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Non-Asymptotic Analysis of Network Coding Delay

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Joint work with
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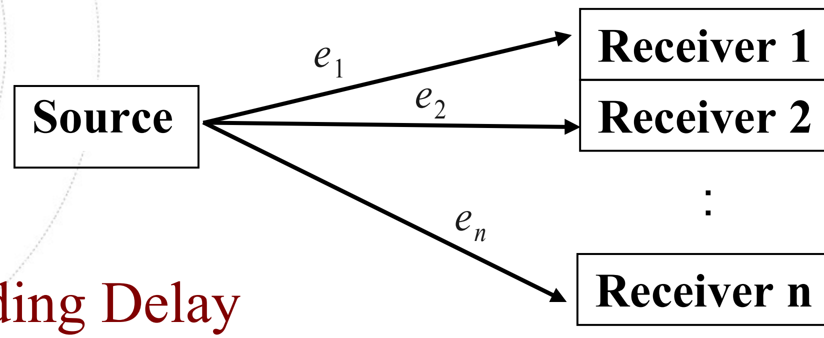
Motivation

- Random Linear Network Coding: one source and multiple receivers.



Broadcast Network

- Analysis of Decoding Delay
  - asymptotic analysis: infinite block lengths
  - **non-asymptotic analysis: finite block lengths**
  - average analysis
  - **complete probability distribution (average and worst-case analysis)**

Analysis of Decoding Delay	Average	Complete Probability Distribution
Asymptotic	e.g. [Fragouli et al. 2006]	e.g. [Ho et al. 2003]
Non-Asymptotic	e.g. [Sundararajanet al. 2008]	?

Motivation

- Brute Force Analysis of Decoding Delay [Nistor et al. 2010]:
  - Non-asymptotic analysis
    - Fix the following parameters: feasible **number of receivers**, sufficient **number of time slots**, feasible **number of symbols** to be encoded, **erasure probability**, small **field size**.
    - Repeat for every experiment: generate **erasure pattern**, generate **all possible packets** (sets of linear combinations of symbols), transmit over **erasure channel**, carry out **Gaussian elimination** on received combination, measure the **decoding delay.**
  - Complete decoding delay distribution for small number of symbols and limited field sizes.
  - Useful for designing systems with strict delay constrains.

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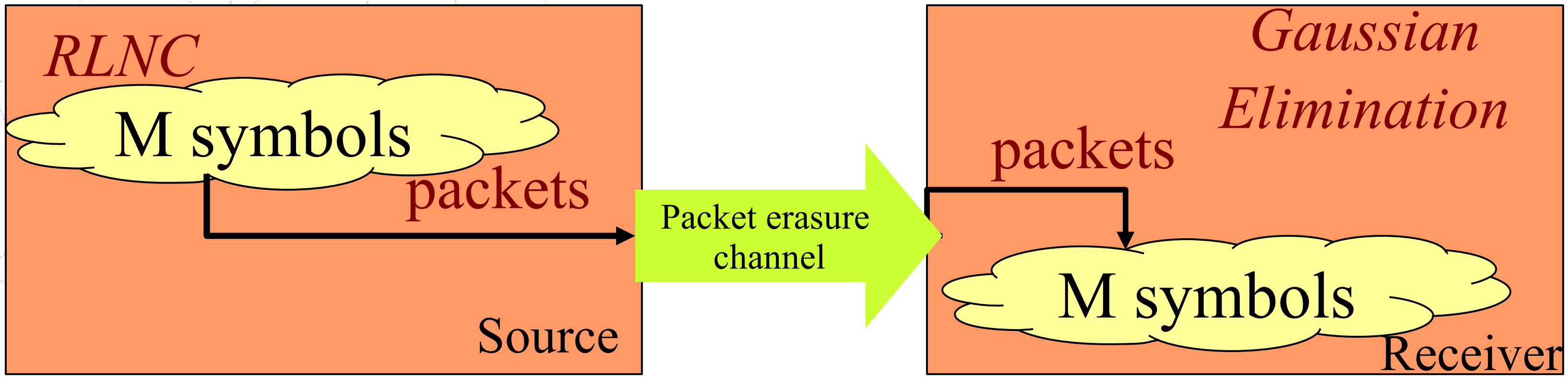
Can we characterize the *decoding delay* behavior of random linear network coding for *any number of symbols* and *any field size*?

Problem Setup and Performance Metric



Decoding Delay (D)

- The total number of time slots required for the receiver to decode all the M symbols.

Goal

- Find the probability distribution of delay $P(D=k)$, where $k \geq M$.

Main Result

- **Separation of the channel effect: - Field size effect**
  - Erasure effect

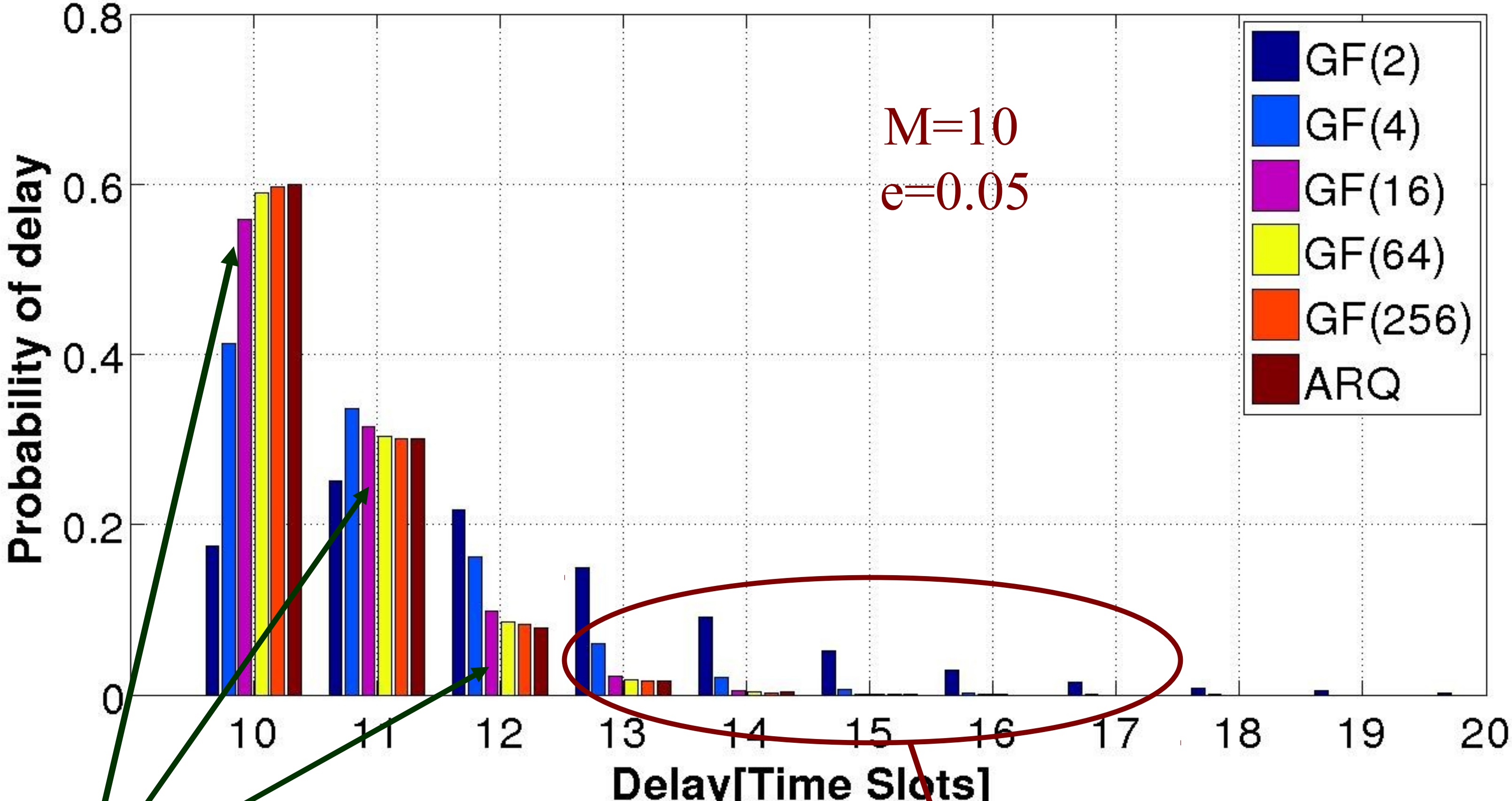
- **The probability distribution of delay *D* for *k*≥*M*:**

$$P(D=k)=\sum_{i=0}^{k-M}\binom{k-1}{i}\frac{q^{M}-q^{M-1}}{(q^{M}-1)^{k-i-1}}C_1(1)\cdot e^{i}(1-e)^{k-i}$$

Where $C_1(1)$ is computed recursively and counts the number of linear independent packets.

- Comparison with ARQ techniques with perfect feedback that achieve optimal throughput.

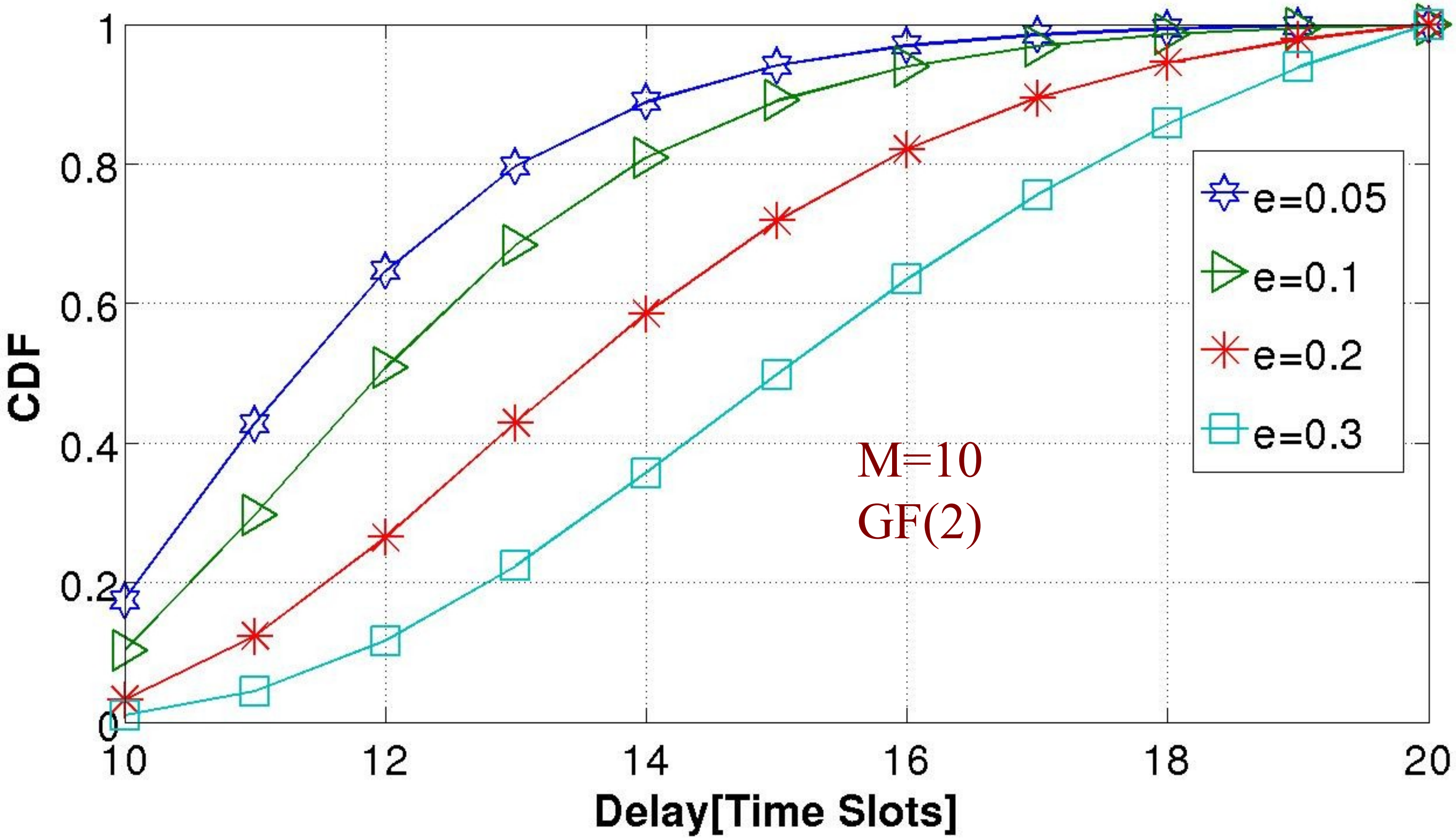
The Effect of Field Size



GF(16) is already very close to ARQ (perfect feedback)

GF(2) has a heavy tail.

The Effect of Erasure Probability



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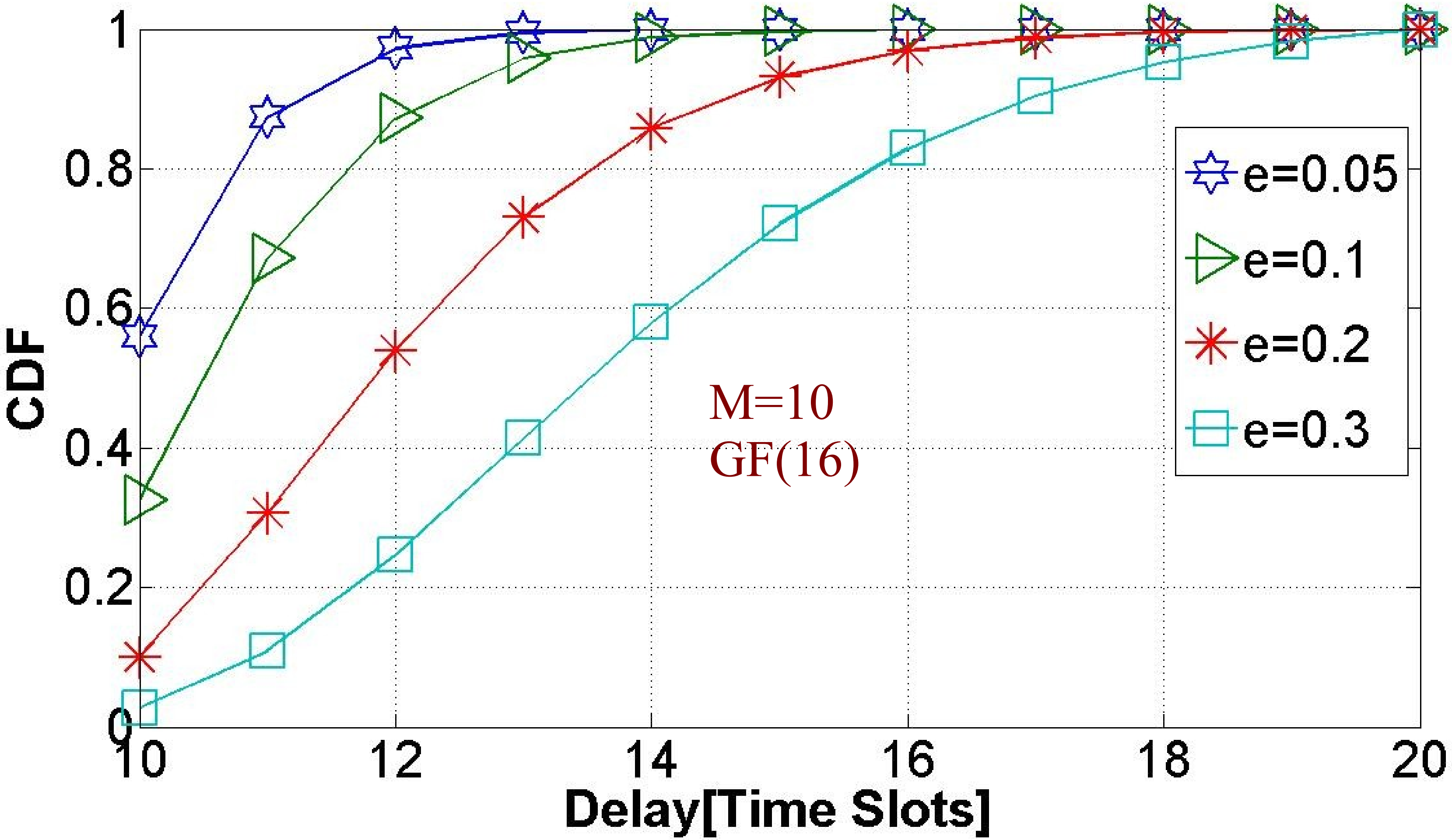
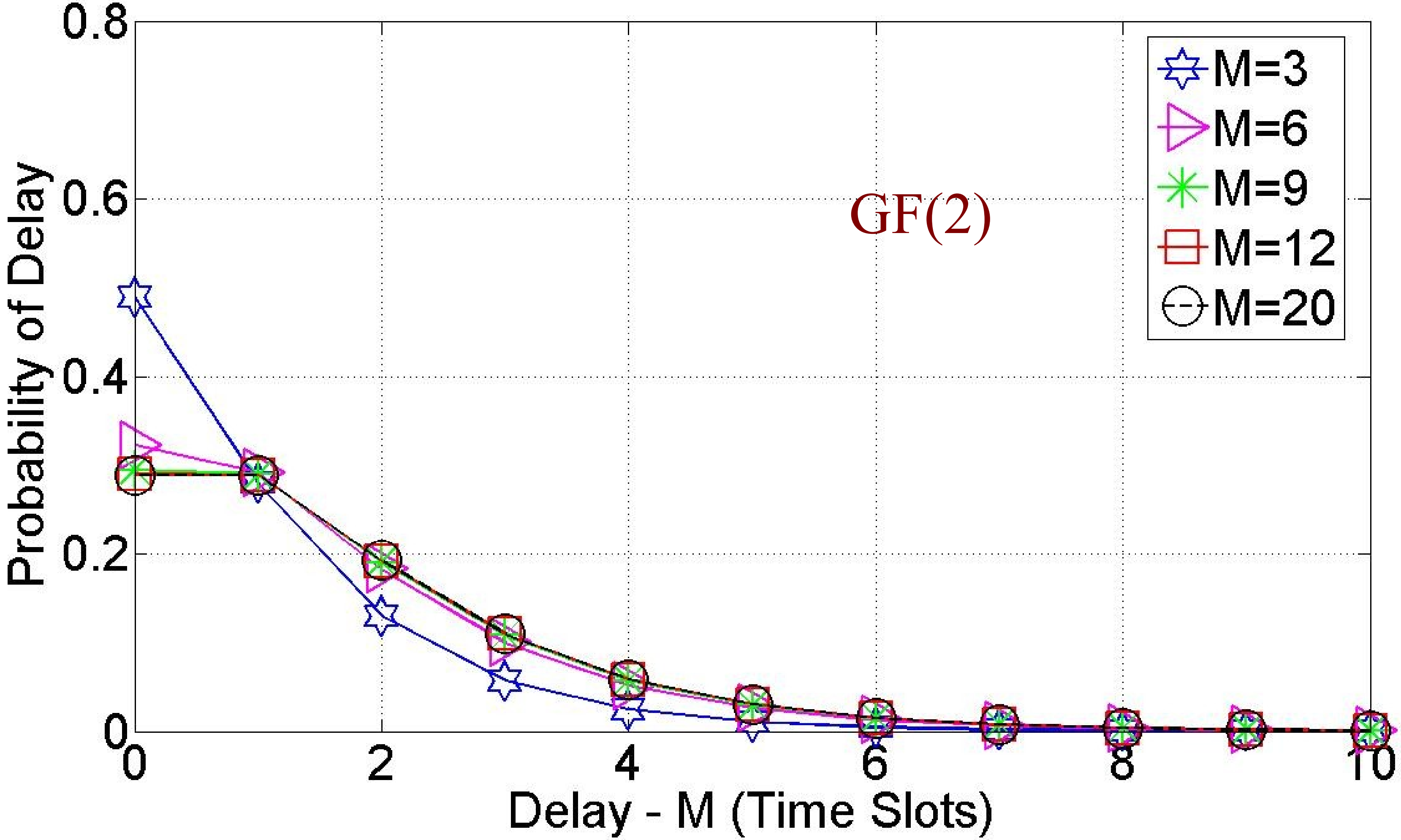
The Effect of Erasure Probability



The Effect of Number of Symbols



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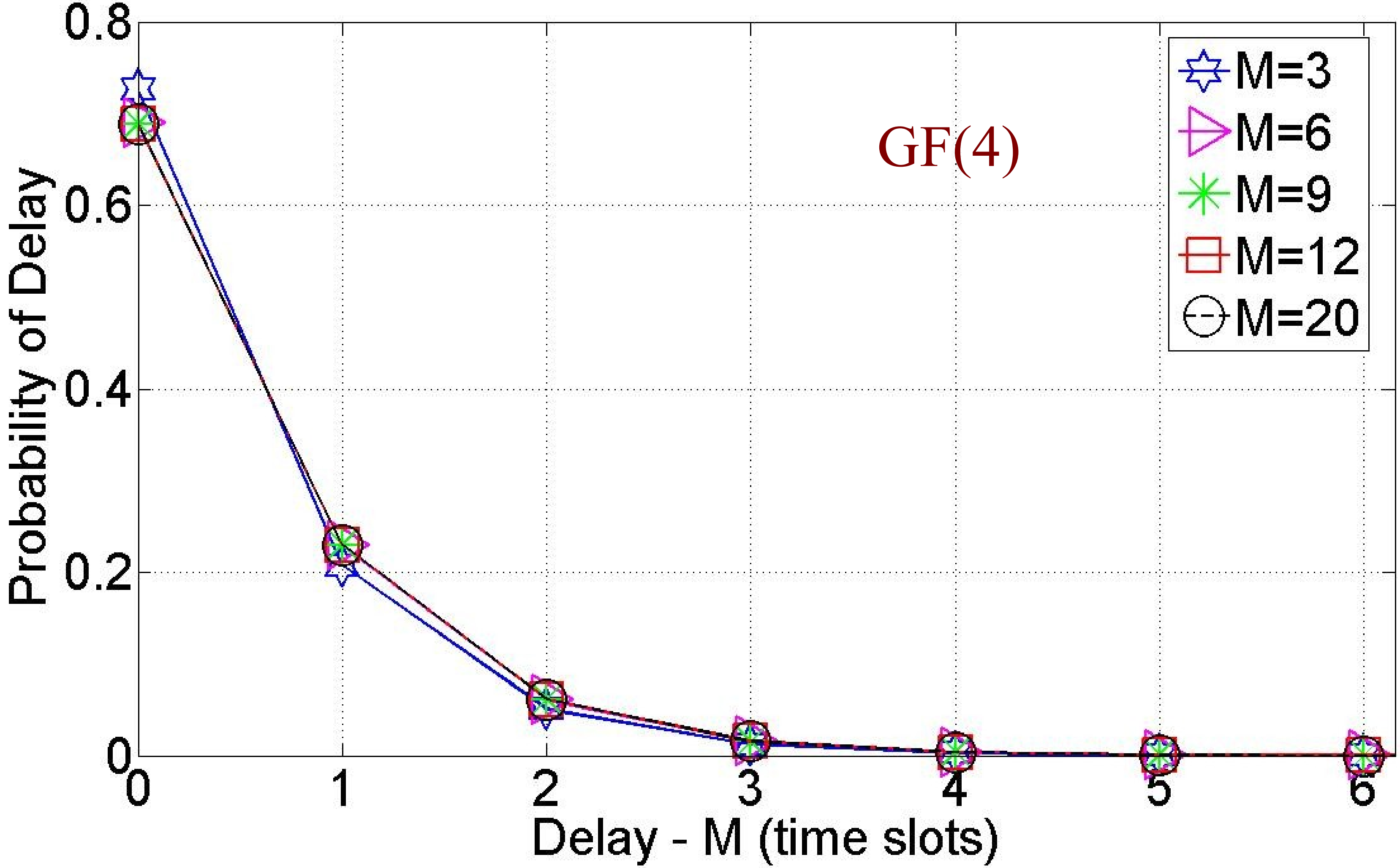

Conclusions

- Delay Distribution is useful for designing systems with stringent deadlines.
- The probability distribution for GF(16) is already very close to ARQ with perfect feedback.
- Network Coding for GF(2) leads to a distribution with heavy tails.
- Future work: extension the analytical model to multiple receivers.