Robótica Móvel

Localization

Part1 - Relative Positioning

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Summary

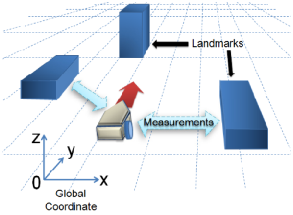
References and further reading

- Webster, J.G., Huang, S. and Dissanayake, G. (2016). Robot Localization: An Introduction. In Wiley Encyclopedia of Electrical and Electronics Engineering, J.G. Webster (Ed.). https://doi.org/10.1002/047134608X.W8318
- RAM and RMI courses at the University of Aveiro

Introduction

Introduction

- Robot localization is the process of determining where a mobile robot is located with respect to its environment.
- More specifically, determine the location (position and orientation) of the mobile robot in a map or a coordinate system.
- In a typical robot localization scenario, a map of the environment is available and the robot is equipped with sensors that observe the environment as well as monitor its own motion.
- The localization problem consists of estimating the robot position and orientation within the map using information gathered from the sensors.
- Robot localization techniques need to be able to deal with noisy observations and generate not only an estimate of the robot location but also a measure of the uncertainty of the location estimate.



The localization challenge

- Localization (positioning) can be done by
  - Internal incremental means (**dead-reckoning**)
  - External absolute references
- Sensors are required in both cases
  - Motion measurements, which are **integrated**
  - Detection of **features with known locations** and calculate the **relative position** to them
  - Reception of **absolute coordinates** from an externally **known** referenced system (like GNSS, etc.)
- Integration of motion measurements exhibits incremental errors and uncertainties, but detection of external features also has uncertainties
- Probabilistic techniques are used to improve the estimation of the localization by using models of motion and perception
  - Kalman filtering
  - Particle filter (or Monte Carlo) localization
  - Markov localization

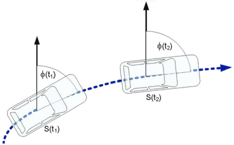
Models

- To perform localization estimation two types of models are required:
    - Robot and motion models
    - Sensor model
- Motion models are the **kinematics equations** specific to each typology of robot
- Sensor model
    - The relationship between the observations from the sensors and the location of the robot in the map.
    - The sensor model is dependent on the characteristics of the sensor mounted on the robot as well as on the way the map of the environment is represented
- The map of the environment is typically defined by:
    - **Coordinates** of known landmarks or features
    - **Occupancy grid** where the status of each grid cell defines whether the area represented by the cell is occupied or free space.

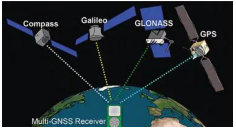
General approaches for localization (positioning)

- Predetermined path (wire guided, line on the pavement, etc. . .)
  - Advantage: the entire route is already delimited.
  - Limitations: low flexibility, precarious installation.
  - Following this path is mainly a technological issue and the problem of localization is simpler!

- Relative positioning systems
  - Determination relative to the last known position based on the measured movement.



- Absolute positioning systems
  - Determination in relation to a fixed coordinate system



- Mixed systems
  - Combine both types

Relative Positioning

Relative Positioning Systems (Dead-reckoning)

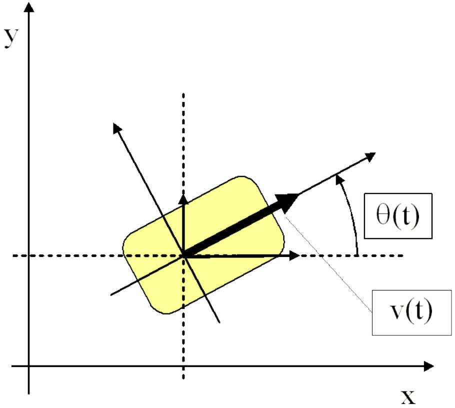
- Principle
  - Obtain the current position (location) based on the knowledge of previous position and the physical evolution of quantities associated with the movement of the mobile system.
  - These are actually incremental positioning systems.
- Examples
  - Odometry
  - Inertial navigation

Odometry

- Count the wheel angular displacements
- Fully self-contained system
- Errors can grow without limit and without control
- Sensors for odometry
    - Encoders
        - Optical (absolute and incremental)
        - Magnetic
        - Capacitive
    - Potentiometers
        - They present a resistance proportional to an angular displacement. Usually limited to systems with few revolutions (less than one sometimes)
    - Tachometers
        - Devices that measure an angular velocity and, through appropriate transformations, a linear velocity.

Odometry

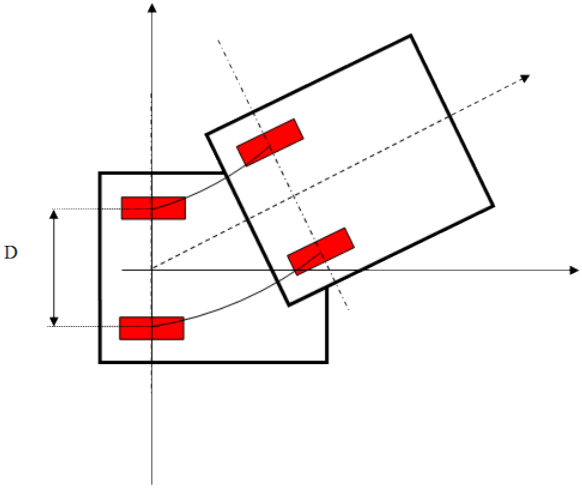
- Start with basic kinematics equations
  - $\dot{x}(t) = V(t)\cos\theta(t)$
  - $\dot{y}(t) = V(t)\sin\theta(t)$
  - $\dot{\theta}(t) = \omega(t)$
- Derive the inverse kinematics continuous model
  - $x(t) = \int_0^t V(\tau)\cos\theta(\tau)\mathrm{d}\tau$
  - $y(t) = \int_0^t V(\tau)\sin\theta(\tau)\mathrm{d}\tau$
  - $\theta(t) = \int_0^t \omega(\tau)\mathrm{d}\tau$



- Establish the discrete version assuming $\Delta\theta_i$ is very small in the measuring interval Δt
  - $\theta_i = \theta_{i-1} + \Delta\theta_i$
  - $x_i = x_{i-1} + \Delta l_i \cos\theta_i$
  - $y_i = y_{i-1} + \Delta l_i \sin\theta_i$
    - Where $\Delta l_i \approx V_i \Delta t$

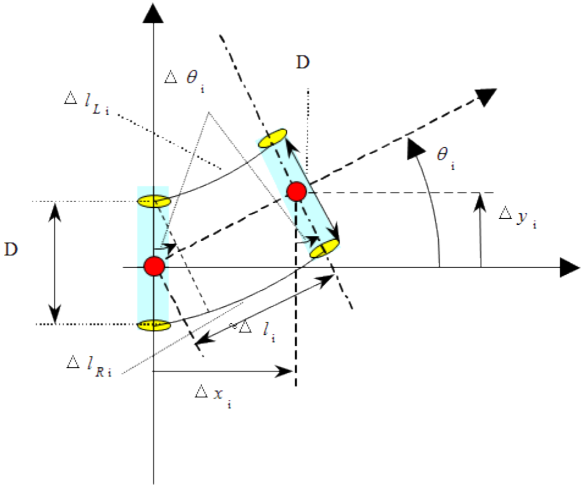
Odometry - details for differential drive

- Consider the constant k
- $k = \frac{\pi d}{n C_e}$
  - d - wheel diameter
  - n - gear ratio (motor/wheel)
  - C_e - number of pulses per turn (encoder)
- At the end of each sampling period T the encoder counters are read: N_L, N_R.
- And we obtain the displacement of each wheel:
  - $\Delta l_L = k N_L$
  - $\Delta l_R = k N_R$



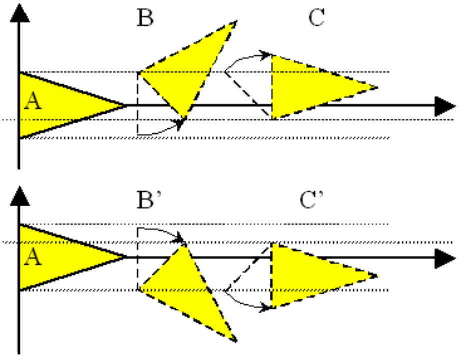
Odometry for differential drive

- Distance travelled by robot center:
  $\Delta l_i = \frac{1}{2}\left(\Delta l_{L_i} + \Delta l_{R_i}\right)$
- From the figure: $\Delta\theta_i = \frac{\Delta l_{R_i} - \Delta l_{L_i}}{D}$
- Approximating the arcs by straight segments (assuming $\Delta\theta_i \ll 1$) we have:
  - $\Delta\theta_i = \frac{\Delta l_{R_i} - \Delta l_{L_i}}{D}$
  - $\Delta l_i = \frac{\Delta l_{R_i} + \Delta l_{L_i}}{2}$
  - $\theta_i = \theta_{i-1} + \Delta\theta_i$
  - $x_i = x_{i-1} + \Delta l_i \cos\theta_i$
  - $y_i = y_{i-1} + \Delta l_i \sin\theta_i$



An exercise with odometry

- Give a counter-example that shows that, in a system with two independent parallel driving wheels, it is not enough to know only the distance traveled by each wheel, from an initial position, to determine the coordinates of a robot in a global reference.

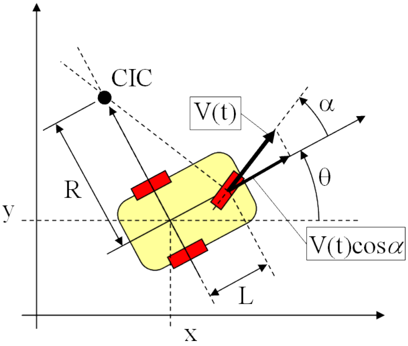


- The right and left wheels move the same amount, but only in the end are the sensors read, showing the same values in both cases but the robot is in different places!

Odometry for the tricyle

- Consider an incremental encoder in the front wheel (to measure its motion and velocity $V(t)$), and an absolute encoder in the steering bar (to measure wheel orientation $\alpha(t)$).

- As seen before, the continuous form of the kinematics is:
  - $$\begin{cases} \theta(t) = \frac{1}{L}\int_0^t V(\tau)\sin\alpha(\tau)\mathrm{d}\tau \\ x(t) = \int_0^t V(\tau)\cos\alpha(\tau)\cos\theta(\tau)\mathrm{d}\tau \\ y(t) = \int_0^t V(\tau)\cos\alpha(\tau)\sin\theta(\tau)\mathrm{d}\tau \end{cases}$$



- Which, for short intervals Δt, can be mapped to:
  - $$\begin{cases} \theta_i = \theta_{i-1} + \frac{V_i}{L}\sin\alpha_i\Delta t \\ x_i = x_{i-1} + V_i\cos\alpha_i\cos\theta_i\Delta t \\ y_i = y_{i-1} + V_i\cos\alpha_i\sin\theta_i\Delta t \end{cases}$$

- Reminding, as previously, that $V_i\Delta t = \Delta l_i$, the distance traveled by the traction wheel!

Sensors for inertial based positioning

Sensors for Inertial Navigation I

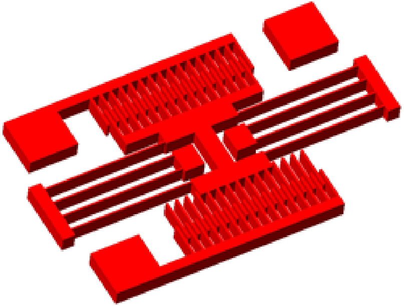
- Accelerometers
  - Indicate acceleration along one direction
  - Basic principle: measurement of a force $F = ma$ through its effects (resistive, piezoelectric, etc.)
  - The acceleration must be integrated twice to obtain the position.
  - In practice, the discrete variant of integration is used: a_i is the acceleration measured at the end of the interval Δt_i and which is normally assumed to be constant during this time interval

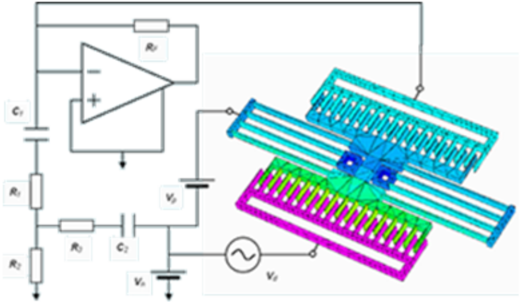
$$s = \iint a\,(t)\mathrm{d}t\mathrm{d}t \qquad s = \sum_{i=1}^{N}\left[\left(\sum_{i=1}^{N} a_i \Delta t_i\right)\Delta t_i\right]$$

- Uncertainties (systematic) in $a(t)$ accumulate and can result in great uncertainty in the final result.

Integrated Accelerometers - MEMS Technology

- Micro-Electro-Mechanical Systems (MEMS)
- The accelerometer is micro-machined
- Based on capacitive or resistive transduction
  - Moving parts affect geometries by changing electrical capacities and/or electrical resistances





Sensors for Inertial Navigation II

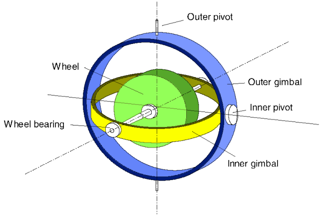
- The Gyroscopes (Gyros)
  - Indicate an angular velocity
  - Integration of these measurements results in absolute heading (orientation)
- Main types of Gyros
  - Mechanics
    - Delicate; expensive; ...
  - Optics
    - No mechanical parts.
    - Based on optical interference from laser beams.
    - More economical with very reasonable accuracies
    - Drift problem
  - Piezoelectric
- Some systems combine accelerometers and gyroscopes - the INS (Inertial Navigation Systems)

Types of Gyroscopes

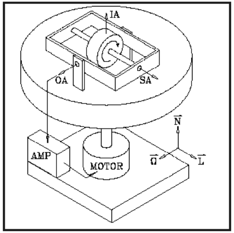
- Mechanical
  - Principle of conservation of angular momentum
  - Precession effect
  - Experience of rotating bicycle wheel when axle position is changed
  - Force measurement / angular deviation
- Optical (Fiber Optical Gyroscope - FOG)
  - Based on Sagnac effect
    - Active or passive resonator
    - Open or closed loop fiber optic interferometer (IFOG)
    - Fiber optic resonator (RFOG)
  - Measurement of phase differences (propagation time)
- Piezoelectric
  - Based on Coriolis force
  - Vibrating material
  - Piezoelectric measurement

Mechanical Gyroscope

- A mass is kept in constant rotation (eg with a precise servomotor)
- The mass rotation axis is supported by a system that can rotate orthogonally
- The entire system is supported by an external structure to which the rotation is applied (on an axis orthogonal to the other two)
- The angular deviation of the second axis indicates the angular velocity of the outer system
- There are other variants but the principle is the same...

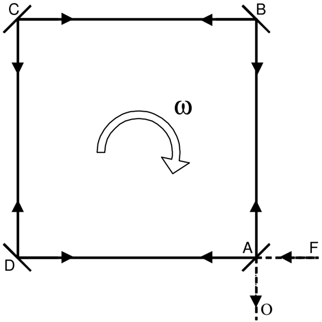


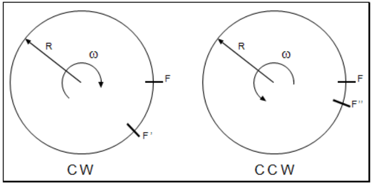
Typical two-axis mechanical gyroscope configuration [Everett, 1995].



- Phenomenon observed by Georges Sagnac (1896-1926) in 1913
- Propagate light along a closed path
- Different propagation times depending on the direction of rotation of the path (clockwise or counterclockwise)

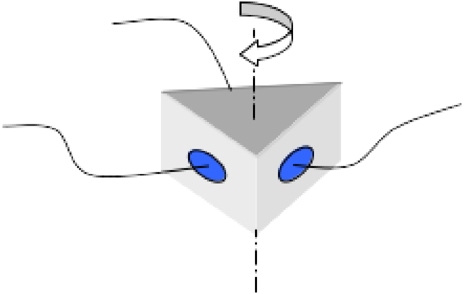
$$\Delta t = \frac{4\pi R v}{c^2 - v^2} \approx \frac{4\pi R^2}{c^2}\omega$$

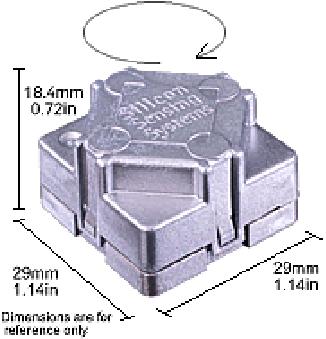




Piezoelectric gyros

- Typically a triangular quartz prism
- Excitations on one side at resonant frequency
- The other two faces have transducers that measure vibrations affected by device rotation
- Coriolis forces affect the propagation of vibrations between faces





Sources of error in optical gyros

- Thermal noise;
- Poor coupling between the light source and the propagation medium;
- Scattering of light in the coupling zone between the source and the propagation medium;
- Light scattering medium of propagation;
- Laser-based ones can have issues of output non-linearity with angular velocity.

Some gyros specification terms

- Random walk
  - Measured in $^\circ/\sqrt{\mathrm{h}}$ or $^\circ/\mathrm{h}/\sqrt{\mathrm{h}}$ or $^\circ/\mathrm{h}/\sqrt{\mathrm{Hz}}$
- Bias instability
  - Measured in $^\circ/\mathrm{h}$
- Drift Rate (bias offset)
  - Measured in $^\circ/\mathrm{h}$ or $^\circ/\mathrm{min}$ or $^\circ/\mathrm{s}$
- Sensitivity axis
- Scale factor
  - Measured in $\mathrm{mV}/^\circ/\mathrm{s}$

Random Walk

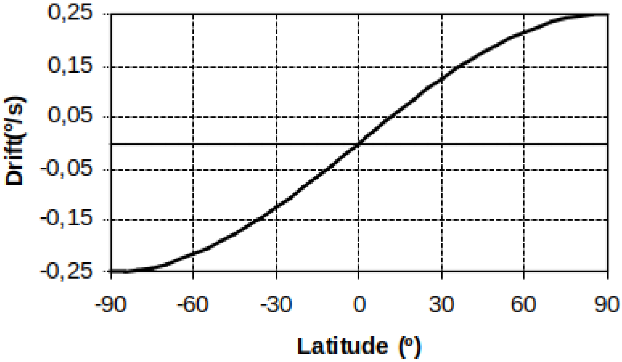
- Refers to the angular error due to the existence of **wideband random noise** (sometimes called white noise) in the output signal.
  - This error is normally expressed, according to IEEE standardization, in $^\circ/\sqrt{\mathrm{h}}$ if the output is an angular value.
  - If the gyroscope output is a rotation rate (angular velocity) the random walk will be expressed in $^\circ/\mathrm{h}/\sqrt{\mathrm{h}}$.
  - It is normal to find in some manufacturers the value expressed in $^\circ/\mathrm{h}/\sqrt{\mathrm{Hz}}$.
- This error decreases with increasing integration (sampling) interval.
- There is therefore a trade-off between the sampling frequency and the associated error!

Bias Instability

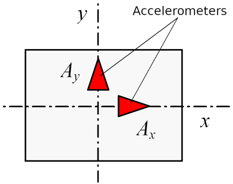
- The output signal from a gyroscope has a component that is not correlated with the applied rotation.
- The calculation of this value over a given time interval gives rise to bias instability.
- This value is determined when the gyroscope is not rotating and has a zero-average oscillatory value, and its RMS value is referred to in the specifications.
- It is typically expressed in degrees per hour ($^\circ/\mathrm{h}$).
- Basically it is the systematic error of the system.

The Drift Question

- Residual indication of angular velocity when it is actually immobilized!
- In high precision applications it is necessary to take into account the earth rotation which is an apparent drift!
  - Its value depends on latitude and is in the worst case $360°$/24h or $0.25°$/min



An exercise with accelerometers



- Robot with two orthogonal accelerometers with readings every $100\,\text{ms}$
- Determine the velocities and position knowing that the robot starts from a stopped position.
- $s = \sum_{i=1}^{N} \left[\left(\sum_{i=1}^{N} a_i \Delta t_i \right) \Delta t_i \right]$

			Velocity			Space run		
			In x	In y	Ampl	In x	In y	Linear
t	A_x	A_y	$\sum A_x$	$\sum A_y$	$\lvert \sum A \rvert$	$\sum \sum A_x$	$\sum \sum A_y$	$\lvert \sum \sum A \rvert$
0.0	0.10	0.00						
0.1	0.05	0.00	0.010	0.000	0.0100	0.0010	0.0000	0.0010
0.2	0.05	0.00	0.015	0.000	0.0150	0.0025	0.0000	0.0025
0.3	0.04	0.05	0.020	0.000	0.0200	0.0045	0.0000	0.0045
0.4	0.04	0.04	0.024	0.005	0.0245	0.0069	0.0005	0.0069
0.5	0.03	0.04	0.028	0.009	0.0294	0.0097	0.0014	0.0098
0.6	0.00	0.03	0.031	0.013	0.0336	0.0128	0.0027	0.0131
0.7	0.00	0.01	0.031	0.016	0.0349	0.0159	0.0043	0.0165
0.8	-0.05	-0.07	0.031	0.017	0.0354	0.0190	0.0060	0.0199
0.9	-0.10	-0.10	0.026	0.010	0.0279	0.0216	0.0070	0.0227
1.0			**0.016**	**0.000**	**0.0160**	**0.0232**	**0.0070**	**0.0242**
			0.018	0.000	0.0176	0.0255	0.0070	0.0265

- Red values would occur if the measurements of A_x had 10% error!