Robótica Móvel

Locomotion

Adapted from the RMI course notes

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Summary

Basics

Mobile robot

- A mobile robot is a combination of various physical and computational units (hardware and software)
- Organized in a set of sub-systems:
  - Sensing
    - measures properties of the robot environment
  - Reasoning
    - maps measurements into high-level action commands
  - Control
    - transforms high-level action commands into low-level actions
  - Actuation
    - transforms low-level action commands into physical actions
  - Locomotion
    - maps physical actions into movement, e.g, defines how the robot moves in its environment
  - Communication
    - provides communication with other robots (or with an external operator)

Locomotion

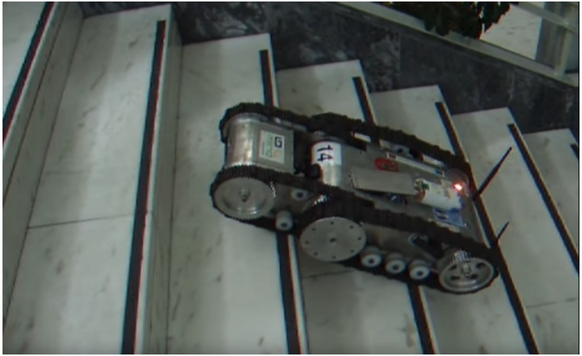
- Locomotion is the deliberate physical process that makes the robot move in its environment from one location to another
- Several solutions available:
    - Tracked locomotion
    - Legged locomotion
    - Wheeled locomotion
- Besides the land-oriented solutions there are also
    - Airborne robots (also known as drones, UAVs, etc.)
    - Marine robots (both surface and underwater)
- These last categories are excluded from this study on locomotion!

Tracked locomotion

- Locomotion system well suited for robots that evolve in very rough terrain (e.g., in natural disaster situations)
- Great traction power - the track contact area with the ground is greater than the one provided by a wheel
- Change of direction is achieved by sliding the tracks, which makes it very difficult to use odometry as a method of localization
- Requires a large amount of power to turn
- The robots that use this type of movement are typically tele-operated

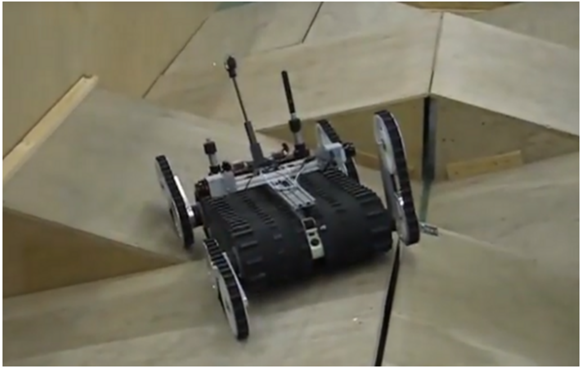
Tracked locomotion - two examples

Raposa (IST - Lisbon)



`https://youtu.be/4UxPvNu5x5s`

Quince Robot - Japan



`https://youtu.be/AkIWxFD2k5Y`
`https://youtu.be/J7GkNF5gLAk`

Legged locomotion

- Locomotion with legs is often based on living beings (as those that move in difficult environments)
- The implementations of this type of locomotion system in robots is complex:
  - Mechanical complexity
  - Stability
  - Power consumption
- Besides mechanical complexity, also kinematics and dynamics are very complex.
  - Their study deserves its own course!

Legged locomotion (some Boston Dynamics robots)

https://youtu.be/fn3KWM1kuAw

Wheeled locomotion

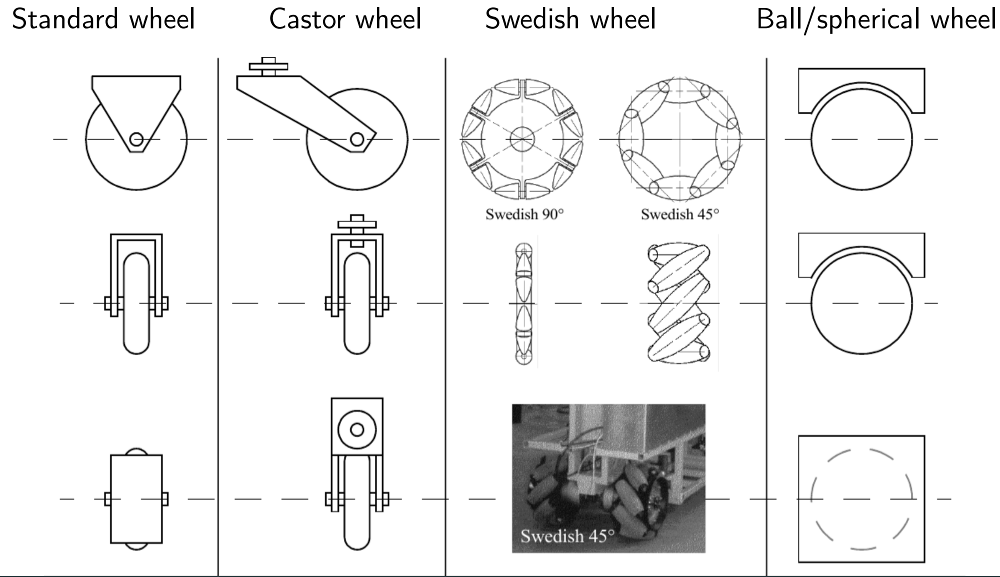
- The wheeled locomotion solution is the most suitable for common applications
  - ... because rolling is very efficient!
- The configuration and type of wheel to use is dependent on the application
- Main constraint: requires flat terrain (or at most slightly irregular)
- Bigger wheels allow the robot to overcome bigger obstacles. However:
  - Motors with larger torque are needed (or gearboxes with bigger reduction ratios, i.e., lower output speed for the same motor)

Wheeled locomotion – static stability

- Two wheels
  - Minimum number of wheels to achieve stability
  - Center of gravity must be below the axle that links the wheels
- Three wheels
  - Stable configuration
  - Center of gravity must be inside the triangle formed by the wheels
- Four wheels
  - Stable configuration
  - Requires a suspension system to compensate for irregularities in the environment where the robot has to move
- More than four wheels
  - Configuration dependent

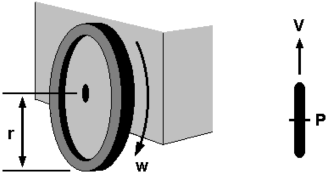
Wheel types

The four wheel types according to Siegwart et al.

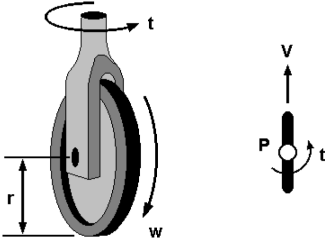


Practical wheel types

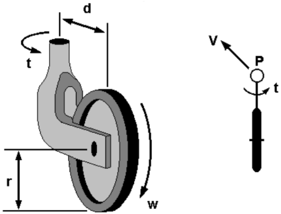
Standard wheel



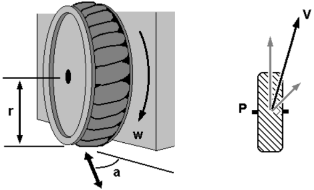
Steered standard wheel



Off-centered orientable wheel (castor wheel)

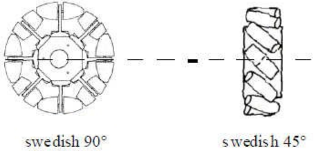


Swedish wheel (omnidirectional)

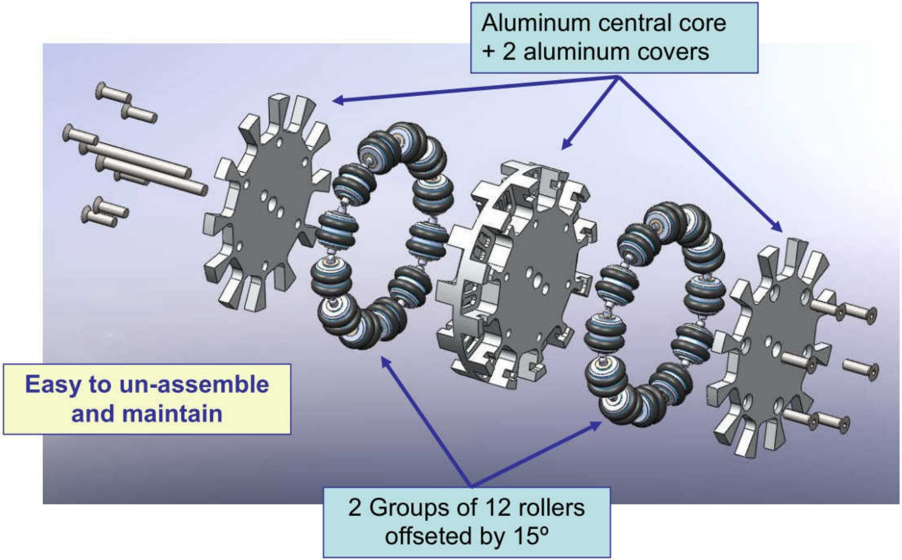


Swedish wheel

- Small rollers around the circumference, perpendicular to the rotation direction
- The wheel can be driven with full force, but will also slide laterally with very low friction
- Omnidirectional property
- Three degrees of freedom:
  - Rotation around the wheel axle (motorized)
  - Around the rollers
  - Around the contact with the ground point

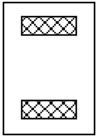


Swedish wheel - mechanical construction



Wheel configurations: 2 wheels

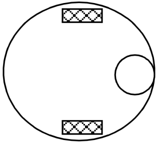
- One steering wheel in the front
- and one traction wheel in the rear



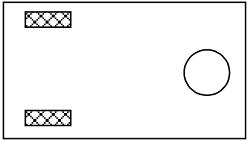
- Two-wheel differential drive with the center of mass below the axle

This slide and next five from: R. Siegwart et al. (2011)

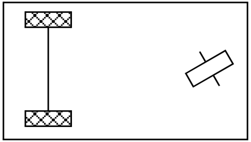
Wheel configurations: 3 wheels



- Two-wheel centered differential drive with a third point of contact

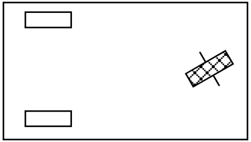


- Two independently driven wheels in the rear/front,
- one steered free wheel (unpowered) in the front/rear

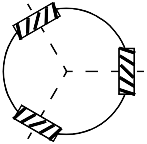


- Two connected traction wheels (differential gear) in rear,
- one steered free wheel in front

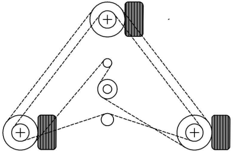
Wheel configurations: 3 wheels



- Two free wheels in rear,
- one steered traction wheel in front

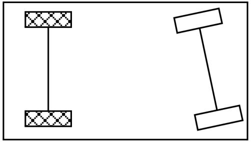


- Three motorized Swedish or spherical wheels arranged in a triangle;
- omnidirectional movement is possible

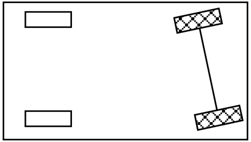


- Three synchronously motorized and steered wheels;
- the chassis orientation is not controllable

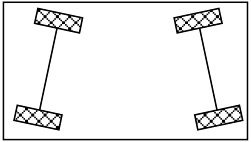
Wheel configurations: 4 wheels



- Two motorized wheels in the rear,
- two steered wheels in the front;
- steering has to be different for the two wheels to avoid slipping/skidding.

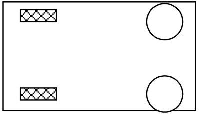


- Two motorized and steered wheels in the front,
- two free wheels in the rear;
- steering has to be different for the two wheels to avoid slipping/skidding.

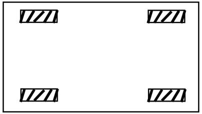


- Four steered and motorized wheels

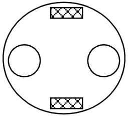
Wheel configurations: 4 wheels



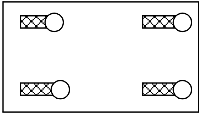
- Two traction wheels (differential) in rear/front,
- two omnidirectional wheels in the front/rear



- Four omnidirectional wheels

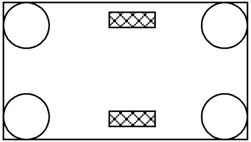


- Two-wheel differential drive with two additional points of contact

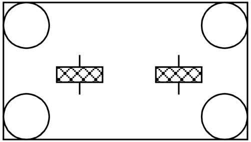


- Four motorized and steered castor wheels

Wheel configurations: 6 wheels



- Two traction wheels (differential) in center,
- one omnidirectional wheel at each corner



- Two motorized and steered wheels aligned in center,
- one omnidirectional wheel at each corner

Non-standard configurations

SHRIMP (EPFL)

Non-standard configurations

SpaceCat (EPFL)

Robot for scientific applications in Mars, developed for ESA

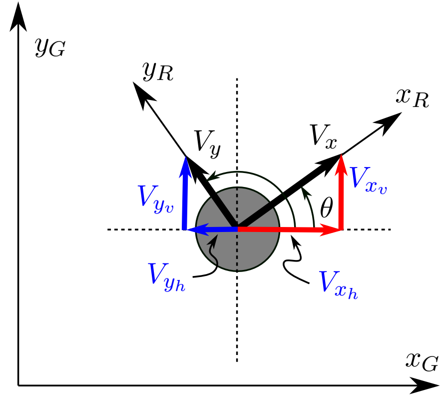
Kinematics

Kinematics

- Locomotion
  - The process that causes the movement of the robot from one location to another
  - In order to produce a motion, forces must be applied to the robot
- Dynamics
  - The study of motion, in which forces, masses and inertias are taken into account
- Kinematics
  - Modeling the motion without considering the forces that affect the motion

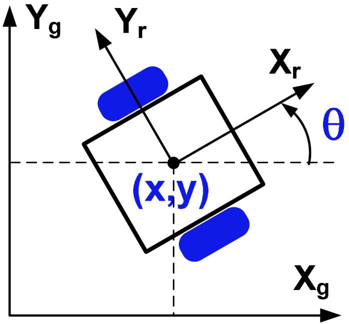
Local and global reference frames

- The **orthogonal rotation matrix** to relate velocities between frames
- R is a local reference frame on the robot and G is the global frame
- Generic coordinates on either frame are given by $\xi = \begin{bmatrix} x & y & \theta \end{bmatrix}^\mathsf{T}$
- $\dot{\theta}_G = \dot{\theta}_R = \dot{\theta}$ and the other velocities relate as shown:



- $\dot{x}_G = V_{x_h} + V_{y_h} = V_x \cos\theta - V_y \sin\theta$
- $\dot{y}_G = V_{x_v} + V_{y_v} = V_x \sin\theta + V_y \cos\theta$
- $\begin{bmatrix} \dot{x}_G \\ \dot{y}_G \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta} \end{bmatrix}$
- $\begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_G \\ \dot{y}_G \\ \dot{\theta} \end{bmatrix}$

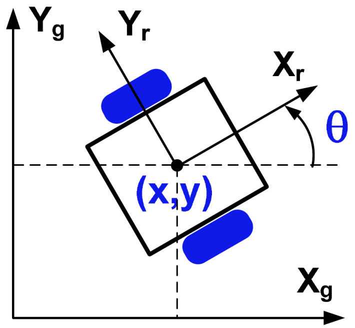
Local and global reference frames



- Mapping velocities:
  - from **global** reference frame to **robot** reference frame
    - Xr, Yr – local (robot) reference frame
    - Xg, Yg – global reference frame
    - $\xi = \begin{bmatrix} x & y & \theta \end{bmatrix}^{\mathsf{T}}$
- $\dot{\xi}_R = R(\theta)\,\dot{\xi}_G = R(\theta)\begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}^{\mathsf{T}}$
- $R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\rightarrow$ the orthogonal rotation matrix.
- $\dot{\xi}_R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$

Local and global reference frames

- Mapping velocities:
  - from **robot** reference frame to **global** reference frame
    - Xr, Yr – local (robot) reference frame
    - Xg, Yg – global reference frame
- $\dot{\xi}_G = R(\theta)^{-1}\dot{\xi}_R = R(\theta)^{-1}\begin{bmatrix} V_x & V_y & \dot{\theta} \end{bmatrix}^\mathsf{T}$

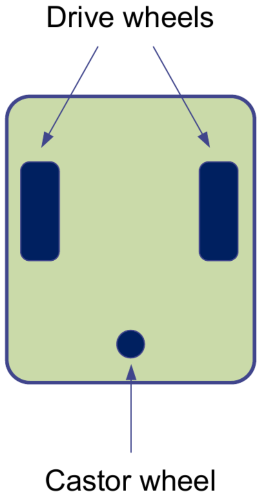


- $R(\theta)^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- $\dot{\xi}_G = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ \dot{\theta} \end{bmatrix}$

Differential drive

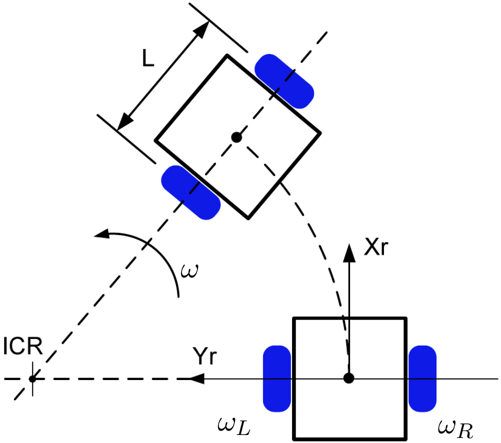
Differential drive

- Common configuration:
  - 2 active independent drive wheels
  - 1 or 2 passive castor wheels
- Robot follows a trajectory which is defined by the speed of each wheel
- Trajectory is sensitive to differences in the relative velocity of the two wheels
  - caused by asymmetries in motors and/or wheels
  - a small error results in a path different from that intended
- Easy mechanical implementation



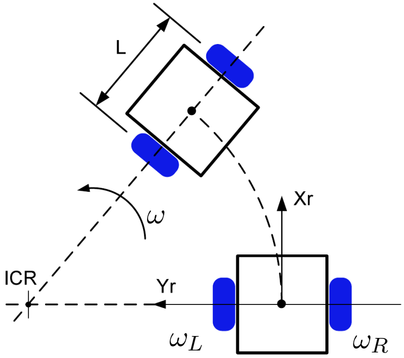
Differential drive – kinematics

- ω_R – angular velocity, right wheel
- ω_L – angular velocity, left wheel
- V_R – linear velocity, right wheel
- V_L – linear velocity, left wheel
- ω – angular velocity of the robot about ICR
- r – wheel radius
- L – distance between wheels



Differential drive – kinematics

- Kinematic model in **local** robot frame
  - $V_R(t) = \omega_R(t) \times r$
  - $V_L(t) = \omega_L(t) \times r$
  - $V_X(t) = \frac{V_R(t)+V_L(t)}{2} = \frac{r}{2}(\omega_L(t) + \omega_R(t))$
  - $V_Y(t) = 0$
  - $\omega(t) = \frac{V_R(t)-V_L(t)}{L} = \frac{r}{L}(\omega_R(t) - \omega_L(t))$



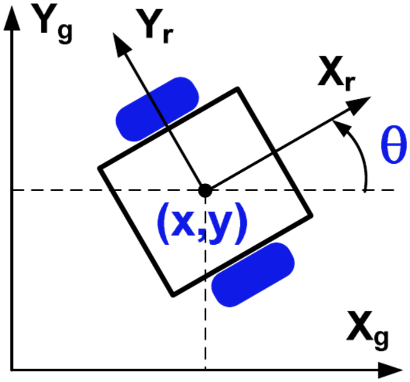
$$\begin{bmatrix} V_X(t) \\ V_Y(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ -r/L & r/L \end{bmatrix} \begin{bmatrix} \omega_L(t) \\ \omega_R(t) \end{bmatrix}$$

Differential drive – kinematics

- Kinematic model in **world** frame

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & -\sin\theta(t) & 0 \\ \sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_X(t) \\ V_Y(t) \\ \omega(t) \end{bmatrix}$$

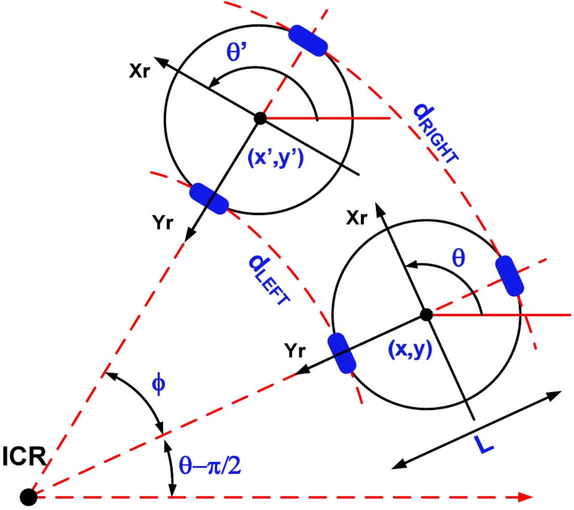
$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V(t) \\ \omega(t) \end{bmatrix}$$



Differential drive – position estimation

- (x, y, θ) – pose (position and orientation) of the robot in the world frame
- Supposing the robot's pose is (x, y, θ), the position estimation consists in finding (x', y', θ') given:
  - d_{RIGHT} - distance travelled by the right wheel
  - d_{LEFT} - distance travelled by left dwheel
    - d_{RIGHT} and d_{LEFT} measured by wheel encoders (odometry)

Robot is moving counterclockwise

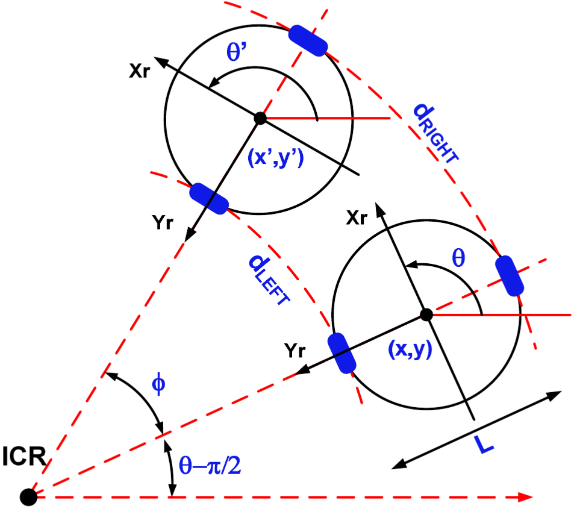


L - distance between robot wheels

Differential drive – position estimation

- Over a small time period, the robot motion can be approximated by an arc
- $d_{CENTER} \approx \dfrac{d_{RIGHT} + d_{LEFT}}{2}$
- $\phi \approx \dfrac{d_{CENTER}}{R}$
- $\phi \times R_{RIGHT} = d_{RIGHT}$
- $\phi \times R_{LEFT} = d_{LEFT}$
- $R_{RIGHT} - R_{LEFT} = L$
- $\phi = \dfrac{d_{RIGHT} - d_{LEFT}}{L}$

Robot is moving counterclockwise

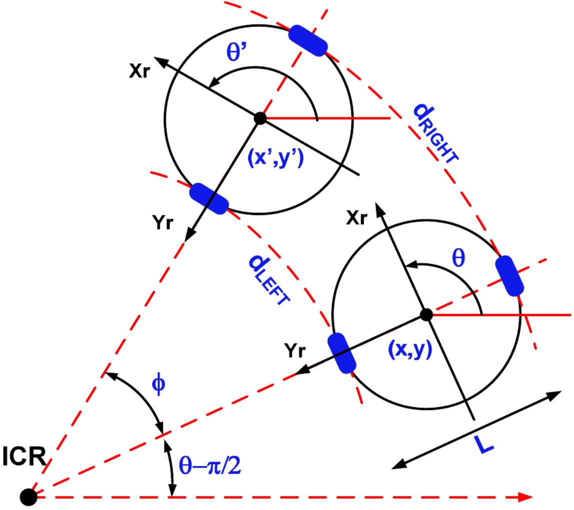


ICR: Instantaneous Center of Rotation

Differential drive – position estimation

- $d_{CENTER} = \dfrac{d_{RIGHT} + d_{LEFT}}{2}$
- $\phi = \dfrac{d_{RIGHT} - d_{LEFT}}{L}$
- For small displacements, such that $\sin(\phi) \approx \phi$ and $\cos(\phi) \approx 0$:
  - $x' = x + d_{CENTER} \times \cos\theta$
  - $y' = y + d_{CENTER} \times \sin\theta$
  - $\theta' = \theta + \phi$

Robot is moving counterclockwise

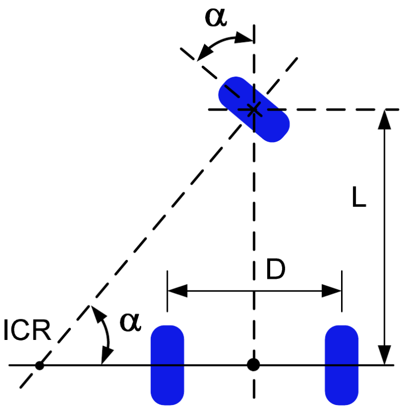


ICR: Instantaneous Center of Rotation

Tricycle drive

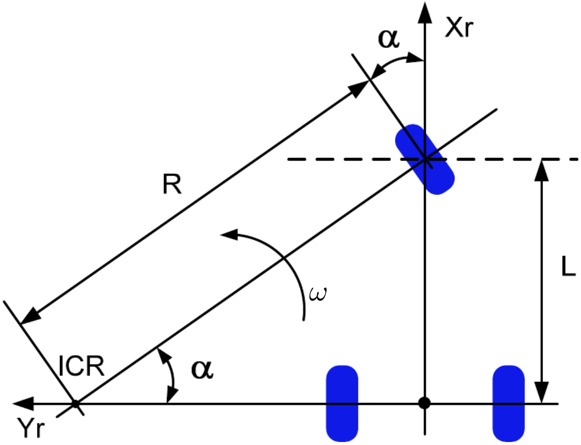
Tricycle drive

- Three wheels: two rear wheels and one (steering) front wheel
- Two possible configurations of traction:
  - Driving wheel on the front (rear wheels are passive) – easier to implement
  - Front wheel is passive - the two rear wheels are driving wheels (must use differential gear)
- Main problems of the front wheel drive configuration:
  - When going uphill, the driving wheel may loose traction due to the displacement of the center of mass
  - The traction contact area with the ground is half of the rear wheel drive configuration



Tricycle drive – kinematics

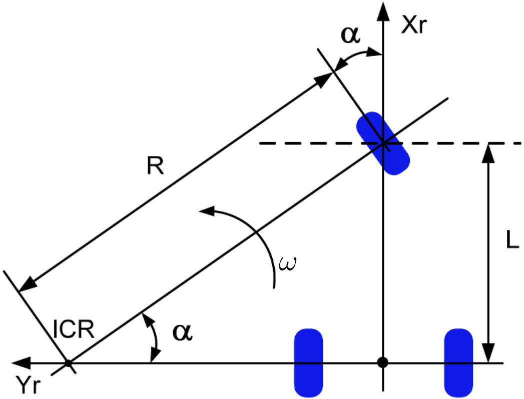
- V_S – linear velocity of the steering wheel
- ω_S – angular velocity of the steering wheel
- r – steering wheel radius
- α – steering angle
- ω – angular velocity of the robot about ICR



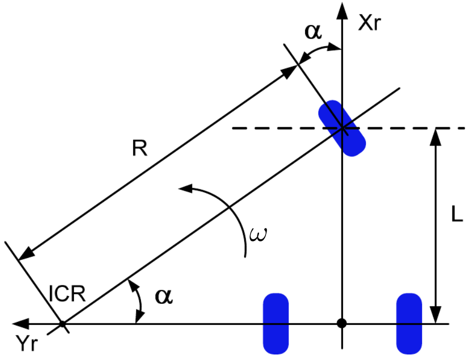
Tricycle drive – kinematics

- $V_S = \omega_S \times r \rightarrow$ linear velocity of the steering wheel
- $\omega_S = \frac{V_S}{r} \rightarrow$ angular velocity of the steering wheel
- $R = \frac{L}{\sin\alpha}$
- $\omega = \frac{V_S}{R} = \frac{V_S \sin\alpha}{L} \rightarrow$ angular velocity of the robot about ICR

- Kinematic model in the **local** frame
  - $V_X(t) = V_S(t) \cos\alpha(t)$
  - $V_Y(t) = 0$
    - because the origin of the coordinate frame does not slide along Y_r!
  - $\omega(t) = \frac{V_S(t)}{L} \sin\alpha(t)$



- Kinematic model in the **world** frame

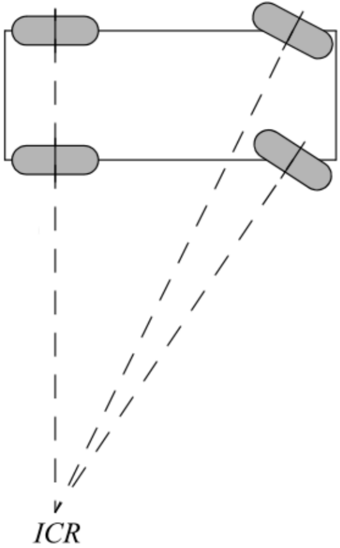


$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & -\sin\theta(t) & 0 \\ \sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_S(t)\cos\alpha(t) \\ 0 \\ \frac{V_S(t)}{L}\sin\alpha(t) \end{bmatrix}$$

Ackerman steering

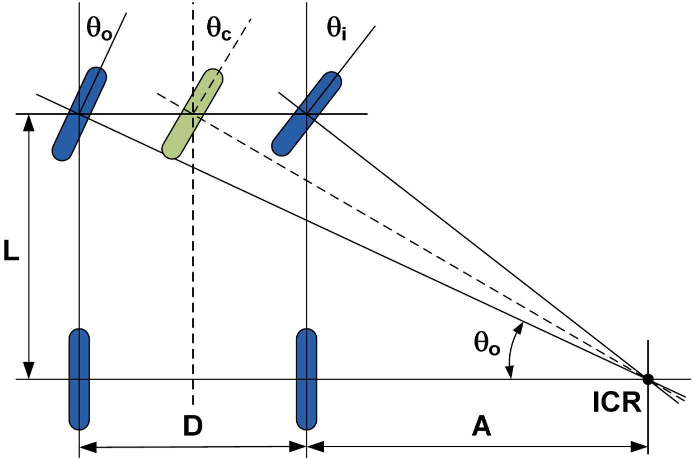
Ackerman steering

- Generally the method of choice for outdoor autonomous vehicles
- The inside front wheel is rotated slightly more than the outside wheel (reduces tire slippage)
- The extension of the axis of the two wheels intersects a common point - ICR
- 4 or 3 wheel system support rear and/or front traction
- A differential gear must be used in the traction axel (unless a single motorized wheel is used in that axel)



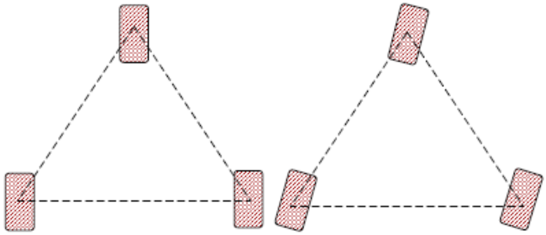
Ackerman steering

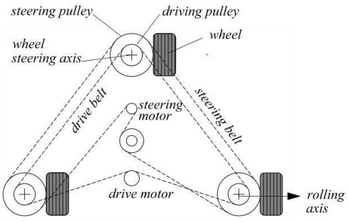
- $\cot \theta_i = \frac{A}{L}$
- $\cot \theta_o = \frac{A+D}{L}$
- $L \cot \theta_o = L \cot \theta_i + D$
- $\cot \theta_c = \cot \theta_o - \frac{D}{2L}$
- $\cot \theta_o - \cot \theta_i = \frac{D}{L}$

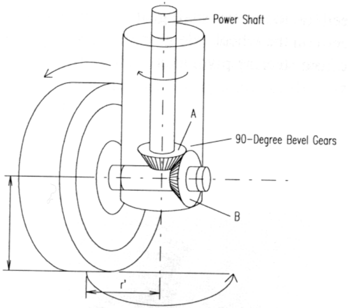


Synchro drive

Synchro drive

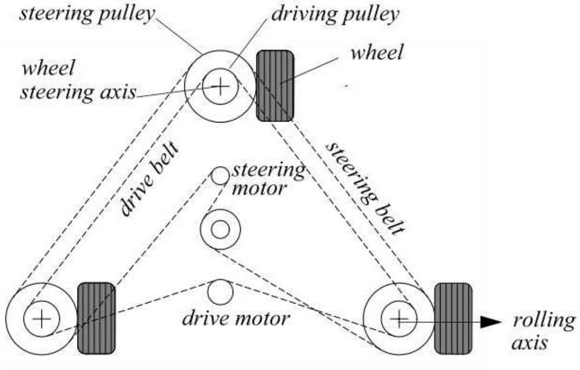






- Three (or more) wheels
- Two motors:
  - Translation motor sets the speed of all three wheels together
  - Steering motor spins all the wheels together about each of their individual vertical steering axes

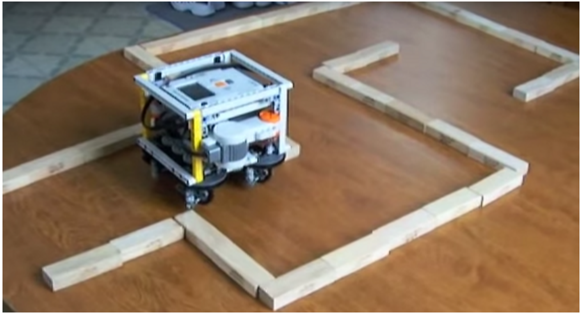
Synchro drive



- The robot can move in any direction
- The robot can always reorient its wheels and move along a new trajectory without changing its footprint
- However, the orientation of the chassis is not controllable (since the wheels are being steered with respect to the robot chassis)

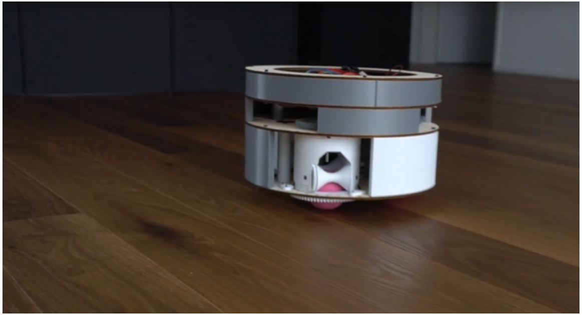
Two examples of Synchro drive

LEGO



https://youtu.be/MFxjIthqXVs

Simple RC piloted



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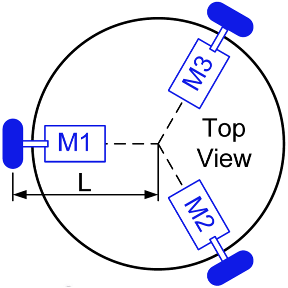
Omnidirectional drive

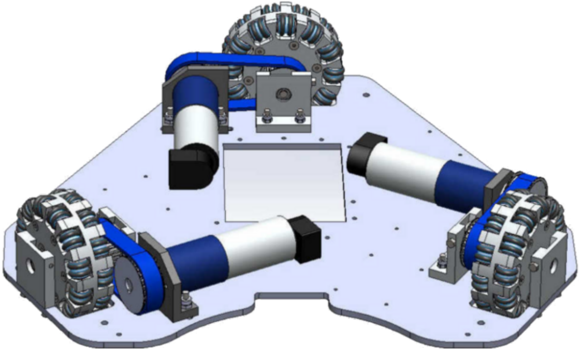
Omnidirectional drive

- Uses swedish wheels
- Each wheel has one independent drive motor
- Allows movement in any direction by setting appropriate speeds in each of the three motors
- Allows complex movements (for instance translation combined with rotation)
- Three wheels configuration:
  - the wheels are spaced $120°$



Omnidirectional drive

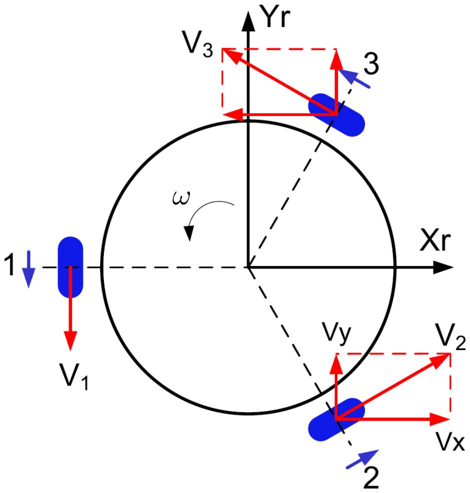
- Allows the generation of complex movements, such as to go straight changing, at the same time, the robot orientation
- Excellent maneuverability
- 4-wheel configuration: greater traction but more sensitive to uneven floors



CAMBADA soccer robot – omnidirectional drive structure

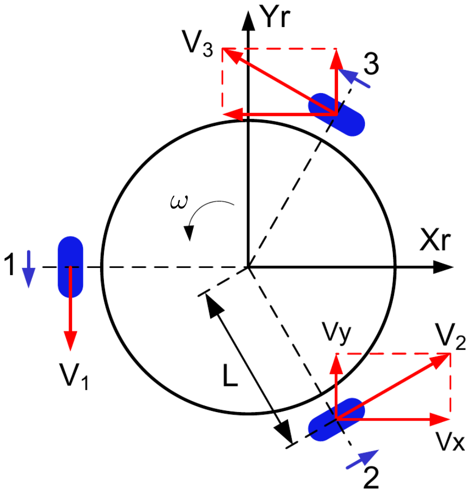
Omnidirectional drive – kinematics

- The translation velocities of the wheels V_1, V_2 and V_3 determine the global velocity of the robot on the environment
- The translation velocity of the wheel hub i (V_i) can be divided in two parts:
  1. pure translation of the robot;
  2. pure rotation of the robot:
     - $V_i = V_{transl,i} + V_{rot}$

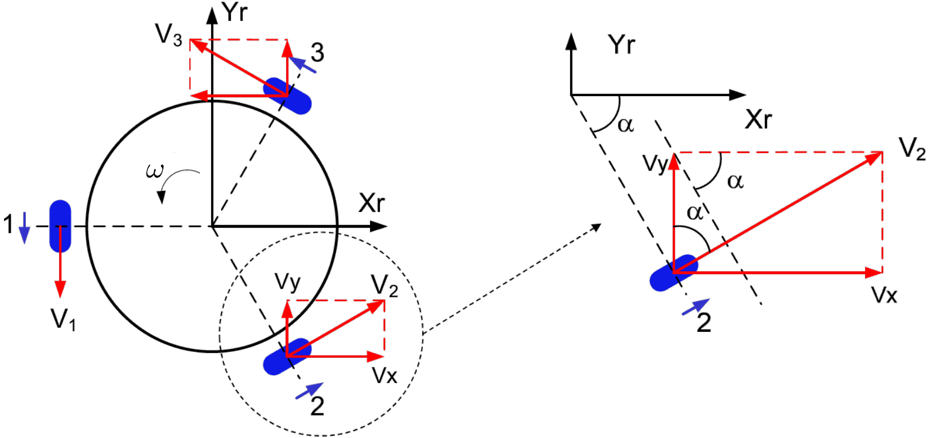


Omnidirectional drive – kinematics

- When the robot performs a pure rotation, the hub "i" velocity becomes
  - $V_i = L\omega$
- where:
  - L: is the distance from the geometric center of the robot to the wheels
  - ω: is the angular velocity of the robot



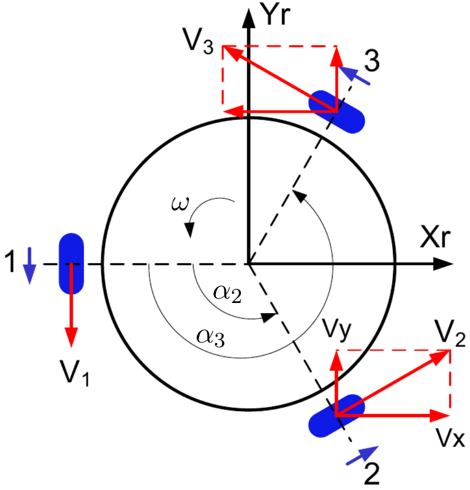
Omnidirectional drive – kinematics



- $V_{transl,1} = \sin(0) - \cos(0)V_y = -V_y$
- $V_{transl,2} = \sin(\alpha)V_x + \cos(\alpha)V_y$
- $V_{transl,3} = -\sin(\alpha)V_x + \cos(\alpha)V_y$

Omnidirectional drive – kinematics

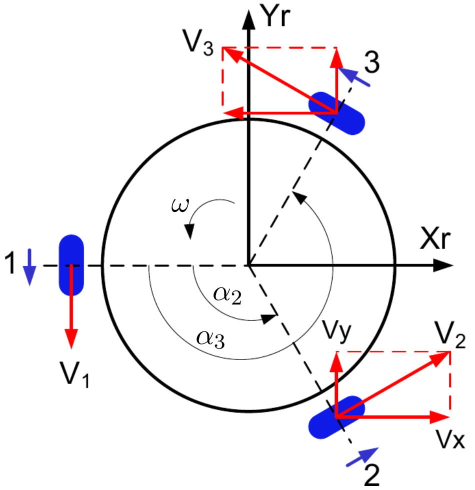
- Taking hub 1 as reference, the hub angles are:
  - $\alpha_1 = 0°, \alpha_2 = 120°, \alpha_3 = 240°$
- The pure translation velocity at wheel hub "i" can then be generalized as:
  - $V_{transl,i} = \sin(\alpha_i)V_x - \cos(\alpha_i)V_y$
- And V_i becomes:
  - $V_i = \sin(\alpha_i)V_x - \cos(i\alpha_i)V_y + L\omega$
  - $V_i = r\omega_i$
    - being r the wheel radius and ω_i the wheel angular velocity



Omnidirectional drive – kinematics

- We can then write:
  - $\omega_1 = \frac{1}{r}\left(-V_y + L\omega\right)$
  - $\omega_2 = \frac{1}{r}\left(\frac{\sqrt{3}}{2}V_x + 0.5V_y + L\omega\right)$
  - $\omega_3 = \frac{1}{r}\left(-\frac{\sqrt{3}}{2}V_x + 0.5V_y + L\omega\right)$
- Solving for V_x, V_y and ω:

$$\begin{bmatrix} V_x \\ V_y \\ \omega \end{bmatrix} = \begin{bmatrix} 0 & \frac{r}{\sqrt{3}} & -\frac{r}{\sqrt{3}} \\ -\frac{2r}{3} & \frac{r}{3} & \frac{r}{3} \\ \frac{r}{3L} & \frac{r}{3L} & \frac{r}{3L} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

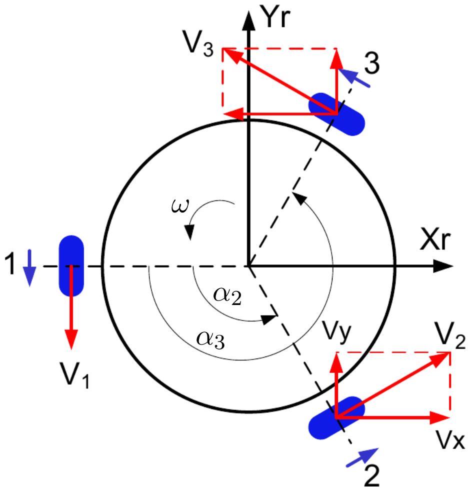


- This is the kinematic model in the **local** frame

Omnidirectional drive - kinematics

- Kinematic model in the **global** frame

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ \omega \end{bmatrix}$$



Omnidirectional drive (Example of CAMBADA robots)

http://y2u.be/PXq89EONEz0