

Distributed and Parallel Computing

Assignment 1 - Performance Evaluation of single
and multi-core implementations

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1 Problem Description

In this analysis, we explored the impact of the memory hierarchy on processor performance during the processing of substantial data volumes, in a single and multi-core context. Our investigation employs matrix multiplication as a case study, a relevant computation prevalent across diverse domains, such as computer graphics or machine learning.

2 Algorithms

In this project we used three distinct algorithms to evaluate the performance of a single processor when processing large data sets. Two parallel versions of one of these algorithms were implemented. These were the algorithms implemented:

2.1 Simple Matrix Multiplication

In our initial setup, we were given a basic C/C++ algorithm for multiplying two matrices. This algorithm multiplies each row of the first matrix with every column of the second matrix to calculate the product. The computational complexity of this method is cubic, denoted as $O(n^3)$.

```
for (i = 0; i < m_ar; i++) {
    for (j = 0; j < m_br; j++) {
        temp = 0;
        for (k = 0; k < m_ar; k++) {
            temp += pha[i * m_ar + k] * phb[k * m_br + j];
        }
        phc[i * m_ar + j] = temp;
    }
}
```

2.2 Line Matrix Multiplication

This variant introduces an enhanced algorithm for computing the matrix product by multiplying each element of the first matrix with the corresponding row in the second matrix. Despite these optimizations, the algorithm still maintains a cubic computational complexity of $O(n^3)$, reflecting the inherent challenge of reducing the computational demands of matrix multiplication without altering the fundamental approach.

```
for (i = 0; i < m_ar; i++) {
    for (k = 0; k < m_ar; k++) {
        for (j = 0; j < m_br; j++) {
            phc[i * m_ar + j] +=
                pha[i * m_ar + k] * phb[k * m_br + j];
        }
    }
}
```

```

    }
}

```

2.3 Block Matrix Multiplication

In our latest adaptation, we have improved the matrix multiplication strategy by segmenting the matrices into sub-matrices or 'blocks'. These blocks are then processed independently before their results are consolidated. This technique, known as block matrix multiplication, operates under the same cubic time complexity, $O(n^3)$, as its predecessors. The primary innovation of this method lies in its potential to enhance memory access efficiency. This approach aims to increase the likelihood of faster data processing by working with smaller blocks of data at a time. The data being processed can reside in faster, lower-level cache memories (such as L1 and L2 caches) more frequently. This optimization reduces the latency associated with retrieving data from higher-level memory stores, theoretically improving overall execution speed by leveraging the hierarchical nature of memory systems for more efficient data access and computation.

```

for (bi = 0; bi < m_ar; bi += bkSize) {
    for (bk = 0; bk < m_ar; bk += bkSize) {
        for (bj = 0; bj < m_br; bj += bkSize) {
            for (i = bi; i < bi + bkSize; i++) {
                for (k = bk; k < bk + bkSize; k++) {
                    for (j = bj; j < bj + bkSize; j++) {
                        phc[i * m_ar + j] +=
                            pha[i * m_ar + k] * phb[k * m_br + j];
                    }
                }
            }
        }
    }
}

```

2.4 Parallel Implementations of Line Matrix Multiplication

Lastly, two different parallel versions of the Line Matrix Multiplication were implemented using OpenMP.

In this first implementation only the outer loop (*i loop*) is parallelized, so each thread is responsible for handling a portion of those iterations, leaving the inner loops (*k and j loops*) executing sequentially in each thread.

```

#pragma omp parallel for private(i, j, k)
for (i = 0; i < m_ar; i++) {
    for (k = 0; k < m_ar; k++) {
        for (j = 0; j < m_br; j++) {

```

```

        phc[i * m_ar + j] += pha[i * m_ar + k] * phb[k * m_br + j];
    }
}

```

In this second version the loop being parallelized is the middle one (k loop). In each iteration of the top loop (i loop), the work of computing the matrix is divided amongst threads based on the k loop.

```

#pragma omp parallel for private(i, j, k)
for (int i = 0; i < m_ar; i++) {
    for (int k = 0; k < m_ar; k++) {
        #pragma omp parallel for
        for (int j = 0; j < m_br; j++) {
            phc[i * m_ar + j] += pha[i * m_ar + k] * phb[k * m_br + j];
        }
    }
}

```

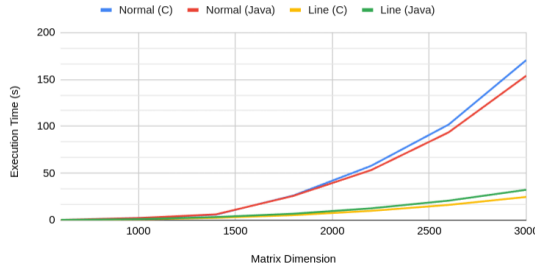
3 Performance Metrics

To measure the performance of algorithms developed in C/C++, we leveraged the Performance API (PAPI), which offers insights into a range of indicators concerning CPU efficiency and how the CPU cache memory is leveraged by the program. This comprehensive evaluation not only included the time taken by the algorithm to execute but also measured the frequency of cache misses at both L1 and L2 cache levels. By averaging these data points, we aimed to smooth out the small inconsistencies that might arise from each run, potentially influenced by the state of the computing environment. The C/C++ programs were compiled using the -O2 optimization flag, a step that significantly refines both the compilation process and the performance of the final executable. Furthermore, this study included a comparative assessment of the execution times for the algorithms when implemented in both C/C++ and Java, offering a dual perspective on their performance efficacy, as well as comparisons between sequential and parallel implementations of specific algorithms.

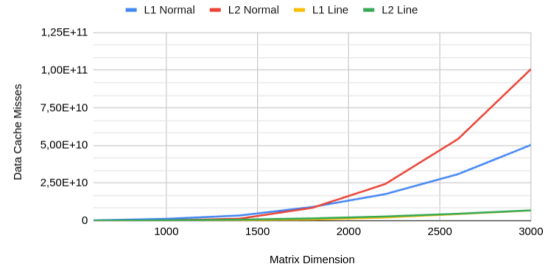
4 Results and analysis

4.1 Comparing Normal and Line Multiplication Algorithms

Execution Time Comparison (Normal and Line Multiplication in C++ and Java)



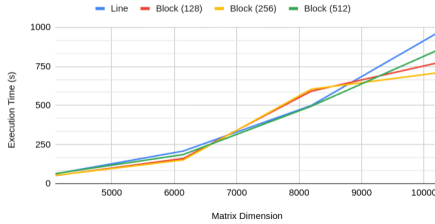
Data Cache Misses Comparison (Normal and Line Multiplication in C++)



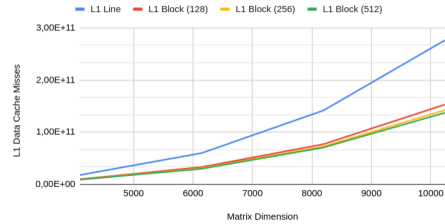
As we expected, the C++ and Java implementations of these algorithms have very similar execution times across all the matrices sizes tested. Also as expected, the line multiplication algorithm has a faster execution time and lower L1 and L2 cache misses, as instead of directly computing and assigning each element of the result matrix, it accumulates the results in the corresponding position of the result matrix, that allows for contiguous and more efficient access to the memory locations of the resulting matrix.

4.2 Comparing Line and Block Multiplication Algorithms

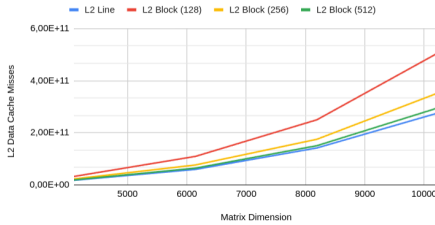
Execution Time Comparison (Line and Block Multiplication in different block sizes in C++)



L1 Data Cache Miss Comparison (Line and Block Multiplication in different block sizes in C++)



L2 Data Cache Miss Comparison (Line and Block Multiplication in different block sizes in C++)

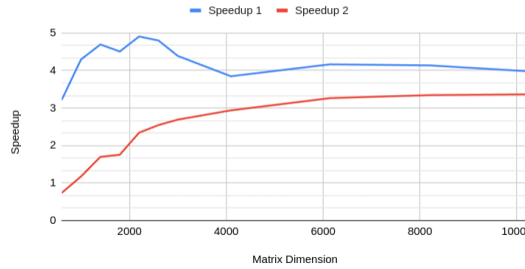


When comparing the Line and Block Multiplication algorithms in different block sizes we recognized that while L1 cache misses tend to be significantly higher in the Line multiplication algorithm, L2 misses also tend to be slightly

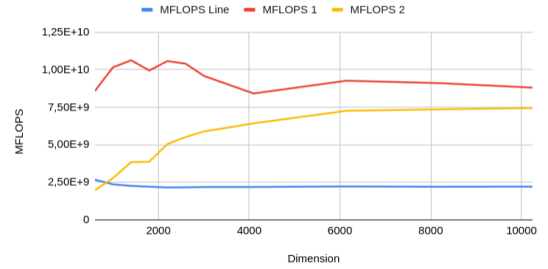
lower. This led to not very interesting results regarding the execution times, as for each matrix dimension the faster algorithm (or block size) varied. However, the 512 Block Size algorithm was consistently the alternative with less data cache misses.

4.3 Comparing Sequential and Parallel implementations of Line Multiplication

Speedup comparison between parallel implementations of Line Multiplication



MFLOPS comparison between sequential and parallel implementations of Line Multiplication



At last, we compared the sequential implementation of the Line Multiplication Algorithm to two parallel implementations. First measuring the speedup, the time of the sequential execution divided by the time of parallel execution. As expected, the speedups for both parallel algorithms were ≥ 1 , meaning both of them were faster than the sequential alternative. The exception is the speedup of the second parallel implementation for smaller matrix dimensions, probably due to the overhead caused by creating multiple threads inside the inner k loop. For small matrices, this overhead caused by this is more costly than the parallelization is effective.

The first parallel implementation showed to be faster because it parallelized the outer loop, allowing each thread to work on different sections of the resulting matrix.

5 Conclusions

This project showed us how memory accesses impact the efficiency of an algorithm. With proper implementation and memory manipulation, apparently similar algorithms can have significant execution times. We were also able to study parallelization, and how the different methods of doing it can affect the efficiency of the algorithms, considering the order of creating the threads and their overhead. Lastly, we learned about methods of performance analysis, using *PAPI* and comparing algorithms through different metrics.

6 Annexes

6.1 Normal Multiplication (C) Measurements

	Dimension	600	
	Time	L1	L2
	0,27	244813494	40744559
	0,268	244798294	40449564
	0,277	244835569	42200261
	0,271	244798351	39624506
	0,278	244806456	41663199
	0,266	244801925	40153630
	0,275	244805558	40954782
	0,272	244819900	41320483
	0,293	244394764	42503690
	0,273	244653151	41685950
Average	0,2743	244752746,2	41130062,4

	Dimension	1000	
	Time	L1	L2
	2,049	1222059445	238066627
	2,474	1227462850	254528600
	2,424	1227483614	277796038
	2,583	1227931798	246159377
	2,294	1227477934	232967869
	2,246	1227441960	277467300
	2,282	1227547055	272923698
	2,295	1227460636	265301141
	2,262	1227435328	268354812
	2,214	1227433399	256915676
Average	2,3123	1226973402	259048113,8

	Dimension	1400	
	Time	L1	L2
	5,82	3408529342	1248742888
	5,936	3429021317	1395997009
	5,912	3429193439	1349871911
	5,938	3428801714	1609801927
	5,906	3429022967	1210182977
	5,702	3428651558	1320733250
	5,695	3428712573	1269961870
	5,787	3428609799	1372045204
	5,828	3428612836	1228764765
	5,838	3428421480	1209824227
Average	5,8362	3426757703	1321592603

	Dimension	1800	
	Time	L1	L2
	26,277	9080293891	8532135607
	26,292	9090315029	8290440863
	26,151	9090415316	8010071361
	26,276	9090552704	8731271025
	26,316	9090198039	8830750213
	26,198	9090367622	8431537498
	26,341	9090225446	8494552580
	26,135	9090463735	8310259749
	26,259	9090579420	8849043835
	26,312	9090041055	8705363239
Average	26,2557	9089345226	8518542597

	Dimension	2200	
	Time	L1	L2
	55,962	17620532070	24383871778
	54,37	17620904117	24270181726
	54,679	17620834279	23380860634
	55,533	17620425525	25088366901
	54,761	17620651517	23448322338
	55,26	17620399590	24557525986
	54,403	17620859777	24432108255
	54,616	17620746915	24798982165
	54,432	17620634075	24646224063
	54,279	17620873043	23925972690
Average	54,8295	17620686091	24293241654

	Dimension	2600	
	Time	L1	L2
	100,218	30899724729	54184095605
	101,456	30900391590	53509711699
	102,012	30900919718	54845379590
	101,757	30901039328	53676078236
	103,05	30900827716	54668166546
	101,639	30899680594	53977955293
	101,861	30901144205	54376850897
	102,508	30900877724	54273680096
	101,943	30900216387	54296474829
	102,347	30901187292	54601164216
Average	101,8791	30900600928	54240955701

	Dimension	3000	
	Time	L1	L2
	171,533	50300666693	101895549363
	171,119	50300090897	99973547955
	170,061	50299837354	100760159900
	170,744	50297336033	100445529209
	170,752	50298265834	100317063432
	170,076	50299446589	100129508008
	169,928	50298483351	101114251486
	170,096	50300617656	102304834650
	171,115	50299797338	99826531920
	170,114	50297496244	99905638908
Average	170,5538	50299203799	100667261483

6.2 Normal Multiplication (Java) Measurements

Dimension	600	Dimension	1000
	Time		Time
	0,302		2,197
	0,291		2,271
	0,264		2,314
	0,262		2,263
	0,298		2,23
	0,282		2,184
	0,271		2,252
	0,269		2,088
	0,262		2,177
	0,259		2,216
Average	0,276	Average	2,2192

Dimension	1400	Dimension	1800
	Time		Time
	6,123		27,026
	6,209		26,358
	6,096		26,535
	6,105		26,25
	6,096		26,763
	6,099		25,846
	6,116		24,716
	6,252		25,734
	6,142		24,549
	6,158		24,692
Average	6,1396	Average	25,8469

Dimension	2200	Dimension	2600
	Time		Time
	55,24		97,324
	52,624		96,157
	53,014		93,656
	52,952		92,492
	53,885		91,681
	55,195		91,832
	53,05		91,746
	52,785		92,414
	52,112		94,19
	52,042		94,41
Average	53,2899	Average	93,5902

Dimension	3000
	Time
	158,063
	156,787
	151,878
	152,986
	152,703
	152,996
	155,151
	152,215
	152,953
	153,184
Average	153,8916

6.3 Line Multiplication (C) Measurements

	Dimension	600	
	Time	L1	L2
	0,16	27144510	57504452
	0,155	27135108	57519128
	0,15	27117526	57448876
	0,161	27154813	57583151
	0,156	27143068	57575527
	0,168	27171113	57387315
	0,166	27144229	57197940
	0,173	27185680	57411535
	0,165	27135740	57203740
	0,163	27176731	57506418
Average	0,1617	27150851,8	57433808,2

	Dimension	1000	
	Time	L1	L2
	0,883	125890043	258052716
	0,892	125970586	256346204
	0,834	125889214	257283982
	0,816	125798270	257409746
	0,828	125815565	256848777
	0,833	125821534	256416554
	0,839	125942298	256051182
	0,843	126242052	256084755
	0,837	126083576	255590414
	0,845	125958805	256182274
Average	0,845	125941194,3	256626660,4

	Dimension	1400	
	Time	L1	L2
	2,509	346667347	699852160
	2,429	346553002	699655547
	2,409	346920621	699544120
	2,397	346419898	699219227
	2,387	346940426	699358638
	2,468	351295314	700632781
	2,41	348086974	699071216
	2,408	347926611	698872510
	2,417	347855383	699376642
	2,396	346934305	699400832
Average	2,423	347559988,1	699498367,3

	Dimension	1800	
	Time	L1	L2
	5,353	751095682	1496936605
	5,334	760105771	1514417844
	5,242	747906035	1537682555
	5,256	749990086	1537775824
	5,306	752442239	1539115481
	5,254	747383409	1537434790
	5,261	749261261	1538246134
	5,27	749267942	1538355651
	5,269	751470483	1538675794
	5,267	750619556	1537906361
Average	5,2812	750954246,4	1531654704

	Dimension	2200	
	Time	L1	L2
	9,815	2084359765	2804154387
	9,883	2110341990	2804840976
	9,937	2110632438	2807508911
	9,888	2120231045	2807649469
	10,019	2128600454	2811859434
	9,907	2123185877	2809615734
	9,863	2102690202	2808867717
	9,805	2098514497	2806448596
	9,759	2101123708	2787013313
	9,877	2104573488	2727037845
Average	9,8753	2108425346	2797499638

	Dimension	2600	
	Time	L1	L2
	16,465	4411117263	4618610924
	16,212	4413766142	4613422809
	16,158	4413648339	4613724185
	16,224	4413732397	4614340753
	16,182	4413692591	4614913944
	16,093	4413463673	4615112059
	16,083	4413302618	4614468474
	16,276	4408874134	4618060893
	16,271	4412418389	4616664261
	16,182	4413709047	4614035163
Average	16,2146	4412772459	4615335347

	Dimension	3000	
	Time	L1	L2
	24,778	6780854930	6873861163
	24,647	6780931765	6876984132
	24,656	6780980366	6875333585
	24,687	6780995719	6874896836
	24,619	6780793299	6878034158
	24,722	6781004390	6874342814
	24,614	6781013367	6876035551
	24,793	6780073414	6876082639
	24,716	6781026383	6874982486
	24,718	6781002931	6874896913
Average	24,695	6780867656	6875545028

	Dimension	4096	
	Time	L1	L2
	63,863	17706215776	17603033674
	62,105	17721178805	17600544872
	62,359	17711793317	17601915282
	62,244	17718337676	17476377324
	62,686	17718606828	17472490775
	62,744	17717814101	17459303362
	62,713	17719004379	17478334613
	62,936	17715476925	17479428270
	63,135	17711829996	17603288597
	63,066	17717696863	17472075572
Average	62,7851	17715795467	17524679234

	Dimension	6144	
	Time	L1	L2
	209,813	59816161833	59121553292
	209,044	59813715314	59363182374
	208,103	59819117807	59432903270
	208,093	59820425152	59470575842
	207,986	59820199142	59467622180
	208,175	59820403506	59409523031
	208,073	59819528309	59427807611
	208,106	59821073811	59443745204
	208,083	59819525498	59401015327
	208,078	59821181355	59455335144
Average	208,3554	59819133173	59399326328

	Dimension	8192	
	Time	L1	L2
	503,573	141480897115	143705437426
	502,944	141477398970	143670462982
	502,49	141471363407	143626116970
	495,946	141705397762	142259619882
	495,515	141656129584	140793232493
	498,278	141501823732	140645280443
	498,519	141496068034	140564848351
	498,56	141495238364	140587253965
	498,932	141501462760	140641328314
	498,221	141500130923	140636294539
Average	499,2978	141528591065	141712987537

	Dimension	10240	
	Time	L1	L2
	972,184	276581965569	278384495537
	976,813	276560028261	274269578489
	975,318	276561634660	274286170048
	972,65	276631046703	274139662124
	966,81	276755102092	273896727855
	971,326	276560017329	274334887174
	965,508	276650824862	276264136850
	964,801	276664430247	276450736692
	965,582	276647028077	278116777542
	967,287	276614953895	282901093901
Average	969,8279	276622703170	276304426621

6.4 Line Multiplication (Java) Measurements

Dimension	600	Dimension	1000
	Time		Time
	0,694		1,062
	0,214		0,997
	0,195		0,854
	0,216		0,893
	0,212		1,081
	0,212		1,057
	0,215		1,108
	0,463		1,085
	0,208		1,078
	0,207		1,083
Average	0,2836	Average	1,0298

Dimension	1400	Dimension	1800
	Time		Time
	3,341		7,068
	3,178		6,876
	3,293		6,87
	3,194		6,899
	3,132		6,808
	3,514		6,78
	3,473		6,638
	3,206		6,777
	3,274		6,827
	3,211		6,8
Average	3,2816	Average	6,8343

Dimension	2200	Dimension	2600
	Time		Time
	12,906		21,032
	12,596		20,665
	12,65		20,831
	12,537		20,677
	12,484		20,798
	12,539		20,711
	12,203		20,711
	12,649		20,784
	12,493		20,707
	12,513		20,809
Average	12,557	Average	20,7725

Dimension	3000
	Time
	32,188
	32,193
	31,947
	31,864
	32,138
	32,07
	33,726
	32,727
	32,12
	32,24
Average	32,3213

6.5 Block Multiplication Measurements

Dimension	4096	Block Size	128
	Time	L1	L2
	56,372	9655942769	32263950240
	49,133	9825753539	32387020502
	50,747	9682736978	32268944924
	50,612	9693295293	32253841215
	52,106	9702009051	32290493321
	50,381	9718597916	32247653156
	49,072	9736628106	32219310774
	49,127	9756488560	32259629613
	54,483	9781564382	32291982687
	54,333	9930464438	32356365771
Average	51.6366	9748348103	32283919220

Dimension	8192	Block Size	128
	Time	L1	L2
	760,288	75558203606	242350209044
	749,382	75577875333	242847327295
	728,943	75700192413	243721937547
	732,378	75731434817	244077008986
	676,086	76090056119	244598882071
	630,8	77975368520	251324350814
	440,883	77671610270	256538924382
	403,056	77920257805	257106276344
	392,71	78054183871	257650788151
	396,786	78073505668	257919821736
Average	591.1312	76835268842	249813552637

Dimension	4096	Block Size	256
	Time	L1	L2
	65,298	9118133192	22561081421
	64,88	9118812725	22614813659
	54,207	9123516041	22767010180
	54,192	9124197280	22757921397
	51,175	9126598777	22777140772
	49,202	9127824064	22841317491
	49,291	9128208926	22735382474
	48,652	9128541271	22850115700
	45,957	9135639052	22864688669
	45,975	9130689422	22860995591
Average	52.8829	9126216075	22763046735

Dimension	8192	Block Size	256
	Time	L1	L2
	643,609	72371697091	173492194573
	634,684	72415235941	173797832053
	629,097	72429274649	173905462730
	621,155	72457374191	174126756633
	606,997	72484273359	174646717474
	595,236	72511922846	175083022474
	589,142	72543838812	176772846122
	583,726	72554313874	177026267240
	572,13	72585047859	175608079731
	565,157	72599917145	175726783824
Average	604.0933	72495289577	175018596285

Dimension	6144	Block Size	128
	Time	L1	L2
	158,839	33030672178	109150914118
	158,878	33029916631	109134068097
	171,139	33062688738	109388290419
	161,748	33036903750	109647199531
	160,035	33032376376	109567201983
	159,414	33032654543	109115181368
	159,031	33031594583	109130508034
	161,127	33039261531	109158595639
	158,918	33033774879	109156210409
	161,45	33040009451	109336515351
Average	161.0579	33036985266	109278468495

Dimension	10240	Block Size	128
	Time	L1	L2
	734,604	152952731332	505831688169
	731,55	152960890025	506335833942
	735,887	152968137684	506280981368
	750,868	153581975475	508520564690
	805,929	152970309715	506010933360
	809,996	152942146755	507755796489
	810,013	152944706496	507689929659
	810,492	152944970713	507708588675
	799,737	152950808053	507676226967
	757,46	154202002026	507761851244
Average	774.6536	153141867827	507157239456

Dimension	6144	Block Size	256
	Time	L1	L2
	151,241	30774339270	77378402199
	152,309	30777015090	77278944966
	151,214	30801920208	77232533822
	152,525	30776206566	77260156182
	152,3	30781608436	76659563530
	149,882	30775019125	75617059083
	151,907	30780410862	75566773108
	150,787	30775576707	75583027163
	152,859	30782942777	75788966097
	150,771	30779521146	75565827081
Average	151.5795	30780456019	76393125323

Dimension	10240	Block Size	256
	Time	L1	L2
	718,4	142500496306	353718340489
	717,422	142500389049	353078078059
	709,784	142487637846	355326233472
	712,294	142492248596	352248260316
	709,866	142487598993	354617976412
	707,738	142492860056	355354605711
	711,95	142495295385	355126760925
	707,192	142496811870	352709671522
	706,476	142495933771	350041547547
	698,915	142476771565	350213769444
Average	710.0037	142492604344	353243524390

Dimension	4096	Block Size		512
	Time	L1	L2	
	62,625	8808574157	18912134656	
	58,797	8775406129	19097842450	
	60,208	8795662403	19201464832	
	55,274	8764576416	18871839514	
	59,046	8796238864	18764264257	
	62,978	8811284676	19017748790	
	65,321	8817121504	19128052413	
	86,027	8903406599	19576457871	
	70,995	8832091241	19006943593	
	61,031	8777163590	18970710003	
Average	64,2302	8808152558	19054745838	

Dimension	8192	Block Size		512
	Time	L1	L2	
	515,549	70640958077	149353505458	
	505,148	70578444890	150352046603	
	475,479	70189505177	150923157633	
	483,416	70445166816	151166317549	
	468,975	70393237302	150826897718	
	457,638	70242107232	150542943540	
	512,537	70666367212	150055536166	
	512,434	70664133970	150084626416	
	510,735	70664597940	150038694783	
	505,78	70662787837	149989925934	
Average	494,7691	70514730645	150333365180	

Dimension	6144	Block Size		512
	Time	L1	L2	
	189,48	29704107873	64498832883	
	185,405	29695952810	64342059203	
	194,228	29733747630	64733016716	
	192,412	29720929719	64428130589	
	184,295	29690491881	63767292301	
	184,115	29644714355	64159861650	
	182,369	29687349126	63767404166	
	182,28	29687441087	63774086132	
	186,425	29692016062	65042682126	
	179,07	29641559193	63677066632	
Average	186,0079	29689830974	64219043240	

Dimension	10240	Block Size		512
	Time	L1	L2	
	1024,671	137723493910	300944492736	
	1006,418	137603681529	300212413523	
	817,62	137260708045	294490633133	
	821,558	137271400864	294824164298	
	818,18	137212307289	294756315182	
	820,593	137267667369	294786151136	
	818,927	137274373621	294794425260	
	818,691	137266464437	294896819832	
	818,607	137225701532	294761682119	
	813,436	137160417520	294643322992	
Average	857,8701	137326621612	295911041952	

6.6 Line Multiplication Parallel Implementation A Measurements

	Dimension	600	
	Time	L1	L2
	0,0526132	2923730	6012588
	0,0452943	3335795	6803308
	0,0506207	3377618	6885462
	0,0535642	3277546	6697324
	0,0479309	3380021	6895075
	0,0460817	3403934	6936843
	0,0532249	3150519	6407274
	0,051657	3228385	6495466
	0,0528572	3159062	6370808
	0,0493365	3217586	6053328
Average	0,05031806	3245419,6	6555747,6

	Dimension	1400	
	Time	L1	L2
	0,509037	40910548	48890709
	0,516273	40281593	47922578
	0,508784	57880773	64371399
	0,514987	40963142	49274870
	0,518407	42734127	50932201
	0,512737	56744174	61115226
	0,51413	47172668	52245930
	0,532248	51123548	58068599
	0,522118	44786467	51398727
	0,513362	50133610	53908400
Average	0,5162083	47273065	53812863,9

	Dimension	1000	
	Time	L1	L2
	0,191789	9252857	19951386
	0,189911	9670097	20247575
	0,200636	10152970	21172752
	0,190748	8888318	19288855
	0,20274	9104724	19531924
	0,199517	11488211	24066549
	0,197844	8864089	19274490
	0,197435	9074653	19546821
	0,201619	8970892	18848582
	0,1953	9077196	19466374
Average	0,1967539	9454400,7	20139530,8

	Dimension	1800	
	Time	L1	L2
	1,11638	141455866	115750777
	1,21263	152494263	136490264
	1,39419	148040517	123015187
	1,22012	168899836	181263953
	1,15782	141882874	112606446
	1,08087	136976095	101342148
	1,27173	144981114	117823104
	1,09943	173857102	182534586
	1,08046	132997089	98096025
	1,08539	131742547	96514627
Average	1,171902	147332730,3	126543711,7

	Dimension	2200	
	Time	L1	L2
	2,00946	257242810	196995609
	2,06046	304384645	301554619
	1,99746	266188859	214908550
	2,00592	284355192	254654626
	1,99386	267666244	218422377
	1,99607	276387352	237093086
	1,9783	258838149	201386629
	2,00323	299810710	291328456
	2,07611	270626046	226503136
	2,00648	258918798	201776712
Average	2,012735	274441880,5	234462380
	Dimension	3000	
	Time	L1	L2
	5,16888	726821805	677850619
	5,76075	721602198	654735782
	5,69825	769247286	747888284
	5,39462	789112373	795578632
	5,16283	667088507	535737111
	5,60336	716945948	634586631
	5,1312	661564366	525088522
	5,19222	656330600	508098919
	6,53351	822740089	841466617
	6,63748	807514308	810483282
Average	5,62831	733896748	673151439,9
	Dimension	6144	
	Time	L1	L2
	45,8711	5841921895	5697038196
	50,5034	6570309964	7558343511
	51,2766	6924996735	8513896599
	51,3495	7117004652	9090042486
	49,1039	5856331381	5731330629
	50,4642	6531128341	7503461059
	50,4199	6948892265	8797289153
	49,8202	6993496453	8915502749
	51,0042	7107428351	9195667239
	50,4949	6453123496	7296025638
Average	50,03079	6634463353	7829859726
	Dimension	10240	
	Time	L1	L2
	240,678	28023928616	33975345845
	236,272	33087895185	52205496671
	241,954	32499349154	49669505252
	241,917	32683403784	51480013367
	244,075	33288572897	53396800146
	248,252	32772206495	50411851767
	243,286	28281249202	34599943832
	245,427	33183663221	52016757658
	249,999	33410549073	53797575939
	245,238	33517539301	54277176101
Average	243,7098	32074835693	48583046658
	Dimension	2600	
	Time	L1	L2
	3,31137	390072707	260569622
	3,343	503169163	495314044
	3,33303	474508222	435901149
	3,28166	507965965	513845083
	3,30841	487203972	468769882
	3,39579	436844389	357100206
	3,57365	418373122	315051586
	3,28986	433513009	346550620
	3,47784	466544960	420252467
	3,47421	442070968	365711317
Average	3,378882	456026647,7	397906597,6
	Dimension	4096	
	Time	L1	L2
	15,2224	1817893844	1596928117
	13,5625	2107258606	2312386724
	14,0548	1797884279	1563363962
	16,5967	2004489261	1980744685
	16,3514	2077061307	2124066226
	19,2409	2160072803	2260414867
	15,7707	2001965766	1982877395
	17,7838	2079085349	2145644128
	18,6411	2154560238	2272819908
	16,0099	1956754677	1892142347
Average	16,32342	2015702613	2013138836
	Dimension	8192	
	Time	L1	L2
	117,485	16261614283	23088630299
	118,236	16498112862	23648060727
	121,077	16146943132	22507234387
	122,055	16981750038	25139512094
	119,832	16299478140	23798432977
	120,835	15989549782	22625366773
	118,609	15615872137	21660725401
	120,798	16340661211	23128338391
	123,214	16168870726	22726192133
	124,974	16403452283	23679664963
Average	120,7115	16270630459	23200215815

6.7 Line Multiplication Parallel Implementation B Measurements

	Dimension	600	
	Time	L1	L2
	0,224466	4996919	7843732
	0,215194	5096542	7685282
	0,224137	4999290	7450600
	0,215968	4902100	8100521
	0,217905	5000203	8036475
	0,222805	4969207	8122814
	0,21785	4961790	8172133
	0,216867	5005687	8161815
	0,222298	4936727	6193848
	0,217362	4949694	7931944
Average	0,2194852	4981815,9	7769916,4

	Dimension	1000	
	Time	L1	L2
	0,659334	32703823	34782939
	0,663953	33906346	36684830
	0,664477	32579541	34914154
	0,764477	31364220	37203608
	0,672664	32748318	36111255
	0,696356	33703802	36711294
	0,712855	31044315	34076438
	0,720421	32305983	33617514
	0,804005	33789277	36419097
	0,823552	32510919	35941435
Average	0,7182094	32665654,4	35646256,4

	Dimension	1400	
	Time	L1	L2
	1,42605	95444553	92902587
	1,41573	89068474	79145589
	1,43502	95491103	91678408
	1,41739	95069738	92054564
	1,4281	97480214	93244419
	1,4333	82196578	65121147
	1,43919	78897149	51620557
	1,41852	83325529	58911784
	1,43014	87402672	77531700
	1,43012	97590096	98408661
Average	1,427356	90196610,6	80061941,6

	Dimension	1800	
	Time	L1	L2
	2,74028	195649590	195671078
	3,10295	202370396	205266055
	2,78094	194200730	185755780
	2,82083	192476050	183681900
	3,02278	194422023	192027035
	3,19046	201070249	206859060
	3,26118	193280820	204769237
	3,43509	200435324	206959464
	3,06041	199205328	200611594
	2,69137	202121039	200169488
Average	3,010629	197523154,9	198177069,1

	Dimension	2200	
	Time	L1	L2
	4,27976	360139931	361021297
	4,20821	338534186	317329460
	4,14641	331854453	280396641
	4,22852	293146340	220593757
	4,21535	343901589	332358351
	4,1518	326821199	290855506
	4,21022	320870485	271079851
	4,25751	315010156	271309995
	4,18588	353401839	355718958
	4,26043	360557626	373560047
Average	4,214409	334423780,4	307422386,3

	Dimension	2600	
	Time	L1	L2
	6,2977	535310400	485023783
	6,30048	528351618	473689173
	6,33596	549017257	520876211
	6,34126	528201017	472106424
	6,36766	564136876	555218152
	6,32775	496649526	411495528
	6,47837	497540899	411379940
	6,41156	599711534	637034243
	6,46524	591612122	620214924
	6,36525	587470141	612641530
Average	6,369123	547800139	519967990,8

	Dimension	3000	
	Time	L1	L2
	9,31976	864848379	886199294
	9,04624	877755900	922729739
	9,06548	878071440	937588016
	9,05053	843565799	852257359
	9,00798	880726194	937055620
	9,21131	897523450	977441316
	9,08417	869280210	930126273
	9,29807	798776553	766382156
	9,29613	693778695	512993659
	9,36613	808358573	786654173
Average	9,17458	841268519,3	850942760,5

	Dimension	4096	
	Time	L1	L2
	20,4262	2143167325	2301005295
	20,4601	1932501394	1801509177
	20,5706	2162867120	2334301052
	21,3031	2225035642	2482088204
	21,1487	2271169784	2555511408
	21,1435	2191099796	2379236847
	24,4082	2264036077	2472566394
	22,1064	2261485233	2504118135
	21,0883	2242296763	2479535827
	21,0776	2306595506	2624728905
Average	21,37327	2200025464	2393460124

	Dimension	6144	
	Time	L1	L2
	60,9454	7101689809	9066179378
	63,7106	7527073867	10111699991
	63,2497	6305367172	6845921647
	64,3836	6862812451	8228866667
	63,0899	5938538967	5947553776
	64,0929	7387387323	9725433592
	63,5624	6734069099	7716832024
	64,7389	6749234163	7821902509
	65,6539	6442056447	7121543879
	65,0043	6290276475	6874203470
Average	63,84316	6733850577	7946013693

	Dimension	8192	
	Time	L1	L2
	146,793	17327347596	25186876023
	151,837	16961510068	24282099788
	148,033	15935201890	21978539165
	150,265	15978125959	21820971721
	149,204	15998488746	22191097224
	149,332	17520192832	27106258054
	149,765	17837206258	28035976057
	148,339	17433480361	27050142845
	151,081	17036311776	25071676446
	148,483	17317472762	26261008057
Average	149,3132	16934533825	24898464538

	Dimension	10240	
	Time	L1	L2
	281,394	33620164785	54685841275
	285,851	34427457924	58725228662
	274,689	30728187778	45485532829
	291,256	33922165594	56707907502
	297,945	34576693149	58764284397
	283,262	34358197586	58964554027
	296,734	33971716463	55285345676
	288,25	33545503840	54782741465
	295,379	34024030319	55470136317
	288,544	32007827446	49957162613
Average	288,3304	33518194488	54882873476