

1. Data types. Arithmetic expressions. Input and output.

1.1

Write a program that reads a letter or other symbol from the keyboard and shows its ASCII code.

1.2

Write a program that reads 3 integer numbers from the keyboard and shows on the screen their mean value and the difference between each number and the mean value. The dialog with the user must be similar to the following:

```
Please, input 3 integer numbers
A ? 23
B ? 47
C ? 30
mean = 33.333
A-mean = -10.333
B-mean = 13.667
C-mean = -3.333
```

Implement different versions of the program: using different types for the variables used to store **A**, **B**, **C** and **mean**. Interpret the results.

1.3

The mass of a sphere is given by the expression $M = 4/3(\rho\pi r^3)$ where **M**, **ρ** and **r** are, respectively, the mass of the sphere, the specific mass of the material from which it is made, and its radius. Write a program that, given the values of **ρ** and **r**, determines the value of **M**. The user must be informed about the units used to represent all the values: Kg/m³, m and Kg for **ρ** , **r** and **M**, respectively. Use a constant to represent the value of π .

1.4

The solution to a system of linear equations in two variables (**x** e **y**)

$$\begin{aligned} a \cdot x + b \cdot y &= c \\ d \cdot x + e \cdot y &= f \end{aligned}$$

is given by

$$\begin{aligned} x &= (c \cdot e - b \cdot f) / (a \cdot e - b \cdot d) \\ y &= (a \cdot f - c \cdot d) / (a \cdot e - b \cdot d) \end{aligned}$$

Write a program that reads the values of **a**, **b**, **c**, **d**, **e** e **f** and determines the solution of the corresponding system of equations. Consider only the cases when there is a solution (it is not an impossible or inconsistent system).

1.5

a) Write a program that reads two times, expressed in hours, minutes and seconds, and determines their sum. The dialog with the user must be similar to the following:

```
Time1 (hours minutes seconds) ? 10 35 50
Time2 (hours minutes seconds) ? 15 59 30
Time1 + Time2 = 1 day, 2 hours, 35 minutes and 20 seconds
```

b) Modify the program so that the user must input a separator between hours, minutes and seconds (ex: 10:35:50). Although the separator usually used is ':', consider that any separator is valid.

1.6

The area of a triangle can be determined using the Heron's formula: $area = \sqrt{s(s-a)(s-b)(s-c)}$ where **s**, **a**, **b** and **c** are, respectively, the semi-perimeter and the length of the 3 sides. Write a program that reads the coordinates of the 3 vertices of a triangle and calculates the area of the triangle, using that formula. Remember that the distance between 2 points whose coordinates are (**x1**,**y1**) e (**x2**,**y2**) is given by $d = \sqrt{(x2-x1)^2 + (y2-y1)^2}$.

2. Control structures: selection and repetition.

2.1.

Solve again problem 1.4 (solution of a system of linear equations in two variables), so that when the system is impossible or inconsistent (a system having infinite solutions) a message is shown to the user: "impossible system" or "inconsistent system".

2.2.

- a) Write a program that reads 3 numbers from the keyboard and determines the largest and the smallest number.
- b) Write a program that reads 3 numbers from the keyboard and writes them on the screen, in descending order.
- c) Write a program that reads 3 positive numbers from the keyboard and determines if they can represent the length of the 3 sides of a triangle (tip: it is not possible to build a triangle if the sum of the 2 smallest lengths is smaller than the largest length). If any of the numbers is not positive the program must show an error message.

2.3.

Write a program that reads 2 integer numbers from the keyboard and tests whether their sum would produce overflow (the result would be greater than **INT_MAX**) or underflow (the result would be lower than **INT_MIN**). If this happens the program must show the message "sum overflow" or "sum underflow", otherwise it should show the result of the sum.

2.4.

The cost of transporting a certain merchandise is determined, depending on its weight, as follows: if the weight is less or equal to 500 grams the cost is 5 euros; if the weight is between 501 grams and 1000 grams, inclusive, the cost is equal to 5 euros plus 1.5 euros for each additional 100 grams or fraction above 500 grams; if the weight exceeds 1000 grams, the cost is 12.5 euros plus 5 euros for each additional 250 grams or fraction above 1000 grams. Write a program that, given the weight of a certain merchandise, determines the cost of its transportation.

2.5.

Write a program to determine the roots of a quadratic equation $Ax^2+Bx+C=0$, the coefficients **A**, **B** and **C** being provided by the user. The program must indicate whether the equation has 2 different real roots, 2 equal real roots or 2 complex roots, and the respective root values, with 3 decimal places.

Example:

```
Solution of Ax^2 + Bx + C = 0
Insert the coefficients (A B C): 2.5 -1 16
The equation has 2 complex roots: 0.200+2.522i and 0.200-2.522i
```

2.6.

Write a program to determine and write the amount that a depositor can withdraw from the bank, after **n** years of depositing an amount **q**, where **j**% is the annual interest rate. The values of **n**, **q** and **j** must be specified by the user. Assume that interest at the end of each year is accrued to the deposited amount.

2.7.

A number **n** is prime if it is divisible only by itself and by one.

- a) Write a program that reads a number from the keyboard and determines if it is prime. Note: it is not necessary to test all divisors in the range **[2..n]**; it is enough to test divisors until the integer part of the square root of **n**.
- b) Write a program that writes on the screen all the prime numbers lower than 1000.
- c) Write a program that writes on the screen the first 100 prime numbers.
- d) Write a program that determines the largest prime number that can be stored in a variable of type **unsigned long**.

2.8.

a) Write a program that displays on the screen a table of sines, cosines and tangents of the angles in the range [0..90] degrees, with intervals of 15 degrees, as shown below (note the particular case of the last line, corresponding to the 90 degree angle).

ang	sin	cos	tan
0	0.000000	1.000000	0.000000
15	0.258819	0.965926	0.267949
30	0.500000	0.866025	0.577350
45	0.707107	0.707107	1.000000
60	0.866025	0.500000	1.732051
75	0.965926	0.258819	3.732051
90	1.000000	0.000000	infinite

b) Change the program you developed in a) so that the range limits and the interval of the value of the angles in the table can be specified by the user (for example, if the range is [0..1] and the increment is of 0.1 degrees, the table for the angles of 0, 0.1, 0.2,..., and 1 degree should be displayed).

2.9.

A palindrome is a word, number, phrase, or other sequence of characters which reads the same backward as forward. For example, the following numbers are palindromes: 12321, 555, 45554 and 11611.

a) Write a program that reads a 3-digit integer and determines whether or not it is a palindrome (suggestion: use the division and module operators to separate the integer into its digits).

b) Generalize the program in a) in order to treat unsigned integers with a greater number of digits. Note: do not use *arrays* or *vectors* to store the digits.

2.10.

Write a program that reads an integer and breaks it down into prime factors (example: $20 = 2 \times 2 \times 5$).

Notes:

- one way to solve this problem would be to start by dividing the number by the first prime number, 2, and continue dividing by 2 until you get non-zero remainder; then divide by the other prime numbers, 3, 5, 7, etc. until the only numbers left are prime numbers;

- an alternative way is to start by dividing the number by 2 and continue dividing by 2 until you get non-zero remainder; then do the same for 3, 4, 5, 6, 7, etc. until the dividend is equal to 1.

2.11.

Write a program to calculate the sum of the first n terms (n being input by the user) for each of the following series.

a) Series giving the value of the mathematical constant π :

$$4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \dots$$

b) Series giving the value of the mathematical constant e :

$$1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots$$

(Tip: Calculate each term from the immediately preceding term)

c) Series giving the value of e^{-x} (com x real positivo previamente definido):

$$1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$$

(Tip: Calculate each term from the immediately preceding term. Note: x must also be input by the user.)

2.12.

Repeat problem 2.11 so that the user can specify the precision with which he wants the result, that is, the maximum variation between the value of the sum of the series, between two consecutive iterations. Note that the variation can be either positive or negative.

2.13.

Write a program that reads a sequence of integer numbers, and determines and writes the sum, the mean, the standard deviation, the smallest and the largest of the numbers, in the following situations:

- a) the length of the sequence is previously indicated by the user;
- b) the end of the sequence is indicated by the value 0 (which is not considered to be part of the sequence); at the end, the program must indicate the length of the sequence;
- c) the end of the sequence is indicated when the user types end of input (**CTRL-Z** in Windows; **CTRL-D** in Linux).

2.14.

a) The square root of a number **n** can be calculated in an approximate way using the following algorithm, due to Heron of Alexandria:

- start from an initial estimate of the root value, **rq**;
- calculate a new estimate, **rqn**, using the formula: $rqn = (rq + n / rq) / 2$;
- repeat this calculation using the **rqn** value as a new estimate of **rq**.

The following table illustrates the evolution of the square root calculation for the number **n** = 20, based on an initial estimate **rq** = 1.

rq	rqn	rqn²	dif = n - rqn²
1	10.500000	110.250000	-90.250000
10.50000	6.202381	38.469532	-18.469532
6.202381	4.713475	22.216844	-2.216844
4.713475	4.478314	20.055300	-0.055300
4.478314	4.472140	20.000001	-0.000039

In general, this algorithm converges quickly to a correct solution, even when the initial estimate is poor. Note that the difference **dif** = **n** - **rqn²**, quickly evolves to zero, so one could use the value of **dif** as a stop criterion, that is, end the iterations when **dif** is less than a small number, **delta** (for example, **delta** = 0.00001). However, it is advisable to limit the number of iterations, repeating, in this case, the calculations until a value of **dif** less than **delta** is reached or a maximum number of iterations, **nMaxIter**, is reached.

Considering the presented example, if **delta** = 0.001 and **nMaxIter** = 3 the final result (**rqn**) would be the one calculated after the 3rd iteration because the maximum number of iterations specified has been reached, although the **delta** value is still greater than 0.001; but if **delta** = 0.001 and **nMaxIter** = 10, the calculation would end after the 5th iteration because in this iteration the absolute value of **dif** is less than 0.001.

Write a program that reads the values of **n**, **delta** and **nMaxIter** and that calculates the square root of **n** using the algorithm described previously. Always use 1 as initial estimate of **rq**.

b) Modify the program in a) in order to present the result with the same number of decimal places used in specifying the **delta** value; for that you should find an algorithm to determine the number of decimal places. Also show the value returned by the **sqrt()** function from the C library and compare the values returned by this function and Heron's algorithm.

2.15.

Write a program to test if the user knows the multiplication tables. The program should generate 2 random numbers (between 2 and 9), present them to the user, and ask for the result of multiplying those numbers. After reading the user's answer, it should present an appropriate message taking into account the correctness of the answer and the time it took the user to give it: if the answer is wrong, the user should receive the message "Very Bad", if the answer is correct and took less than 5 seconds, the user should receive the message "Good", if you answer is correct but took between 5 seconds and 10 seconds (inclusive) the user should receive the message "Satisfactory", otherwise the user should receive the message "Insufficient".

2.16.

Write a program to emulate the operation of a basic calculator. In addition to performing the 4 fundamental algebraic operations, addition, subtraction, multiplication and division, the calculator must have a memory where the value shown on the display can be saved, using the **M** command. It must also be possible to clear the contents of the memory, using the **MC** command, add or subtract the current value on the display to the memory, using the **M+** and **M-** commands, or show the contents of the memory on the display using the **MR** command. The command to clear the contents of the display is **C**.

3. Functions.

3.1.

Rewrite the program of problem **1.6**, for calculating the area of a triangle using the Heron formula, in order to use the following functions:

- **double area(double x1, double y1, double x2, double y2, double x3, double y3)** to calculate the area of a triangle whose vertices have coordinates (x1,y1), (x2,y2) and (x3,y3);
- **double distance(double x1, double y1, double x2, double y2)** to calculate the distance between two points whose coordinates are (x1,y1) and (x2,y2).

3.2.

Rewrite the programs of problem **2.7**, for determining prime numbers, in order to use a function **isPrime()**, that determines whether the number that it receives as parameter is prime or not. Choose suitable types for the parameter and the return value of the function.

3.3.

Rewrite the program of problem **2.14** in order to use a function to calculate the square root, using the method indicated in that problem. Note that this function must have 3 parameters: the value whose square root is to be calculated, the precision and the maximum number of iterations.

3.4.

In the C/C++ libraries, there is no function to round a decimal number to a specified number of decimal places. However, it is possible to achieve this using the **floor()** function. For example, to round the value of **x** to 2 decimal places), you can use the following operation:

$$\text{floor}(x * 100 + 0.5) / 100$$

Write a function whose prototype is

double round(double x, unsigned n)

that rounds a floating point number, **x**, to a given number of decimal places, **n**, returning the rounded value.

Develop a program for testing the function; it must ask the user for the values of **x** and **n** and show the result of **round(x,n)**.

3.5.

Euclid's iterative algorithm for determining the greatest common divisor (GCD) of two numbers is based on the following findings:

- if the two numbers are equal, the GCD is given by the value of either one;
- if one of the numbers is zero, the GCD is the other number;
- the GCD of two numbers is not changed if the smaller of the two numbers is subtracted from the larger; by repeatedly applying this rule until the two numbers are equal, the GCD is obtained.

Write a function which returns the GCD of two numbers which it receives as parameters. This function does not write anything on the screen. Implement two different versions of the function, having different prototypes: one in which the GCD is the return value of the function; another in which the GCD is returned through a parameter of the function.

3.6.

Write a function whose prototype is **time_t timeElapsed()** that returns the time (in seconds) that has elapsed since the first time it was called. For example, if the function is called 3 times, at 10:59:55, at 11:00:25 and at 11:00:45, the returned values should be, respectively, 0, 30 and 50. Suggestion: use a local static variable to keep the time of the first call.

3.7.

Write a function **bool readInt(int &x)** that tries to read a valid integer number from the keyboard. If the input is a valid number, it must be returned through parameter **x**, and the return value of the function must be **true**. If the input is invalid, the function must return **false**; in this case the value of **x** has no significance. In both cases, the input buffer must be cleaned before the function returns. Note: if the input contains other characters beyond those that make up a single integer number (ex: 122a or 123 45), it must be considered invalid.

3.8.

The C/C++ language does not have the type "fraction" nor, obviously, operators or functions to manipulate fractions. You must develop a set of functions that allow the manipulation of fractions, represented by independent variables that represent the numerator and denominator of each fraction (later you will use a **struct** to represent a fraction).

a) Write a function whose prototype is

bool readFraction(int &numerator, int &denominator)

that reads a fraction, written in the format **numerator/denominator** (ex: **2/3** or **5/12**). The return value of the function must be **true** if the values entered for the numerator and the denominator are valid, and the separator is '/', or false otherwise. In the latter case, the values returned, for the **numerator** and **denominator**, must be zero. Note: the function must not write anything to the screen; any input message must be written before the function is called.

b) Write a function whose prototype is

void reduceFraction(int &numerator, int &denominator)

which reduces the fraction whose **numerator** and **denominator** are passed as parameters, dividing them by their greatest common divisor. Suggestion: use the function developed in problem 3.5 to calculate the greatest common divisor of the **numerator** and **denominator**.

c) Write functions to perform the basic operations (addition, subtraction, multiplication and division) on fractions, presenting the result in reduced form. Choose suitable prototypes for these functions.

d) Write a program to test the developed functions.

3.9.

The final result of this problem will be a program to show, on the screen, the calendar of a given year. The development of this program will be done modularly, using a bottom-up approach.

a) A leap year is a year that meets any of the following conditions: it is divisible by 4 but not divisible by 100; however, years divisible by 400, despite being divisible by 100, are considered leap years (eg, the year 2000 was a leap year but the year 2100 will not be). Write a function that has as parameter an integer representing a year and returns a Boolean value, indicating whether the year is leap or not (true if it is and false if it is not). Write a program to test this function.

b) Write a function that has as parameters two integers, representing a month and a year, and returns the number of days of that month, in that year. Note that only the month of February has a variable number of days, depending on whether the year is leap or not.

c) In November 2004, Sohael Babwani published an article in the Mathematical Gazette in which he describes a formula for calculating the day of the week (Sunday, Monday, ...) corresponding to a certain date in the Gregorian calendar. The formula is as follows:

$$ds = \left(\left\lfloor \frac{5 \cdot a}{4} \right\rfloor + c + d - 2 \cdot (s \% 4) + 7 \right) \% 7$$

where

- $\lfloor \rfloor$ – operator that calculates the integer contained in its operand
- $\%$ – operator that calculates the remainder of integer division
- ds – day of week
- d – day of month
- m – month number (1-January, 2-February, ...)
- s – two first digits of the year (ex: if the year is 2010, s will take the value 20)
- a – two last digits of the year (ex: if the year is 2010, a will take the value 10)
- c – month code, given by the following table, where m represents the month number (1-January, 2-February, ...); note that the c depends on whether the year is leap or not but it only differs for the months of January and February.

month	m	c	
		leap	non-leap
January	1	6	0
February	2	2	3
March	3	3	3
April	4	6	6
May	5	1	1
June	6	4	4

month	m	c	
		leap	non-leap
July	7	6	6
August	8	2	2
September	9	5	5
October	10	0	0
November	11	3	3
December	12	5	5

The result, ds , should be interpreted as follows: 0 = Saturday, 1 = Sunday, 2 = Monday, etc.
For example, applying the formula to "1 of January of 2011" you get:

$$ds = \left(\left\lfloor \frac{5 \cdot 11}{4} \right\rfloor + 0 + 1 - 2 \cdot (20 \% 4) + 7 \right) \% 7 = 0$$

indicating that the corresponding day of the week is Saturday.

Write a function that has as parameters 3 integer numbers, representing a date (year, month, day), and that returns an integer indicating the corresponding day of the week. Write a program that reads a date and, using this function, writes the name of the corresponding day of the week (Sunday, Monday, ...).

d) Write a function that, using the function developed in the previous paragraph, shows on the screen the calendar of a month/year specified by the user, in a format similar to the following:

January/2011						
Sun	Mon	Tue	Wed	Thu	Fri	Sat
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

e) Finally, write a program that, using the previously developed functions and others that you consider necessary, shows on the screen the calendar of all months of a year indicated by the user.

3.10.

a) Write a function, **factorial_ite(unsigned int n)**, that determines the factorial of a number using an iterative algorithm. Declare the variable that will contain the result as being of type **unsigned long long**. Start by determining which is the largest integer that can be represented in such a variable. Also, determine which is the largest number whose factorial is less than that integer. Note: there is a constant, **ULLONG_MAX**, defined in **<climits>** that contains the value of the largest integer that can be represented in a variable of type **unsigned long long**.

b) Write a recursive function, **factorial_rec(unsigned int n)**, that determines the factorial of a number. The result must also be of type **unsigned long long**.

3.11.

Euclid's recursive algorithm for determining the greatest common divisor (GCD) of two numbers, **m** and **n**, is the following:

- if **m** is divisible by **n**, then **GCD(m,n)** is **n**;
- otherwise, **GCD(m,n)** is given by **GCD(n,remainder of the division of m by n)**

Write a recursive function that determines the greatest common divisor of 2 numbers that it receives as parameters. Write a program for testing that function.

3.12.

Write and test a program that overloads a function, **area()**, that can be used to calculate:

- the area of a circle, given its radius;
- the area of a triangle, given its 3 vertices (remember problem 3.1);
- the area of a rectangle, given 2 opposite vertices.

3.13.

Consider the following function:

```

int rollDie(int low = 1, int high = 6)
{
    assert(high >= low);
    return (rand() % (high - low + 1)) + low;
}

```

- Explain what it does. Also, explain the effect of the **assert()** statement.
- The function has 3 different signatures. Explain why.
- Is it possible to overload this function with a function whose prototype is **int rollDie()**? Explain why or why not.

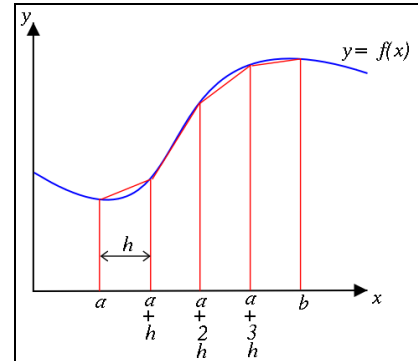
3.14.

The "trapezoidal rule" is a numerical method for calculating an approximate value for the definite integral. To calculate the integral

$$\int_a^b f(x) dx$$

the area under the curve $y=f(x)$ is divided into n regions, each with a width $h = (b-a) / n$. The area of each region is approximated by the area of a trapezoid. The sum of the areas of all the trapezoids gives an approximate value of the definite integral. The area of the i -th trapezoid ($i = 1, 2, \dots$) is given by

$$\frac{h}{2} (f(a + (i-1)h) + f(a + ih))$$



In general, the estimate of the integral improves with the decrease of h .

- Write a function whose prototype is

double integrateTR(double f(double), double a, double b, int n),

with the parameters $f, a, b \in \mathbb{R}$, above mentioned, that calculates the integral of a function, using this method.

- Write a program to calculate the following two integrals:

$$g(x) = x^2 \quad \text{when } a = 0, b = 10 \quad \text{e} \quad h(x) = \sqrt{4 - x^2} \quad \text{when } a = -2, b = 2.$$

The function h defines a semi-circle with radius 2. Compare the estimate obtained with the real area of the semi-circle. Try with different values of n .

4. Arrays. Vectors. Pointers and Dynamic memory allocation.

4.1.

- a) Write a function, whose prototype is **void readArray(int a[], size_t nElem)**, which reads from the keyboard the elements of an array of integers, **a[]**; the number of elements to be read is **nElem**. The space for the array must be allocated statically, before calling the function. Before reading each element of the array, the respective index must be displayed on the screen.
- b) Write a function, whose prototype is **int findValueInArray(const int a[], size_t nElem, int value, size_t pos1, size_t pos2)** which searches the **nElem** elements of the array, between indexes (positions) **pos1** and **pos2**, inclusive, for the first occurrence of **value**. If **value** is found, the function must return the index (position) of the corresponding element of the array; otherwise it must return **-1**.
- c) Write a program for testing the functions **readArray()** and **findValueInArray()**.
- d) In C++, it would be possible to specify default values for the parameters **pos1** and **pos2** so that all the elements of the array are searched when **pos1** and **pos2** are not specified as arguments of the call. How would you modify the prototype of **findValueInArray()** to do this?
- e) Modify the function **findValueInArray()** to obtain a new function **size_t findMultValuesInArray(const int a[], size_t nElem, int value, size_t pos1, size_t pos2, size_t index[])** that, if there are multiple occurrences of **value** in **a[]**, returns the indexes of those occurrences through parameter **index[]**. The value returned by the function must be the number of occurrences, that is, the number of valid elements of **index[]**. In C language, is it possible that the name of this function is also **findValueInArray**, like the previous function? And in C++ language?
- f) Change the program developed in exercise **4.1.c)** in order to use the function **findMultValuesInArray()**. Note: the space for array **index[]** must be allocated before calling the function **findMultValuesInArray()**, taking into account the expected maximum number of occurrences (in the limit, it must have **nElem** elements).

4.2.

- a) Repeat problem **4.1**, using STL vectors to store the numbers, instead of C arrays. The prototypes of the functions to be developed are, in this case, the following:
- **void readVector(vector<int> &v, size_t nElem)**; (Note: in this case, it is not necessary to pre-allocate space for the values to be read)
 - **size_t findValueInVector(const vector<int> &v, int value, size_t pos1, size_t pos2)**;
 - **void findMultValuesInVector(const vector<int> &v, int value, size_t pos1, size_t pos2, vector<size_t> &index)**;
- b) Modify function **readVector()** so that the number of elements to be read does not need to be specified as a function parameter; reading must end when the user closes the standard input stream, by typing **CTRL-Z** (in Windows) or **CTRL-D** (in Linux).
- c) Alternative prototypes of functions **readVector()** and **findMultValuesInVector()** could have been used, so that both return the resulting vectors, instead of **void**. Implement these functions and modify the developed code in order to use the new functions. Do you find any advantage/disadvantage of using these new versions? Would it be possible to modify the function **readArray()** in problem **4.1.a)** so that it returns an array?

4.3.

- a) The *bubblesort* method, for sorting the elements of a vector with **N** elements, consists of the following:
- scan the vector, starting at the 1st element (index=0), comparing the elements of indexes **i** and **i + 1** of the vector, and changing their position if they are out of order; after this step, the largest (or smallest, depending on whether the sorting is done in ascending or descending order) will be in the correct position (the last position of the vector);
 - repeat the previous step, considering in each iteration only the elements not yet in the correct position; note that, after the 2nd iteration the last 2 elements will be in the correct position, after the 3rd iteration the last 3 elements will be in the correct position, and so on.

Write a function **void bubbleSort(vector<int> &v)** that implements this sorting method to sort a vector of integer values in ascending order.

b) Write a program for testing the **bubbleSort()** function.

c) Improve the function developed in **4.3.a**, in order to optimize the implementation, by stopping the algorithm when no swap is done during a scan of all the elements, which means that the elements are sorted.

d) Improve the function developed in **4.3.a**, by adding an additional parameter of type char, that allows the user of the function to sort the values in ascending or descending order (parameter equal to 'a' or 'd', respectively).

e) Modify the function developed in **4.3.a**, so that the additional parameter is a function **bool f(int x, int y)** that determines the sorting order (ascending or descending): **void bubbleSort(vector<int> &v, bool f(int x, int y))**. This new version of **bubbleSort()** could be called in the following way: **bubbleSort(v,ascending)** or **bubbleSort(v,descending)**, where **ascending()** and **descending()** are the functions that determine the sorting order. The function **ascending(int x, int y)** must return **true** when **x<y** and the function **descending(int x, int y)** must return **true** when **x>y**.

Note: both the C library and the C++ Standard Template Library (STL) have sorting functions that will be presented and used later.

4.4.

a) "Binary search" is an algorithm that can be used to search for a value in a sorted array/vector by repeatedly dividing the search interval in half until the value is found. In pseudocode, for searching a value in an array/vector sorted in ascending order, it can be described as follows:

- assign the index of the first element of the array/vector to **first**;
- assign the index of the last element of the array/vector to **last**;
- make variable **found** equal to **false**;
- repeat until **found** is **true** or **first** is greater than **last**:
 - make **middle** equal to index of the middle position between **first** and **last**;
 - if the value of the array/vector in **middle** position is equal to the searched value, make **found** equal to true otherwise, if the searched value is lower than the value of the array/vector in **middle** position
 - make **last** equal to **middle-1**;
 - otherwise
 - make **first** equal to **middle+1**.

Write a function whose prototype is **int binarySearch(const vector<int> &v, int value)** that applies this search algorithm to a vector **v**. The function must return the index of the element whose value is **value** if it exists in **v** or **-1**, if it does not exist.

b) Write a program for testing the **binarySearch()** function.

4.5.

a) Write a void function **void removeDuplicates(vector<int> &v)** that eliminates the repeated elements of vector **v**. The original ordering of the vector elements must be maintained. Suggestion: the elimination of an element can be done "in-place" (that is, without using an auxiliary vector) by moving all the elements that occupy the next positions of the vector to the position before the position they occupy and changing the size of the vector, using the **resize()** method of the vector class, to decrease the size of the vector. Note: there are member functions of the STL vector class that can be used to do this task; these functions will be studied later.

b) Write a program for testing the **removeDuplicates()** function.

4.6.

a) Write two functions whose prototypes are

void vectorUnion(const vector<int> &v1, const vector<int> &v2, vector<int> &v3);

void vectorIntersection(const vector<int> &v1, const vector<int> &v2, vector<int> &v3);

that do, respectively, the union and the intersection of the elements of vectors **v1** and **v2**, returning the result through vector **v3**. The resulting vector must not have repeated elements and these must be sorted in ascending order. Suggestion: use the functions **bubbleSort()** and **removeDuplicates()** from the previous problems.

b) Write a program for testing the **vectorUnion()** and the **vectorIntersection()** functions.

4.7.

a) An element of a matrix is said to be a local maximum if its value is greater than the value of all its neighbors. Write a function whose prototype is `void localMax(const int a[] [NE])` that displays on the screen the position (row and column) and value of all the local maxima of a 2D matrix with `NExNE` of elements of type `int`, considering that only the elements of the matrix that have 8 neighbors can be considered local maxima. Using this rule in the example below, only the elements colored in red would be considered local maxima. Note: since `NE` must be a global constant, and the number of lines is equal to the number of columns, it is not necessary to pass the number of lines and columns as parameters.

	0	1	2	3	4
0	7	3	4	1	3
1	2	9	6	2	1
2	1	3	5	1	4
3	6	5	2	7	5
4	4	2	1	3	6

b) Write a test program for the function developed, using the matrix above as an argument to the call. Suggestion: to avoid reading the elements of the matrix from the keyboard, on every run, use a declaration with initialization of the 2D matrix, in `main()`.

c) Rewrite the code of `localMax()` and of the test program with the following modifications:

- use a 2D vector (from STL), instead of a 2D array;
- use an additional parameter, in `localMax()`, to let the user decide if he wants to consider that any element of the matrix can be a local maximum, regardless of the number of neighbors it has; in this case, the elements colored in magenta, in the example, would also be considered local maxima.

4.8.

Write a program that reads the pluviosity observed in every day of a given year and determines the following statistical data: the average daily pluviosity; the average monthly pluviosity; the date(s) of maximum pluviosity; the date(s) in which the pluviosity was above the average daily pluviosity.

The pluviosity values must be stored in a 2D STL vector, indexed by month and day (note: the number of days is not constant for every month; the vector should have the number of elements strictly necessary). The program must have independent functions to "read" the pluviosity (note: instead of reading the values, generate them randomly) and to calculate each one of the statistical values. The results must be shown on the screen by the `main()` function. A function for calculating the number of days of each month of the given year must be used (see problem 3.9.b).

4.9.

Consider the two program fragments:

```
int x = 1, y = 2;
int &ref_x = x, &ref_y = y;
ref_x = ref_y;
cout << "x = " << x << "; y = " << y << endl;
cout << "ref_x = " << ref_x << "; ref_y = " << ref_y << endl;
```

```
int x = 1, y = 2;
int *ptr_x = &x, *ptr_y = &y;
ptr_x = ptr_y;
cout << "x = " << x << "; y = " << y << endl;
cout << "ptr_x = " << ptr_x << "; ptr_y = " << ptr_y << endl;
cout << "*ptr_x = " << *ptr_x << "; *ptr_y = " << *ptr_y << endl;
```

Without running the code, say which will be the output of each fragment.

4.10.

Consider the program fragment:

```
int values[] = { 2, 3, 5, 7, 11, 13 };
int *p = values+1;
cout << values[1] << endl;
cout << values+1 << endl;
cout << *p << endl;
cout << *(values+3) << endl;
cout << p+1 << endl;
cout << p[1] << endl;
cout << p-values << endl;
```

Explain the meaning of the following expressions:

```
values[1]
values+1
*p
*(values+3)
p+1
p[1]
p-values
```

4.11.

Standard C library provides **qsort()** function that can be used for sorting an array. This function uses quicksort algorithm (<https://en.wikipedia.org/wiki/Quicksort>) to sort the given array. The prototype of **qsort()** is

void qsort (void* base, size_t num, size_t size, int (*comparator)(const void*,const void*))

The meaning of the parameters is:

- **base** – pointer to the first element of the array to be sorted;
- **num** – number of elements in the array pointed by base;
- **size** – size in bytes of each element in the array;
- **comparator** – function that compares two elements; this function must return:
 - **<0** when the element pointed by the 1st parameter goes before the element pointed by the 2nd parameter
 - **0** when the element pointed by the 1st parameter is equivalent to the element pointed by the 2nd parameter
 - **>0** when the element pointed by the 1st parameter goes after the element pointed by the 2nd parameter.

Write a program that uses this function to:

- a) Sort all the elements of an array in ascending order.
- b) Sort all the elements of an array in descending order.
- c) Sort the elements in the first half and in the second half of the array, independently, in ascending order.

4.12.

a) Redo problem **4.1** so that the space for the array is allocated dynamically, after asking the user for the effective number of the elements to be read and before calling the function **readArray()**. In **findMultValuesInArray()**, the space for the resulting array must also be allocated dynamically. The new prototype of the functions must be:

- **void readArray(int *a, size_t nElem)**

- **int findValueInArray(const int *a, size_t nElem, int value, size_t pos1, size_t pos2)**

- **size_t findMultValuesInArray(const int *a, size_t nElem, int value, size_t pos1, size_t pos2, size_t *index)**

b) Modify the solutions of **4.12.a** so that **findValueInArray()** and **findMultValuesInArray()** have the following prototypes:

- **int findValueInArray(const int *pos1, const int *pos2, int value)**

- **size_t findMultValuesInArray(const int *pos1, const int *pos2, int value, size_t *index)**

where **pos1** and **pos2** are pointers to elements of the array.

4.13.

Adapt the program presented in the lecture notes for computing the average score for each student and the average score for each quizz, using a 2D array, so that the number of students and the number of questions can be specified in run-time, by the user of the program. Use dynamic memory allocation for the 2D array.

5. Strings. Structs. Stringstreams. Files.

5.1.

In cryptography, a Caesar cipher is one of the simplest and most widely known encryption techniques. It is a type of substitution cipher, in which each letter in the plaintext is replaced by a letter some fixed number of positions up (a right shift) or down (a left shift) the alphabet. For example: with a right shift of 3, **A** would be replaced by **D**, **B** would become **E**, ..., and **Z** would become **C**; with a left shift of 3, **a** would become **x**, **d** would become **a**, and so on. The method is named after Julius Caesar, who used it in his private correspondence (*adapted from Wikipedia*).

a) Write a function **char encryptChar(char c, int key)** that encrypts a character using the Caesar cipher, where **c** is the character to encrypt and **key** is the shift, or encryption key. A positive or negative value of the parameter **key** indicates a right or left shift, respectively.

b) Write a function **string encryptString(string s, int key)** that encrypts a **C++ string** using the above mentioned technique.

c) Write a program that reads a string from the keyboard and shows its encryption, using **encryptString()**.

Examples of execution:

- **s** = "The quick brown fox JUMPS over the lazy dog" / **key** = 10
output = "Dro aesmu lbygx pyh TEWZC yfob dro vkji nyq"
- **s** = "The quick brown fox JUMPS over the lazy dog" / **key** = -10
output = "Jxu gkysa rhemd ven ZKCFI eluh jxu bqpo tew"

d) Redo the program, using a **C string** instead of a **C++ string** parameter. Modify the prototype of **encryptString()** accordingly.

5.2.

a) Write a function **void bubbleSort(vector<string> &v, char order)** that implements the bubblesort method to sort a vector of **C++ strings**, in lexicographic (ascending) order or in reverse lexicographic (descending) order, depending on the value of parameter **order** being '**a**' or '**d**'. Tip: adapt the code from problem **4.3.d**.

b) Write a program that reads, from standard input, a list of person names (each name having more than one word), stores them into a **vector<string>**, and sorts them using the **bubbleSort()** function.

5.3.

Redo problem **5.2** using an **array** of **C strings** to store the names and the library function **qsort()** to sort the names. Tip: adapt the code from problem **4.11**. Note: in order to use the library function **qsort()** the space allocated for each name must be the same (for example, 100 bytes), although the effective length of the names may be different.

5.4.

Write a function a **bool sequenceSearch(const string &s, int nc, char c)** that checks whether string **s** has a sequence of **n** consecutive characters equal to **c**. Example: **sequenceSearch("abcddeedddf", 3, 'd')** must return **true**, but **sequenceSearch("abcddeedddf", 4, 'd')** must return **false**.

Write a program for testing the function. Implement 2 different versions of the program:

a) Using your own algorithm.

b) Using one of the **find** methods of class **string**. Tip: build a string made of **nc** characters equal to **c**.

5.5.

Write a function **string normalizeName(const string &name)** to "normalize" a Portuguese name as follows: spaces at the beginning or at the end are suppressed; multiple spaces between two words are replaced by a single space; lowercase are converted to uppercase; the particles "DE", "DO", "DA", "DOS", "DAS", and "E" are removed. The return value of the function is the "normalized" name. Example: the result of normalizing the name " Maria da Felicidade

dos Reis e Passos Dias de Aguiar " should be "MARIA FELICIDADE REIS PASSOS DIAS AGUIAR". Write a program for testing the function. Tips: use an **array** or **vector** of **strings** to store the particles to be removed; use auxiliary functions to implement each one of the steps (they can be useful in other occasions, for example, to remove spaces at the beginning or end of the string).

5.6.

Redo problem 3.8 using **struct**'s to represent fractions:

```
struct Fraction {  
    int numerator;  
    int denominator;  
};
```

Adapt the prototype of the functions accordingly; for example, the prototype of **readFraction()** must now be **bool readFraction(Fraction &fraction)**.

5.7.

The following data type is used to represent dates:

```
struct Date {  
    unsigned int year, month, day;  
} Date;
```

- a) Write a function **void readDate(Date *d)** that reads a date from the keyboard, in the format year/month/day. For simplicity, consider that the user always inputs a valid date.
- b) Write a function **void writeDate(const Date *d)** that writes a date on the screen, in the format YYYY/MM/DD, where YYYY, MM and DD represent, respectively, the year, the month, and the day (MM and DD with 2 digits).
- c) Write a function **int compareDates(const Date *d1, const Date *d2)** that compares two dates, returning -1, 0 or 1, depending on whether the date **d1** is before, equal to, or after date **d2**.
- d) Write a function **void sortDates(Date *d1, Date *d2)** that compares and sorts the dates **d1** and **d2** so that **d2** is after **d1**.
- e) Write a program that reads 2 dates, sorts them, and shows the sorted dates on the screen.

5.8.

The process of betting on EuroMillions requires that the player chooses five or more "main numbers" from 1 to 50 and two or more "lucky stars" from 1 to 12. The key of EuroMillions is made of five "main numbers" and two "lucky stars". So both a bet and a key can be represented by the following **struct**:

```
struct EuroMillionsBet {  
    vector<unsigned> mainNumbers;  
    vector<unsigned> luckyStars;  
};
```

Write a program that, using variables of type **EuroMillionsBet** to represent bets and keys, does the following:

- reads the bet of a player;
- generates a random key;
- shows the result of the bet, by computing the number of coincident values in "main numbers" and the "lucky stars" in the bet and the key. Tip: reuse the code of problem 4.6.

5.9.

- a) Declare a **struct** that can be used to represent a street address. For simplicity, consider that the only elements of the address are: street, door number, and city.
- b) Declare a **struct** that can be used to represent a person. For simplicity, consider that the only attributes of the person are: name, age, gender and address. Use the **struct** declared in a to represent the address.
- c) Write a program that reads, from the keyboard, the data relative to a set of persons, including their address, and then shows the name of the persons that live in the same street of a given city (input by the user). Suggestion: use functions to read and to write an address and the person's data.

5.10.

The area of any polygon, given the coordinates of its vertices and assuming that the vertices are stored either clockwise or counter-clockwise, is given by the formula: $\left| ((x_1y_2 - y_1x_2) + (x_2y_3 - y_2x_3) + \dots + (x_ny_1 - y_nx_n)) / 2 \right|$, where (x_i, y_i) are the coordinates of the i-th vertex and $|\dots|$ represents the absolute value.

- a) Write a program in **C++** that reads the number of vertices of a polygon, as well as the corresponding coordinates, and then calculates the area of the polygon. The program must use a **struct Point** to represent a point and a **struct Polygon** to represent a polygon.
- b) Rewrite the program in "pure **C**".

5.11.

Redo problem **5.2.b** so that the person names are read from a text file, whose name is entered by the user. Each line of the text file contains a person's name. Build two versions of the program, in which:

- a) the sorted names are written to the standard output (the screen);
b) the sorted names are written to a file having the same name as the input file name, with the suffix "_sorted";
example: if the input file name is "**names.txt**" the output file name must be "**names_sorted.txt**".

5.12.

A binary image is an image made of black and white pixels. It can be represented in a text file using, for instance, character 'b' to represent black pixels and character 'w' to represent white pixels. For example, the binary image on the right could be represented by the text file shown below (**A**); the 2 numbers at the beginning of the text file (10 10, in this case) represent the number of lines and the number of columns of the image.

Run-length encoding (RLE) is a form of lossless data compression in which "runs" of data (sequences in which the same data value occurs in many consecutive data elements) are stored as a single data value and count, rather than as the original run; this is most useful on data that contains many long runs. Using RLE, the pixels of the image in the example could be represented

by the sequence illustrated in **B**, resulting in a compression from 100 characters to 72 characters.

Example:

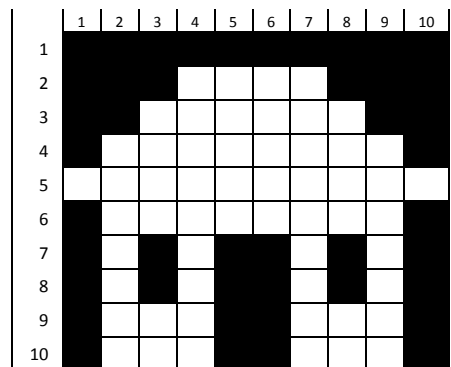
A) Contents of an original image file:

```
10 10 bbbbbbbbbbbbbwwwwbbbbbbwwwwwwbbwwwwwwwwwwbwwwwwwwwwwbwwwwwwwwbwbwbbwbwbbwbwbbwbwbbwwwwbbwwwwbbwwwwbwwwwb
```

B) Compressed file (using RLE) resulting from the original file above:

10	10	13b4w5b6w3b8w1b10w1b8w2b1w1b1w2b1w1b1w2b1w1b1w2b1w1b1w2b3w2b3w2b3w2b3w1b
----	----	--

- a) Write a program that takes as input a binary image file, in the format illustrated in **A**, and compresses it, using RLE, producing a file in the format illustrated in **B**. The names of the original file and of the compressed file must be passed as command line arguments. Being **compress** the name of the executable code, it must be run using the command **compress <uncompressed image> <compressed image>**; example: **compress img1 img1c**. The program must end immediately if the number of arguments is not valid or the original image file does not exist.
- b) Write a program, that takes as input a file representing a compressed image and produces the corresponding uncompressed image. It must be executed using the command **uncompress <compressed image> <uncompressed image>**.



5.13.

Remember problem **5.8**. Consider that the bets for a given draw are stored in a text file, using the format illustrated on the right, consisting of the name of the players, followed by the bets they have done. Develop a program that reads, from the keyboard, the key of that draw, and produces another file containing the name of the players, their bets, the number of "main numbers" and "lucky stars" of each bet, as well as the number of correct "main numbers" and "lucky stars" for each bet. The output must be formatted so that: the key is written at the top of the file; every number occupies a field 2 characters wide. Suggestion: read each bet into a single string and use stringstreams to extract the numbers of the bet.

Example of input file contents:

Rosa Ramalhete	13	18	29	39	50	-	5	12
Modesto Patacas	1	8	12	21	23	35	50	-
Zeferino Fortunato	3	13	20	39	49	-	2	9
	9	18	19	25	30	-	11	12

Example of output file contents:

KEY = 3 13 20 39 50 - 5 12
 Rosa Ramalheite
 13 18 29 39 50 - 5 12 => 5- 2 | 3- 2
 Modesto Patacas
 1 8 12 21 23 35 50 - 6 8 => 7- 2 | 1- 0
 Zeferino Fortunato
 3 13 20 39 49 - 5 9 => 5- 2 | 4- 1
 9 18 19 25 30 - 11 12 => 5- 2 | 0- 1

5.14.

Repeat problem **5.5**, using stringstream to do the normalization. Tip: use an **istringstream** to decompose the name into its components and an **ostringstream** to build the normalized name.

5.15.

Write a program that keeps a phone list in a random-access file. Each record must contain the name of a person and his/her phone number. Implement functions for adding/removing persons to/from the phone list. You need not keep persons in sorted order. To remove a person, just fill the corresponding record with an empty name. When adding a new person to the file, try to add it into one of the empty records first, before appending it to the end of the file. The program must show a menu to the user, so that he/she can choose the operation to be done, among a set of possible operations, such as: add a new person, remove the record of an existing person, modify the phone number associated with an existing person or search for the phone number of a given person.