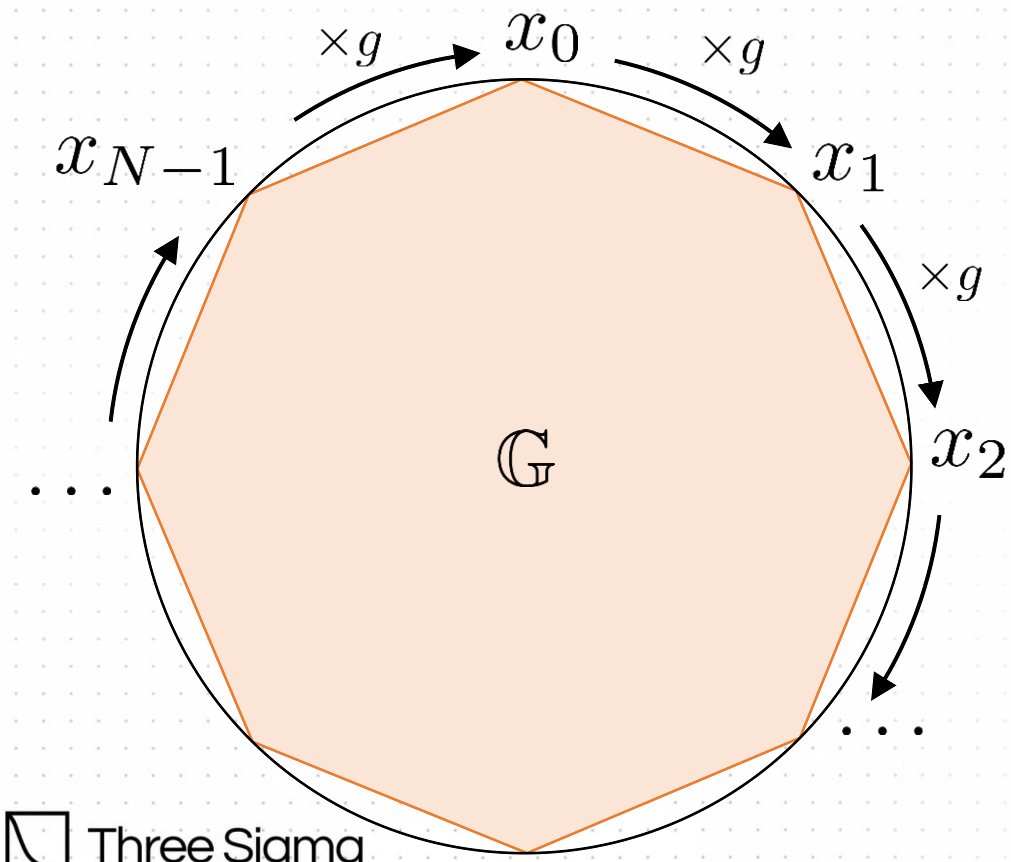
is a polynomial of degree $\deg(p_0) = 2\deg(f) - 5$.

POLYNOMIAL

$$p(x) = \sum_{n=0}^{N-1} c_n x^n$$

$$\underbrace{\begin{bmatrix} p(x_0) \\ p(x_1) \\ \vdots \\ p(x_{N-1}) \end{bmatrix}}_{\mathbf{p}} = \begin{bmatrix} 1 & x_0 & \dots & x_0^{N-1} \\ 1 & x_1 & \dots & x_1^{N-1} \\ \vdots & \vdots & & \vdots \\ 1 & x_{N-1} & \dots & x_{N-1}^{N-1} \end{bmatrix} \underbrace{\begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_{N-1} \end{bmatrix}}_{\mathbf{c}}$$

FAST FOURIER TRANSFORM TO EVALUATE POLYNOMIALS



$x \in \mathbb{G}$

$\mathbf{p} = \mathrm{FFT}(\mathbf{c})$

$\mathbf{c} = \mathrm{iFFT}(\mathbf{p})$

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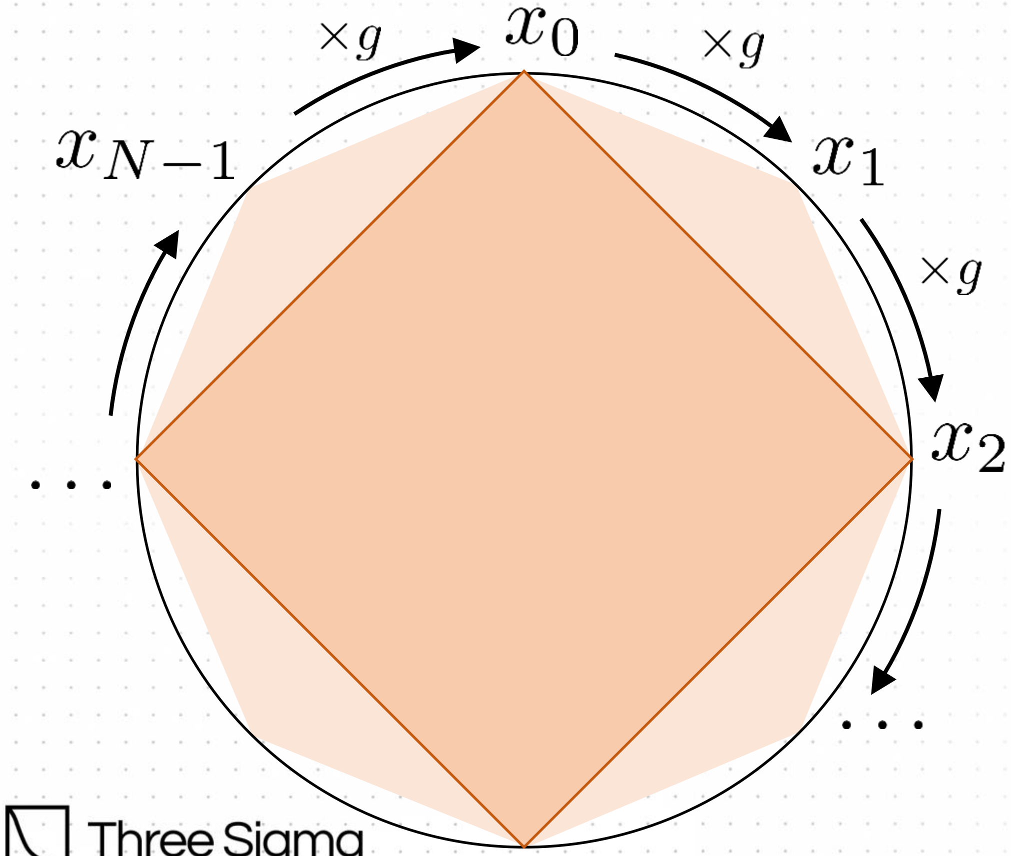
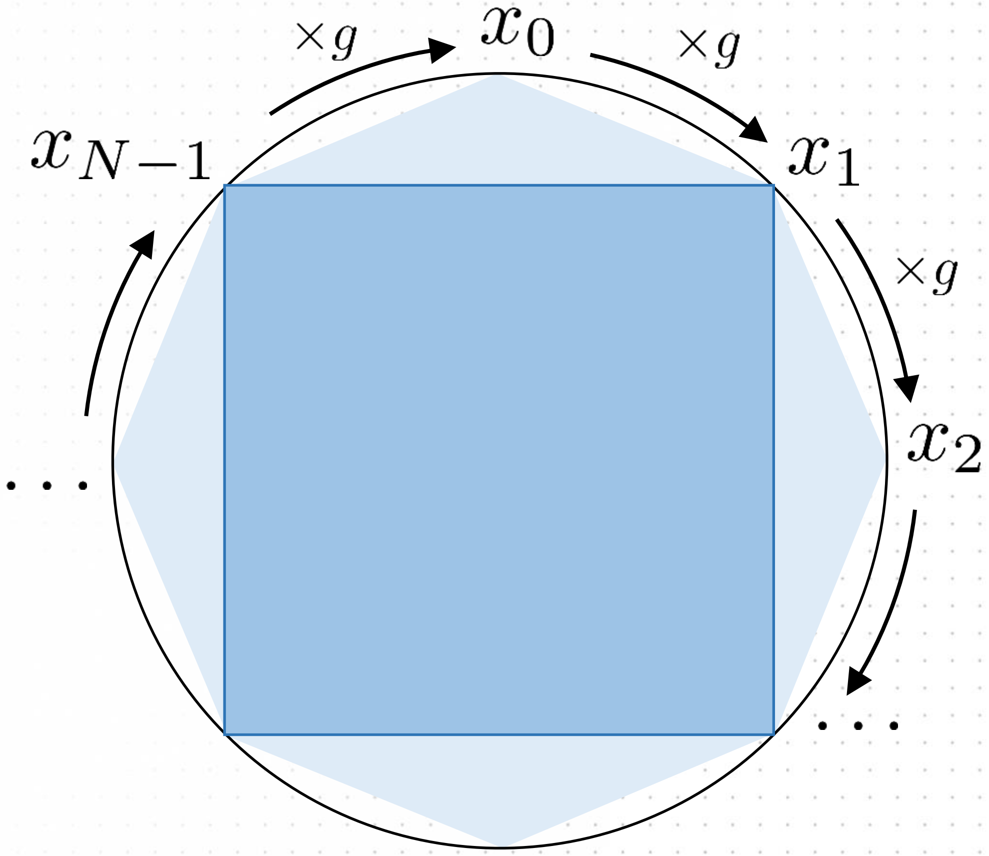
FAST FOURIER TRANSFORM to EVALUATE POLYNOMIALS



$x \in \mathbb{G}$



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HANDS-ON!

Files:
Base.ipynb
FFT for function evaluation.ipynb

https://github.com/tiagomartins-threesigma/SINFO2023FRI4STARKs

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Section Break

Let's FRI

DEGREE RESPECTING PROJECTION

$$
\begin{array}{rlllllll}
\mathbf{c} = & [c_0, & c_1, & c_2, & c_3, & \ldots, & c_{N-2}, & c_{N-1} \quad] \\
\mathbf{c}_{\text{even}} = & [c_0, & & c_2, & & \ldots, & c_{N-2} & \quad] \\
\mathbf{c}_{\text{odd}} = & [& c_1, & & c_3, & \ldots, & & c_{N-1} \quad]
\end{array}
$$

$$
\begin{aligned}
\operatorname{len}(\mathbf{c}) &= N \\
\operatorname{len}(\mathbf{c}_{\text{even}}) &= N/2 \\
\operatorname{len}(\mathbf{c}_{\text{odd}}) &= N/2
\end{aligned}
$$

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DEGREE RESPECTING PROJECTION

$$
\begin{array}{rl}
\mathbf{c} = & [c_0, \quad c_1, \quad c_2, \quad c_3, \quad \ldots, \quad c_{N-2}, \quad c_{N-1}] \\
\mathbf{c}_{\text{even}} = & [c_0, \quad c_2, \quad \ldots, \quad c_{N-2}] \\
\mathbf{c}_{\text{odd}} = & [c_1, \quad c_3, \quad \ldots, \quad c_{N-1}] \\
\mathbf{c}_{\text{new}} = & [c_0 z_{\text{even}} + c_1 z_{\text{odd}}, \quad c_2 z_{\text{even}} + c_3 z_{\text{odd}}, \quad \ldots, \quad c_{N-2} z_{\text{even}} + c_{N-1} z_{\text{odd}}]
\end{array}
$$

$$\text{len}(\mathbf{c}_{\text{new}}) = N/2$$

$$\mathbf{c}_{\text{new}} = \mathbf{c}_{\text{even}}\, z_{\text{even}} + \mathbf{c}_{\text{odd}}\, z_{\text{odd}}$$

```
c_new = c[::2]*z_even + c[1::2]*z_odd
```

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FRI COMMIT PHASE

For each FRI layer $i \geq 0$ such that $d_i > 1$:

1. the verifier selects random $z_{i,\text{even}}$ and $z_{i,\text{odd}}$,
2. the prover defines $p_{i+1}(x)$ through the degree respecting projection $\mathbf{c}_{i+1} = \mathbf{c}_{i,\text{even}}\, z_{i,\text{even}} + \mathbf{c}_{i,\text{odd}}\, z_{i,\text{odd}}$,
3. the prover commits to the polynomial evaluation $\mathbf{p}_{i+1} = \texttt{FFT}(\mathbf{c}_{i+1})$,
4. the degree bound reduces as $d_{i+1} = d_i/2$.

At the end, the prover commits to a constant C allegedly equal to the last polynomial.

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HANDS-ON!

Files:
FRI Commit.ipynb

https://github.com/tiagomartins-threesigma/SINFO2023FRI4STARKs

DRP wo/ COEFFICIENTS

$$p_i(x) = \sum_{n=0}^{N-1} c_{i,n}\, x^n \qquad p_i(-x) = \sum_{n=0}^{N-1} (-1)^n\, c_{i,n}\, x^n$$

$$p_{i+1}(x) = \sum_{n=0}^{N/2-1} c_{i+1,n}\, x^n \quad \text{where} \quad \mathbf{c}_{i+1} = \mathbf{c}_{i,\text{even}}\, z_{i,\text{even}} + \mathbf{c}_{i,\text{odd}}\, z_{i,\text{odd}}$$

$$p_{i+1}(x^2) = \underbrace{\frac{p_i(x) + p_i(-x)}{2}}_{g(x^2)} z_{i,\text{even}} + \underbrace{\frac{p_i(x) - p_i(-x)}{2x}}_{h(x^2)} z_{i,\text{odd}}$$

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FRI QUERY PHASE

The verifier samples a random x. The verifier queries for $p_0(x)$, and, subsequently, for each FRI layer $i \geq 0$ such that $d_i > 1$:

1. the verifier queries for $p_i(-x)$ and $p_{i+1}(x^2)$,
2. the verifier computes $g(x^2)$ and $h(x^2)$ from $p_i(x)$ and $p_i(-x)$,
3. the verifier checks whether $p_{i+1}(x^2) = g(x^2)\, z_{i+1,\text{even}} + h(x^2)\, z_{i+1,\text{odd}}$,
4. set $x \leftarrow x^2$.

At the end, the verifier checks whether the last value equals the constant C.

HANDS-ON!

Files:
FRI Query.ipynb

https://github.com/tiagomartins-threesigma/SINFO2023FRI4STARKs

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