Toolkit for SOLEs in Maths

Introduction

During this research, one of the things I stumbled was the teacher's difficulty in making questions, (e.g. a teacher stated as a limit for SOLEs: *"[to make] the right question in order to get what the students need"*). Most questions made before this research on Maths were focused on direct questions for concepts or applications ('What/Who…?'). These are good questions for verbalization of mathematical concepts, and probably for children whom searching information or mathematical skills are still low.

However, if empathy and novel situations drive learning (Papert, 1993; Davis, 1996; Piaget and Cook, 1952; Inamdar and Kulkarni, 2007), the deep and long conversations come from the empathy/curiosity children feel towards the question. If the question was not made by the children and beforehand the reasoning/curiosity are not theirs, the question's design should attract children into these sessions.

Besides some 'Big Question examples' in some of Dr. Sugata Mitra's articles and in the toolkit (Mitra, 2013), there is not much guidance for making them. This is worse for Maths, as there are almost no examples.

For these reasons, I wrote this toolkit based on the research of my dissertation. I hope it may be of assistance.

What are SOLEs?

SOLEs acronym stands for Self-Organized Learning Environments. It is a model of learning in which students' research and self-organize in groups next to computers with internet access (Mitra, 2013) researching on 'big questions', with minimal facilitator support. A big question is, as defined by in the School-in-the-Cloud (Mitra, 2014a) website:

> *Big questions are the ones that don't have an easy answer. They are often open and difficult; they may even be unanswerable. The aim of them is to encourage deep and long conversations, rather than finding easy answers.*

This pedagogy aims to look at how children may learn collaboratively and to think for themselves, anywhere around the globe, through the Internet and the power of big questions (Mitra, 2014b). For it, the facilitator must not teach or interfere during the students' exploration phase, however present either on site or online.

These sessions are divided in 3 parts (Mitra, 2013):

- Question: Before giving the question, simple rules are given to ensure students are aware of the activity they are undertaking: students are free to…
    - Use all materials at their disposal (e.g. computers, internet, paper, pens, pencils, whiteboards, etc.);
    - Talk and discuss with each other;
    - Make their own groups;
    - Walk around;

    - Look at what other groups are doing, discuss with them and incorporate what they have learned back into their own group;
    - Switch groups;
    - In the end, every group must present their work;

    (Optional) A student 'police' may be selected to maintain law and order during the session.

- Investigation: During this time based on the rules previously introduced, students self-organize in groups (of usually 3-5) around a computer and research for 30-90 minutes, usually without facilitator's interference. In the end, they should have produce a short report of their work.

- Review/Conclusion/Discussion[1]: Students' groups present their answers in a conference setting, in which arguments and counter-arguments are presented. The facilitator listens and praises their work. In the end, he discusses with the students the different points made and summarises the session. He should also try to[2]:
    - Find connections and disagreements between students' groups
    - Ask how students have felt during the session, and what can be done better next time

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[1] Throughout the paper I use these three terms for the last stage.

[2] If a facilitator tries to do all these points at once, a great amount of time is spent and students may lose their focus, and also the sense of fun and engagement. Additionally, the complexity of this review naturally varies with the children's age and how they are used to do SOLEs.

- Ask how the investigation process was; what they have looked into (e.g. websites, other groups' work) and what they have discussed with other groups.
- Discuss about the new 'difficult' words and what do they mean?
- See what other questions came up throughout the whole session
- Ask them another big question to leave them wondering about the subject more

The setting and the review/conclusion are of incredible importance. Papert (1993) stated children thinking about their own thinking stimulates genuine development, and the teacher/facilitator intervention guides them in the process (Dolan *et al.*, 2011) and praises their progress (Mitra, 2013)[3].

[3] Papert (1993) observed this last part of discussing with students about their "thinking about thinking about something" it is often unexplored in schools, as it is not integrated within the school curriculum.

Motivation to use SOLEs in Maths in your classroom

According to an enactivist perspective, the way a child learns mathematics is deeply connected with the context that learning occurs, and one is inseparable of the other (Davis, 1996).

SOLEs offer other dynamic features that no other pedagogy yet does: continuous possibilities of collaboration, information search and retrieval, autonomy, and adaptability of a whole group of children towards a unique common purpose or goal (answering the 'Big Question') (Dolan *et al.*, 2013).

As in Maths, SOLEs always start with an inquiry and many ideas are imagined and thought. During these sessions, children build false theories as they reject them, fold back to learn previous concepts and go back again, instead of just doing the right answer. It also does not look for a specific notation, thus children either create their notation or they learn how to read mathematical reasoning from others online and adapt their thinking.

Thus, if the focus is on doing Maths, SOLEs may change the way maths is researched, learned and explored. Currently, most math exercises are designed either the way mathematicians usually imagine it (simple problems with powerful ideas) [what I called in my dissertation: 'the Lockhart's (2009) way'], or the way schools usually give math problems (step-by-step).

However, if children are not in the mood for simple imaginary things or applying directly formulas initially, it will be hard to inspire them to do any of these things. On both cases, maths may lack personal meaning and the spectrum of students is reduced (Papert, 1993). Unfortunately, as Dr. Droujkova in her interview stated, due to the scope of the maths *curriculum* (and other mathematical fields) it is very hard for children to learn all by themselves without some external guidance.

Hence, a different approach should be looked at. Papert (1993) throughout his work advocated the importance of computers to debug mathematical thinking, by letting children program meaningful things for them. In addition, Dr. Hamilton and Dr. Droujkova work (MathPickle (2013) and Natural Maths (2014), respectively) involve children in mathematical problems with hidden powerful ideas through games and activities adapt to the children's language.

SOLEs and the way how 'Big Questions' are designed, can be another challenging alternative, adapted for children and with powerful mathematical ideas. As in Papert's work (1993), debugging/self-correcting of the maths occurs, due to the dynamic features of SOLEs in which they are based on. And as in Hamilton's and Droujkova's work, the facilitator can start the sessions with a story, game, or funny problem adapted to the children.

An advantage to these other approaches the facilitator observes first-hand the evolution of his group of children. In after-sessions the facilitator can continue to search the themes explored, by looking to mathematically *simplifying* them (e.g. in the Lockhart's way) and/or applying to different contexts, and/or incorporating new concepts from the *curriculum* or from the children's curiosity.

In sum, the flexibility that SOLEs provide in Maths may challenge the children's understanding of mathematical concepts and their ability to apply them in a new manner that stimulates their perseverance and curiosity.

Types of Big Questions for SOLEs in Maths

I would argue for now there are 6 types of questions that can be applied in SOLEs:

(1) **Direct mathematical questions for concepts or applications** (‘What/Who…?’) – These are good questions for verbalisation of mathematical concepts, and probably for children whom searching information or mathematical skills are still low, as these questions will likely have the answer available online. From these type of questions, especially with older students, it is possible they may only write the information displayed without critically thinking what they are searching. However, in the discussion phase, with good probing questions, new activities may emerge that stimulate new learning and new activities.

(2) **“Big questions”** that do not look related to mathematical concepts (e.g. How does an iPad find where it is? How can you measure a mountain?) – From these questions children wonder and they are ready to engage. In after-sessions children may engage with questions from type (1) and (3).

(3) **Why…? Questions** – to get mathematical proofs and probe on specific topics.

(4) **Think like a machine:** Inspired in Papert (1993) and Resnick’s (2003) work, this type of questions help children learn ecological strategies to approach exercises. Examples in the last chapter.

(5) **Mathematical ‘Big problems’** (e.g. “Can you draw a straight line on a circular object?’ / ‘How many rolls do we need to wrap up Mars?’). These questions can be Mathematical unsolved problems (e.g. MathPickle exercises (2013)), open problems (e.g. Fermi problems, tessellating shapes…), or problems adapted to children with powerful mathematical ideas, in a naïve language.

In the latter, the goal is for children to play with powerful mathematical ideas. In these questions (and/or):

a. they do not know the concepts exist yet;

b. it would take a long time to be taught in a structured lesson

c. Or perhaps they have learned, but they do not yet grasp the applicabilities.

You can start an exercise from a real world example, something more absurd (e.g. 'How many toilet paper rolls would we need to wrap up Mars?'), or a really difficult math problem (e.g. MathPickle (2013)). This exercise should have plenty of underlying concepts to explore and it should be adapted for a child mindset so children will likely engage. After the session, as a facilitator you pick up the concepts students explored or propose a bigger question.

E.g. from the case above: '*For any planet and toilet paper roll, I wonder if there is a way of doing this exercise immediately without all these values…', 'Does this work only for planets and toilet paper rolls? What if it is a wool hank?'*.

This way you might be introducing new mathematical concepts, and reinforcing those they are just learning.

(6) **Questions that children do:** The thing about questions is a student in these sessions stated: *"Wow, we enter here with a question and we leave with even more!"*. A suggestion would be to create a wonderwall of questions (e.g. Hocking's work, (2014)), on which students see the questions and make new ones, from which the teacher selects.

Note: Other ways of doing SOLEs include going ‘around the Internet’, by providing images or something else, also using other technologies/alternatives to explore maths (e.g. computer science, 3D printing…) and connect maths with other modules and teachers’ works. However, this option have its limits as new technology is developed for searching and finding information.

Interesting suggestions to add to your SOLE in Maths:

A. A “concepts wall” in which the concepts discovered in the students own words is fixed around the class, and in some points in the SOLE session. Inspired in (Dolan *et al.*, 2013) language wall;

B. Older students as SOLE mentors (in Australia) to introduce SOLEs to younger and new students, for them to get used and help them as facilitators (1 mentor per 4 unexperienced children).

C. Let students record themselves instructional videos (as a tutorial for other children), follow-up sessions using iPads and do ‘scavenger hunts’ (these activities were suggested by Paul Kenna and Brett Millot from Australia)

D. In the UK, groups of children are aiding teachers to incorporate SOLEs and help them in designing better questions

E. The use of 3D Printing as a maker-space, for children to tinker with shapes and in the making of objects without adult intervention (Krish, 2014).

In case of ‘trouble’

During the sessions

Despite all these alternatives, there might be a possibility that children cannot achieve the result on their own in one session. If they get stuck or start going in a wrong direction, these tricks displayed in the figure below from Bob Kaplan’s playingwithmath.org website, and questions like: ‘***Have you talked with other groups? Is there a way of proving that you are correct? Can you think of another way to verify your claims? Can you create a way that makes this systematic for all cases?’*** are great examples to let children continue to explore. The important thing is to try not to interfere during the research phase, as children are exploring maths on their own.

Becoming Invisible, by Bob Kaplan and friends

Students are used to following the teacher, and it’s hard to change that dynamic. Here are some responses you might try…

Holding your cards close:

- I don’t know, I’m just the secretary.
- What a good way to put it.
- Why does that work?
- What an interesting idea. Why?
- That’s great, but are you really sure about that? Is 19 really less than 18?
- Sounds good, sounds right, it could work, but how could you convince a Martian or a skeptic?
- This may not work, but it might!
- That’s a good point.
- That’s a good thing you’re doing.
- Ah!
- Hey, that’s terrific.
- This is great thinking, by the way.
- Your question really clarifies things, thanks.
- I’m in complete doubt – let’s work it out.
- Oh, nice idea.
- Why? I’m sure you’re right, I just don’t see it.
- The numbers 12 and 24 are both in the same family, so they’re both good guesses.
- You’ve found an economical way of thinking about it.
- You can guess - take a risk and be wrong. Sometimes it’s fun to be wrong.
- How were we thinking about it?

Thinking Prompts:

- What do you think is going on with this?
- Do you see what the previous speaker was saying?
- Can we make this simpler?
- Take a wild guess: 17? 31/2?
- What would a harder problem be?
- I have a terrible memory for these things, so I’m going to put them on the board.
- I’m bothered that this is an odd number.
- Wait, can I just check?
- Are these expressions the same? Anyone think no?
- I’m getting confused – we have too many examples up here.
- That’s an interesting discovery: you can’t have both of these at once, can you?
- I’m not convinced….
- Wait, you’re going too fast for me.
- What’s a way to be systematic in exploring this?
- Exactly. Give us the argument again - why?
- What stayed the same? What changed?
- Where did we start?
- What seems significant?
- How does this representation show our thinking?
- Can we generalize?
- How can we capture that thinking in writing?
- Can you come up and explain or walk through your thinking?
- Could we show it another way?

On the Design of ‘Big Questions’ for Maths

Dr. Droujkova suggested creating a point-scale or a list of check-points for teachers, students and others (e.g. online community) to evaluate themselves and their progress. These are some points, but future research should pinpoint the key-questions that SOLE designers’ should check quickly, before/after/during delivering a SOLE session and act accordingly:

- What is the type of question (concepts, probing, applications of maths, mathematical big problem)?
- Do you know the outcome of the activity?
- How much children know about the topic before the activity?
- Can the answer be reached easily online?
  - Will children immediately understand it?
- Does your ‘5 year-old inner child’ feel wonder about this question?
  - What about your colleagues?
- What are the children’s skill in information search and retrieval?
  - And working in SOLEs?
- Have children genuinely enjoyed the question?
- What is their feedback?

Examples of Designed SOLE sessions

Topic 1: Probabilities, Combinatory and Statistics

Session A:

Inspiration on - http://www.buzzle.com/articles/math-riddles-and-puzzles.html

Context: Every child individually answer - How can you add eight 8s to get the number 1,000 using only addition?

SOLE Question: I wonder how can I know all the possibilities for this exercise without counting every option? [A mathematician friend told me it is possible]

Session B: **Gauss**

Setting: A bright mathematician when we was a kid he discovered the result of adding every number from 1 to any integer in seconds.

SOLE Question Option A: How?

SOLE Question Option B: Who this person was and why is he still relevant today?

Topic 2: Surface Areas

SOLE Question: How many toilet paper rolls would we need to wrap up Mars?

After-session questions: Considering the amount of trees we cut per year, in how many years would we be able to do that (if all cut trees were used for toilet paper for this single purpose)?

Mathematical after-question: (Abstract) → For any planet and any toilet paper roll, how would you show everyone how to solve the exercise?

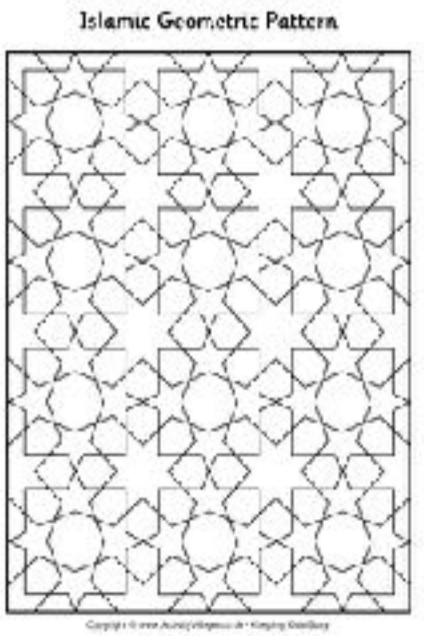
Topic 3: Catapulting things into space

Materials needed: Scale [to weight a person]

SOLE Question: How much strength would we need for this person to be catapulted against a satellite 18km away?

After-session questions: If this person did not die, could it become a satellite rotating around the Earth? How/Why?

Topic 4: Philosophical Questions – Taylor series



Context: This photo. Take students to look at big patterns, then small patterns, and the lines.

SOLE Question: What is the math behind these lines, then the connecting lines, then the patterns and so on?

Topic 5: Math Art – Focal Point and Leonardo da Vinci

Inspiration: https://www.youtube.com/watch?v=DFL_2L634Zw and

SOLE Question: How can you stretch a painting in a way that if you see it straight it looks weird but, from another perspective it looks normal? (inspired in Leonardo Da Vinci work in Annunciation) [excerpt from Youtube video shown – min: 4:56 to 5:15]

After-question: show video of a painting which the eyes follow the viewer. Could these be related? How/Why?

After-question N.2: How can we guarantee to draw this in a perfect angle?

Topic 6: Magic number

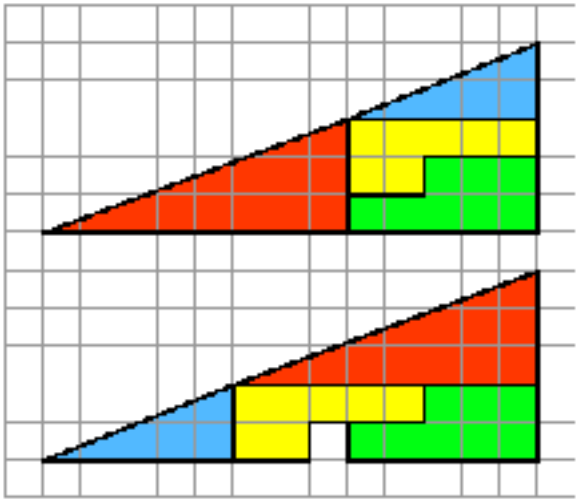
Each student: Write down their age. Multiply it by 1/5 of 100. Add on today's date (e.g. 2 if it's the 2nd of the month). Multiply by 20% of 25. Now add on their shoe size (if it's a half size round to a whole number). Finally subtract 5 times today's date.

SOLE Question: How does this pattern reoccurs?

Topic 7: Coffee Pot

SOLE Question: I wonder how can I count the number of coffee beans inside this pot, without spending 'all my life doing it'?

Topic 8: Illusion



SOLE Question: How does this happen?

Topic 9: 2 sessions/2 classes

Up to the facilitator: Tell students about the 2nd session or not, to evaluate competition between classes or self-organised intrinsic/extrinsic learning motivation

Cryptography - Session 1

Setting: We want to send an important message into a satellite in space! But the Russians are very good at intercepting messages!

Mission: What do can we do to make our message only be understood by the people in the satellite and us?

End session: Decide best code in the class to be sent to the other class.

Session 2

Mission: Decipher the other class code.

Minecraft / Castles – Medieval times - Session 1

Materials: ruler, compass, paper, pens, all sorts of things, paper, protactors. Give paper with the text/images. **Or: Minecraft/ 3D Printing (right now it is a bit expensive but it would be super interesting)**

Setting: You are a town growing into a city. Your opponents know that the city is getting a lot of money and they want to take it. A spy gets this information and informs you need to build protection fast. The town is in a plain, its square and of the size 8 acres. With this protection your city still needs to function well and trade needs to be done with other people in the kingdom. You have enough bricks to surround the town 150 times. The height of the bricks is 3 1/8’’ (79mm) / converted measure for Minecraft. Cannons were not invented yet.

Game: Build the best fort possible in this location.

Session 2

Evaluation: A spy steals a blueprint. Find the weak spots in the castle. How would you break in? [No cannons]

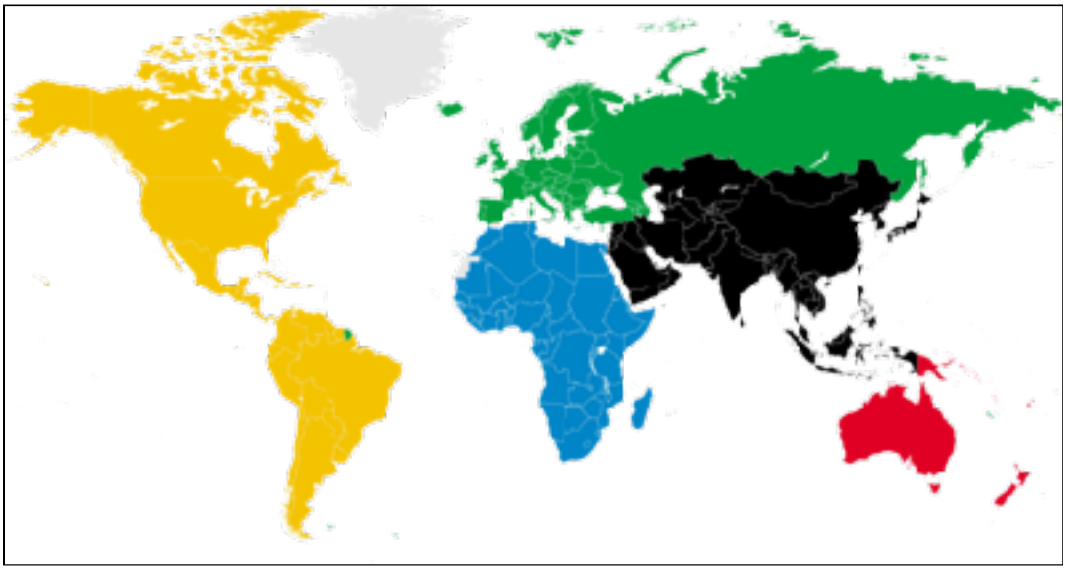
Topic 10: Random things

SOLE Question: How does a robot learn how to walk?

SOLE Question(s): How would you teach a computer/calculator how to add / subtract / multiply / divide?

SOLE Question: If the sun is at the same distance of the earth during the day, how come our shades change and the sun changes colours?

SOLE question: Both these maps are considered good. But shouldn't just there be one?



Question: Why do sports ball, are all round but have different shapes on the outside?

Question: What is the fastest way to calculate these sort of calculations in your head?

15 + 98 + 25 + 10 +72 + 53 + 32 + 86 + 62 + 97 + 40 + 52 +12 + 63 +42 + 19

476 * 32

Topic 11: Mathematical Concepts

Curves

Story: Long long time ago, people did not know the formulas of area and perimeter of circles that you know (they actually did not know how to do what you do).

SOLE question: I wonder… how did they measure curve things?

Triangles in semi-circles

Story: Imagine a triangle… Now imagine the Moon squeezed into an enormous of paper and cut in half.

SOLE question: How many ways can we have a triangle with at least two perpendicular sides and the 3 'tips' are touching the outside of the Moon?

After-question (no Internet): If I draw this half-moon (semi-circle), and I decide the semi-circle side is the also the triangle. Why if I choose any other point outside of the moon, the other two sides of the triangle are going to be perpendicular?

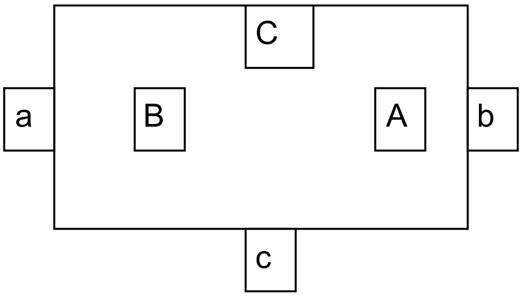
This is exactly the same as: Why a triangle in a semicircle has always a right angle?

But if you google this, the answer appears right in front of you.

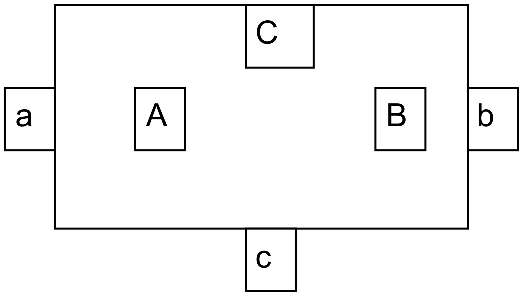
Topology 1

Material: Puzzle B (at least)

Puzzle B



Puzzle A



Option A:

- **(Optional) Stage 1 - no computers:** Is it possible to link A to a, B to b, C to c, without any line crossing each other?
- **Stage 1 or 2 – SOLE Question:** A mathematician can solve this exercise [explained in Stage 1], without having ever looked at it and doing any lines, in 2 seconds. How?

Option B [prefer option A]: SOLE Question: Why for mathematicians, by being able to solve puzzle A it means they can solve puzzle B, without having to do it? (link A to a, B to b, C to c)

Sieve of Eratosthenes

http://upload.wikimedia.org/wikipedia/commons/b/b9/Sieve_of_Eratosthenes_animation.gif

[Remove the word prime numbers from it]

SOLE question: I wonder… how would it be for 213?

After-questions: What is this exactly doing? What is a prime number? Do they keep going on forever? **How can I apply this to any number in the fastest way possible?** Is this the best way to find all prime numbers?

Topic 12: Form your own

I suggest a game. See this video, it has some really powerful mathematical ideas (https://www.youtube.com/watch?v=V1gT2f3Fe44) (Lockhart, 2012). Based on this and what I wrote, transform his questions into SOLEs.

Notes

It is important to note the making of questions has not been effectively explored.

Despite good results on temporary students' motivation and engagement in mathematical ideas, neither long-term nor after-session effects have been studied. These points need further research.

On another hand, it is possible that as more tools for maths education are developed and integrated throughout time, the better the children's ability to do, imagine and question mathematics may become.

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