Tarefa 01

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1) a) $27_{10} = ?_2$

$27 \,|\underline{2}$

$(1)\ 13 \,|\underline{2}$

$(1)\ 6 \,|\underline{2}$

$(0)\ 3 \,|\underline{2}$

$(1)\ 1 \,|\underline{2}$

$(1)\ 0 \leftarrow$ pode parar

$27_{10} = 11011_2$

b) $11011_2 = ?_{10}$

$1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0 =$

$16 + 8 + 0 + 2 + 1 = 27_{10}$

c) código em anexo, verificar impl de dec2bin/bin2dec

2) $b = 10,\ t = 4,\ e \in [-5, 5]$

a) $m = ?\ /\ M = ?$

$m = 0{,}1000 \times 10^{-5} = 0{,}000001$

$M = 0{,}9999 \times 10^{5} = 99.990$

b) $100.000 \in I$?

$100000 = 0{,}1 \times 10^{6}$, como $6 \notin [-5, 5]$, não é possível representá-lo nesse sistema

c) round(357,26)

valor arredondado $= 357{,}3 = 0{,}3573 \times 10^{3}$

d) trunc(357,26)

valor truncado $= 357{,}2 = 0{,}3572 \times 10^{3}$

e) $\varepsilon_a = ?$, $\varepsilon_r = ?$

$\varepsilon_a = |357{,}3 - 357{,}2| = 0{,}1 \Rightarrow \varepsilon_r = 0{,}1 / 357{,}3 = 0{,}2798769 \times 10^{-3}$

3) a) $f_x + g_x = 357{,}26$; $f_x = ?$; $g_x = ?$

$357{,}26 = 357{,}2 + 0{,}06 = \underbrace{0{,}3572 \times 10^{3}}_{f_x} + \underbrace{0{,}6 \times 10^{-1}}_{g_x}$

b) $\varepsilon_a = ?$; $\varepsilon_r = ?$; trunc

$0{,}35726 \times 10^{3} = 0{,}3572 \times 10^{3} + 0{,}6 \times 10^{-1} \Rightarrow x = \bar{x} + \varepsilon_a \Rightarrow$

$\varepsilon_a = 0{,}6 \times 10^{-1} \Rightarrow \quad x - \varepsilon_a \leqslant \bar{x} \leqslant x + \varepsilon_a \Rightarrow$

$0{,}35726 \times 10^{3} - 0{,}6 \times 10^{-1} \leqslant \bar{x} \leqslant 0{,}35726 \times 10^{3} + 0{,}6 \times 10^{-1} \Rightarrow$

$357{,}2 \leqslant \bar{x} \leqslant 357{,}32$

$$\varepsilon_r = \frac{\varepsilon_a}{x} = \frac{0{,}6 \times 10^{-1}}{0{,}35726 \times 10^{3}} = 0{,}1679449 \times 10^{-3}$$

$x(1 - \varepsilon_r) \leqslant \bar{x} \leqslant x(1 + \varepsilon_r) \Rightarrow$

$0{,}35726 \times 10^{3}(1 - 0{,}1679449 \times 10^{-3}) \leqslant \bar{x} \leqslant 0{,}35726 \times 10^{3} \cdot (1 + 0{,}1679449 \times 10^{-3})$

$\bar{x} = x(1 - \varepsilon_r)$

$0{,}3572 \times 10^{3} = 0{,}35726 \times 10^{3} \cdot (1 - 0{,}1679449 \times 10^{-3})$

c) $\varepsilon_a = ?$; $\varepsilon_r = ?$; round

$g_x \geqslant 0{,}5 \times 10^{-1} \Rightarrow f_x = 0{,}3573 \times 10^{3}$

$\varepsilon_a = 0{,}4 \times 10^{-1}$

$x - \varepsilon_a \leqslant \bar{x} \leqslant x + \varepsilon_a$

$0{,}35726 \times 10^{3} - 0{,}4 \times 10^{-1} \leqslant \bar{x} \leqslant 0{,}35726 \times 10^{3} + 0{,}4 \times 10^{-1}$

$\bar{x} = x + \varepsilon_a \Rightarrow \bar{x} = 0{,}35726 \times 10^{3} + 0{,}4 \times 10^{-1}$

$$\varepsilon_r = \frac{\varepsilon_a}{x} = \frac{0{,}04}{357{,}26} = 0{,}1119633 \times 10^{-3}$$

$x(1 - \varepsilon_r) \leqslant \bar{x} \leqslant x(1 + \varepsilon_r)$

$0{,}35726 \times 10^{3}(1 - 0{,}1119633 \times 10^{-3}) \leqslant \bar{x} \leqslant 0{,}35726 \times 10^{3} \cdot (1 + 0{,}1119633 \times 10^{-3})$

$\bar{x} = x(1 + \varepsilon_r) \Rightarrow \bar{x} = 0{,}35726 \times 10^{3}(1 + 0{,}1119633 \times 10^{-3})$

4) a) $u = (m+n)w/\sigma$, round, $\varepsilon_{rk} = 0{,}5 \times 10^{-t+1}$, $\varepsilon_r = ?$

i) $m+n$

$$\varepsilon_{r\,x+y} = \varepsilon_{rx}\left(\frac{\bar{x}}{\bar{x}+\bar{y}}\right) + \varepsilon_{ry}\left(\frac{\bar{y}}{\bar{x}+\bar{y}}\right)$$

Mas $\varepsilon_{rx} = \varepsilon_{ry} = \varepsilon_r$

$$\varepsilon_{r\,m+n} = \varepsilon_r\left(\frac{m}{m+n}\right) + \varepsilon_r\left(\frac{n}{m+n}\right)$$

$$\varepsilon_{r\,m+n} = \varepsilon_r\left[\frac{m}{m+n} + \frac{n}{m+n}\right]$$

$$\varepsilon_{r\,m+n} = \varepsilon_r = 0{,}5 \times 10^{-t+1}$$

ii) $(m+n)w$

$$\varepsilon_{rxy} = \varepsilon_{rx} + \varepsilon_{ry} = 0{,}5 \cdot 10^{-t+1} + 0{,}5 \cdot 10^{-t+1} = 10^{-t+1}$$

iii) u

$$\varepsilon_{rx/y} = \varepsilon_{rx} - \varepsilon_{ry} = 10^{-t+1} - 0{,}5 \cdot 10^{-t+1} = 0{,}5 \cdot 10^{-t+1}$$

iv) avaliando os erros residuais,

$3 \times 0{,}5 \cdot 10^{-t+1}$

v) $\varepsilon_r = \varepsilon_{r,\text{parcelas}} + \varepsilon_{r,\text{residuais}}$

$\varepsilon_r = 0{,}5 \times 10^{-t+1} + 3 \cdot 0{,}5 \times 10^{-t+1}$

$$\boxed{\varepsilon_r = 2 \times 10^{-t+1}}$$

b) Se os números são representados exatamente, $\nexists\, \varepsilon_{r,\text{parcela}}$, então $\varepsilon_r = \varepsilon_{r,\text{residual}}$. Para truncamento, temos $\varepsilon_{r,\text{residual},op} = 10^{-t+1}$,

$\varepsilon_{r,\text{residual}} = \boxed{3 \cdot 10^{-t+1} = \varepsilon_r}$

c) $u = (m+n)w/\sigma$

$u = (10+20) \cdot 30/40$

$u = 22{,}5$

Considerando $t=4$, da questão anterior, $10^{-t+1} = 10^{-3}$

i) $22{,}5 \times 2 \cdot 10^{-3} = 0{,}045 \Rightarrow \boxed{\varepsilon_r \leq 0{,}045}$

ii) $22{,}5 \times 3 \cdot 10^{-3} = 0{,}0675 \Rightarrow \boxed{\varepsilon_r \leq 0{,}0675}$