PERFORMANCE COMPARISON OF MINIMUM VARIANCE SINGLE CARRIER AND MULTICARRIER CDMA RECEIVERS

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ABSTRACT

This work provides comparisons between CDMA-based multiple access systems in a single and multicarrier fashion. Both zero padding and cyclic prefix types of guard intervals are considered. Comparisons include different performance measures such as signal-to-interference plus noise ratio (SINR), bit error rate (BER), and robustness against channel order overestimation. We also address the effects of finite number of samples when estimating the detector filter. In order to allow a fair comparison, blind detection based on the minimum variance is assumed for all considered systems. It is shown through computer simulations that multicarrier CDMA performs better than single carrier CDMA and is more robust against channel order overestimation.

Index Terms— Block Transmission, Minimum Variance Receivers, SC-CDMA, MC-CDMA.

1. INTRODUCTION

Wireless communication systems have to support a wide range of multimedia services such as speech, image, and data transmission with different and variable bit rates, QoS and latency. These requirements cannot be entirely covered by the second generation systems, primarily designed for speech transmissions. An important challenge for third generation systems is the selection of an appropriate multiple access scheme to meet these demands. Wideband CDMA (W-CDMA) is a strong contestant for third generation systems that include the IMT–2000 standard and UMTS standard [1]. As a result of this, block transmission systems in single or multiple carrier fashion became very active research area in communications [1].

In recent years, single carrier (SC) block transmission and OFDM systems have been widely studied in multiple user access schemes, such as CDMA. Single carrier CDMA (SC–CDMA) is the well known CDMA with a guard interval between each symbol [2], [3]. The chips are transmitted sequentially over the whole bandwidth allocated for that user. Multicarrier CDMA (MC–CDMA) is based on the concatenation

of DS spreading and OFDM technique. The data symbol of a user is spreaded and the chips are simultaneously transmitted, each one over a narrowband subchannel by the multicarrier modulation (frequency domain spreading). The combination of Direct Sequence Code Division Multiple Access (DS–CDMA) and multi–carrier modulation was first proposed in 1993 [4],[5].

MC-CDMA combines the main advantages of CDMA and MC transmission, namely, robustness against selective fading and interference resistance. Furthermore, MC-CDMA multipath suppression does not depend of the spreading factor, so high data rates may be accommodated in a smaller bandwidth [6]. On the other hand, SC-CDMA does not present some of the problems that affect multicarrier systems such as peak-to-average power ratio (PAPR) and sensitivity to phase noise and frequency offset [3].

The aim of this work is to provide a comparison between these multiple access systems (SC, MC–CDMA). Both zero padding (ZP) and cyclic prefix (CP) types of guard intervals are considered. Comparison includes different performance measures such as signal-to-interference plus noise ratio (SINR), bit error rate (BER), and robustness against channel order overestimation. We also address the effects of finite number of samples when estimating the detector filter, based on the work of Xu[7]. In order to allow a fair comparison between them, we restrict our study to constrained minimum variance (CMV) blind detectors.

This paper is organized as follows: Section 2 describes a unified framework of a block transmission system under which both SC-CDMA and MC-CDMA are described. In section 3 the blind receiver based on the minimum variance principle is presented. Section 4 addresses the effects of finite-data-sample parameter estimation on system performance. Section 5 presents the results obtained through computer simulation and, finally, section 6 gives the conclusions.

2. BLOCK TRANSMISSION SYSTEM MODEL

Let us consider a discrete model of a block transmission system depicted in Fig. 1. We want to transmit a data block,

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denoted by the $M \times 1$ vector s(i). Here, the block contains the chips of a BPSK-modulated symbol b_k spreaded by a code c_k .

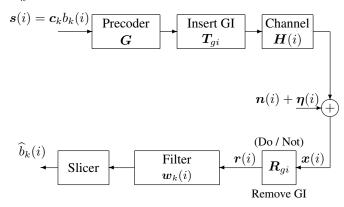


Fig. 1. General Structure of a Block Transmission System

The block is precoded by the $M \times M$ matrix G that represents an arbitrary linear operation performed before transmission. After that, a guard interval of length G is inserted to allow for interblock interference (IBI) suppression at the receiver 1 . This operation is represented by a $P \times M$ matrix T_{gi} , where P = M + G. For the particular cases of zero padding and cyclic prefixing the guard interval insertion is implemented by the matrices T_{zp} and T_{cp} given below.

$$m{T}_{zp} = \left[egin{array}{c} m{I}_M \ m{0}_{G imes M} \end{array}
ight] \qquad m{T}_{cp} = \left[egin{array}{c} m{0}_{G imes M-G} \,|\, m{I}_G \ m{I}_M \end{array}
ight]$$

where I_k represents a $k \times k$ identity matrix and $\mathbf{0}_{m \times n}$ represents an $m \times n$ null matrix.

The block is then transmitted through a multipath channel, modeled here as a FIR filter with L taps whose gains are samples of the channel impulse response complex envelope. Assuming that during the i-th block duration the multipath channel impulse response remains constant, that is, $h(i) = [h_0(i) \dots h_{L-1}(i)]^T$, the transmission through the multipath channel can be represented by a $P \times P$ lower triangular Toeplitz convolution matrix H(i), whose first column is $[h_0(i) \dots h_{L-1}(i) \ 0 \dots 0]^T$.

The transmitted signal is corrupted by a complex white Gaussian noise vector $\mathbf{n}(i) = [n_0(i) \dots n_{P-1}(i)]^T$ whose covariance matrix $\mathsf{E}\left[\mathbf{n}(i)\mathbf{n}^H(i)\right] = \sigma^2\mathbf{I}_P$, where $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian transpose, respectively. The operator $\mathsf{E}\left[\cdot\right]$ stands for ensemble average. The transmitted block is also subject to interference, represented by a term $\boldsymbol{\eta}(i)$ that accounts for multiple access interference (MAI) plus IBI, in a multiuser scenario.

The received signal, represented by the $P \times 1$ vector $\boldsymbol{x}(i)$ can be written as

$$x(i) = H(i)T_{gi}Gs(i) + \eta(i) + n(i)$$
 (1)

For block transmission with CP, the receiver must remove the guard interval from the received signal to eliminate IBI. In the ZP case, these removal is not necessary. We can, therefore, represent the input to the detection filter by

$$r(i) = R_{gi}H(i)T_{gi}Gs(i) + R_{gi}[\eta(i) + n(i)]$$
 (2)

where $R_{gi} = R_{zp} = I_P$ for ZP systems and $R_{gi} = R_{cp} = [\mathbf{0}_{M \times G} | I_M]$ for CP systems.

As will be detailed below, for all flavors of CDMA systems studied in this work the observation vector r(i) for a synchronous K-user system can be written as

$$\boldsymbol{r}(i) = \sum_{k=1}^{K} \boldsymbol{C}_k \boldsymbol{h}_k(i) b_k(i) + \boldsymbol{n}(i)$$
 (3)

where C_k is a code related matrix for user k and b_k its transmitted symbol.

Parameters for each system are summarized in Table 1, where F is a $M \times M$ matrix that implements a M-point FFT, normalized such that, $F^H F = F F^H = I_M$. Having these parameters, we next determine the structure of C_k for each system.

Table 1. Block Transmission System Considered

Transmission System	s	\boldsymbol{G}	$m{T}_{gi}$	$oldsymbol{R}_{gi}$
SC-CDMA-CP	$b_k c_k$	I_M	T_{cp}	$oldsymbol{R}_{cp}$
SC-CDMA-ZP	$b_k c_k$	I_M	T_{zp}	$oldsymbol{R}_{zp}$
MC-CDMA-CP	$b_k c_k$	$oldsymbol{F}^H$	$m{T}_{cp}$	$oldsymbol{R}_{cp}$
MC-CDMA-ZP	$b_k c_k$	$oldsymbol{F}^H$	$m{T}_{zp}$	$oldsymbol{R}_{zp}$

2.1. SC-CDMA-ZP

In this case, $\eta(i)$ in (2) represents only MAI because IBI is avoided by the zero-padding. Also, because we focus on downlink scenarios, users experience the same channel conditions. Hence

$$r(i) = H(i)T_{zp}c_1b_1(i) + n(i) + \eta(i)$$

$$= \sum_{k=1}^{K} H(i)T_{zp}c_kb_k(i) + n(i)$$
(4)

Here $H(i)T_{zp}c_k$ can be written as $C_kh(i)$, where C_k is an $P \times L$ Toeplitz matrix containing shifted versions of the spreading sequence of the kth user padded with G zeros.

2.2. SC-CDMA-CP

In this case, $\eta(i)$ in (2) represents MAI and IBI, but after guard interval removal, the only source of interference is MAI.

$$r(i) = \mathbf{R}_{cp}\mathbf{H}(i)\mathbf{T}_{zp}\mathbf{c}_{1}b_{1}(i) + \mathbf{R}_{cp}\left[\mathbf{n}(i) + \mathbf{\eta}(i)\right]$$

$$= \sum_{k=1}^{K}\mathbf{H}_{C}(i)\mathbf{c}_{k}b_{k}(i) + \mathbf{n}'(i)$$
(5)

 $^{^{1}\}mathrm{The}$ guard interval length G must be at least the channel order to avoid IBI.

Where $H_C(i)$ is a $M \times M$ circulant matrix whose first column is $[h_0(i) \dots h_{L-1}(i) \ 0 \dots 0]^T$. Here $H_C(i)c_k$ can be written as $C_k h(i)$, where C_k is an $M \times L$ circulant matrix containing circularly-shifted versions of the spreading sequence of the kth user and n'(i) is the noise vector with its first G components removed.

2.3. MC-CDMA-ZP

As in the SC case, $\eta(i)$ in (2) represents only MAI because IBI is avoided by the zero-padding.

$$r(i) = \mathbf{H}(i)\mathbf{T}_{zp}\mathbf{F}^{H}\mathbf{c}_{1}b_{1}(i) + \mathbf{n}(i) + \mathbf{\eta}(i)$$

$$= \sum_{k=1}^{K} \mathbf{H}(i)\mathbf{T}_{zp}\mathbf{F}^{H}\mathbf{c}_{k}b_{k}(i) + \mathbf{n}(i).$$
(6)

Here $H(i)T_{zp}F^Hc_k$ can be written as $C_kh(i)$, where C_k is an $P \times L$ Toeplitz matrix containing shifted versions of the transformed spreading sequence of the kth user, F^Hc_k , padded with G zeros.

2.4. MC-CDMA-CP

As in the SC case, $\eta(i)$ in (2) represents MAI and IBI, but after guard interval removal, the only source of interference is MAI.

$$r(i) = \mathbf{R}_{cp} \mathbf{H}(i) \mathbf{T}_{zp} \mathbf{F}^H \mathbf{c}_1 b_1(i) + \mathbf{R}_{cp} \left[\mathbf{n}(i) + \mathbf{\eta}(i) \right]$$
$$= \sum_{k=1}^K \mathbf{H}_C(i) \mathbf{F}^H \mathbf{c}_k b_k(i) + \mathbf{n}'(i)$$
(7)

Where $H_C(i)$ is a $M \times M$ circulant matrix whose first column is $[h_0(i) \dots h_{L-1}(i) \ 0 \dots 0]^T$. Here $H_C(i)F^Hc_k$ can be written as $C_kh(i)$, where C_k is an $M \times L$ circulant matrix containing circularly-shifted versions of the kth user spreading sequence transform F^Hc_k . Again, and n'(i) is the noise vector with its first G components removed. Note that the FFT usually present in the multicarrier case at the receiver was not applied. Actually, this operation is embedded in the receiver filter that will be derived in the next section.

3. MINIMUM VARIANCE RECEIVERS

Considering an observation vector r(i) in the form of (3), the design of a receiver filter $w_k(i)$ based on the MV criterion [8] corresponds to the minimization of the MV cost function:

$$J_{MV} = \boldsymbol{w}_k^H \boldsymbol{R} \boldsymbol{w}_k \tag{8}$$

where $R = \mathsf{E}\left[r(i)r^H(i)\right]$ subject to the linear set of constraints $C_k^H w_k = g$, that avoids the trivial solution w = 0 and anchor the desired user signal. Using the method of Lagrange multipliers, the optimum parameter vector is obtained as

$$\mathbf{w}_{k,o} = \mathbf{R}^{-1} \mathbf{C}_k (\mathbf{C}_k^H \mathbf{R}^{-1} \mathbf{C}_k)^{-1} \mathbf{g}. \tag{9}$$

and the resulting minimum variance, given q, is

$$J_{MV}(\mathbf{w}_o) = \mathbf{w}_o^H \mathbf{R} \mathbf{w}_o = \mathbf{g}^H (\mathbf{C}_k^H \mathbf{R}^{-1} \mathbf{C}_k)^{-1} \mathbf{g}.$$
 (10)

For simplicity, we dropped the user index from the optimum parameter vector. The optimization of the constraint vector g proposed in [8] maximizes (10)

$$g_o = \arg \max_{||g||=1} g^H (C_k^H R^{-1} C_k)^{-1} g$$
 (11)

whose solution g_o is the eigenvector corresponding to the maximum eigenvalue of $(C_k^H R^{-1} C_k)^{-1}$ or to the minimum eigenvalue of $(C_k^H R^{-1} C_k)$.

It was noted later [9] that, in fact, g, given by (11) provides an estimate of the channel impulse response h, since it corresponds to a special case (m=1) of the power method for blind channel estimation proposed in [9], which is based on the eigen-decomposition² of $(C_k^H R^{-m} C_k)$. Even though the performance of the channel estimator and consequently of the receiver can be improved by increasing m, in this work we restrict ourselves to the case m=1. Note that since g_o is an eigenvector of $(C_k^H R^{-1} C_k)^{-1}$, the minimum variance receiver filter parameter vector given by (9) is proportional to

$$\boldsymbol{w}_o = \boldsymbol{R}^{-1} \boldsymbol{C}_k \boldsymbol{g}_o. \tag{12}$$

4. PERTURBATION ANALYSIS

The design results in the previous section assume an exact knowledge of the observation signal correlation matrix. In realistic situations the receiver uses an estimate of the correlation matrix. At the i-th signaling interval a typical estimate $\widehat{\mathbf{R}}(i)$ is obtained, for example, using the time average of N observations (N transmitted symbols).

$$\widehat{R}(i) = \frac{1}{N} \sum_{j=0}^{N-1} r(i-j)r^{H}(i-j)$$
 (13)

Assuming that a large number of samples had been processed, i.e., large N, $\widehat{R}^{-1}(i)$ can be approximated using first-order Taylor's expansion:

$$\widehat{\boldsymbol{R}}^{-1}(i) = (\boldsymbol{R} + \delta \boldsymbol{R}(i))^{-1} = \boldsymbol{R}^{-1} - \boldsymbol{R}^{-1} \delta \boldsymbol{R}(i) \boldsymbol{R}^{-1}$$

Defining $A = (C_k^H R^{-1} C_k)$, we then have that the perturbation in $\widehat{A}(i)$ due to the use of $\widehat{R}(i)$ is given by:

$$\delta \boldsymbol{A}(i) = -\boldsymbol{C}_k^H \boldsymbol{R}^{-1} \delta \boldsymbol{R}(i) \boldsymbol{R}^{-1} \boldsymbol{C}_k$$

Perturbations due to the use of covariance matrix estimates affect the eigenvalues and eigenvectors of $(C_k^H R^{-1} C_k)$. Using the results from [10], it can be shown that:

$$\delta \boldsymbol{g}_o \approx -(\boldsymbol{A} - \lambda_{\max} \boldsymbol{I})^{-1} \delta \boldsymbol{A}(i) \boldsymbol{g}_o$$

 $^{^2}$ It is important to note that this subspace–based method limits the maximum number of users to $K \leq M-L$ and $K \leq M-L+G$ for CP and ZP guard intervals, respectively.

where λ_{max} denotes the largest eigenvalue of A^{-1} .

By expressing $\widehat{\boldsymbol{w}}_o(i)$ as in (12), we obtain the perturbation $\delta \boldsymbol{w}_o(i)$ suffered by the optimum receiver filter

$$\widehat{\boldsymbol{w}}_o(i) \approx \boldsymbol{w}_o - \boldsymbol{R}^{-1} \delta \boldsymbol{R}(i) \boldsymbol{R}^{-1} \boldsymbol{C}_k \boldsymbol{g}_o + \boldsymbol{R}^{-1} \boldsymbol{C}_k \delta \boldsymbol{g}_o(i)$$
 (14)

Substituting $\delta g_o(i)$ and $\delta A(i)$ in (14) results that the deviation from the k-th user optimum receiver filter due to errors in the estimation of the correlation matrix R can be approximated by

$$\delta \mathbf{w}_o(i) = \mathbf{A}_k \delta \mathbf{R}(i) \mathbf{w}_o \tag{15}$$

where
$$m{A}_k = \left[m{R}^{-1} m{C}_k (m{A} - \lambda_{\max} m{I})^{-1} m{C}_k^H - m{I} \right] m{R}^{-1}.$$

The effect of $\delta w_o(i)$ in performance will be evaluated through the signal to interference plus noise ratio (SINR) which is defined as:

$$\widehat{\mathsf{SINR}}_k(i) = \frac{\mathsf{E}\left[\|\widehat{\boldsymbol{w}}_o^H(i)\boldsymbol{s}_k\|^2\right]}{\mathsf{E}\left[\|\widehat{\boldsymbol{w}}_o^H(i)\boldsymbol{R}_I\widehat{\boldsymbol{w}}_o(i)\right]} \tag{16}$$

where $s_k = C_k h$ is the effective spreading sequence of the desired user and $R_I = R - s_k s_k^H$ is the covariance matrix due to the interferers and the additive noise. In terms of $\delta w_{opt}(i)$, (16) is written as:

$$\widehat{\mathsf{SINR}}_{k}(i) = \frac{\|\boldsymbol{w}_{o}^{H}\boldsymbol{c}_{k}\|^{2} + \mathsf{E}\left[\delta\boldsymbol{w}_{o}(i)^{H}\boldsymbol{c}_{k}\boldsymbol{c}_{k}^{H}\delta\boldsymbol{w}_{o}(i)\right]}{\boldsymbol{w}_{o}^{H}\boldsymbol{R}_{I}\boldsymbol{w}_{o} + \mathsf{E}\left[\|\delta\boldsymbol{w}_{o}^{H}(i)\boldsymbol{R}_{I}\delta\boldsymbol{w}_{o}(i)\right]}$$
(17)

After substituting $\delta w_o(i)$ in (17), both expectations became functions of $\delta R(i)$.

$$E\left[\delta \boldsymbol{w}_{o}(i)^{H}\boldsymbol{c}_{k}\boldsymbol{c}_{k}^{H}\delta \boldsymbol{w}_{o}(i)\right] = \boldsymbol{w}_{o}^{H}E\left[\delta \boldsymbol{R}(i)\boldsymbol{A}_{k}^{H}\boldsymbol{c}_{k}\boldsymbol{c}_{k}^{H}\boldsymbol{A}_{k}\delta \boldsymbol{R}(i)\right]\boldsymbol{w}_{o}$$

$$(18)$$

$$E\left[\|\delta \boldsymbol{w}_{o}^{H}(i)\boldsymbol{R}_{I}\delta \boldsymbol{w}_{o}(i)\right] = \boldsymbol{w}_{o}^{H}E\left[\delta \boldsymbol{R}(i)\boldsymbol{A}_{k}^{H}\boldsymbol{R}_{I}\boldsymbol{A}_{k}\delta \boldsymbol{R}(i)\right]\boldsymbol{w}_{o}$$

$$(19)$$

In this work, these expectations were evaluated using computer simulation. An analytical evaluation of (18) and (19) for conventional DS-CDMA systems can be found in [7].

5. SIMULATION RESULTS

The simulation results presented are for BPSK synchronous MC- and SC- CDMA systems that employ Gold sequences of length N=31. Because we focus on a downlink scenario, users experience the same channel conditions. The channel is modelled as an FIR filter with L=4 coefficients. Unless stated otherwise, the guard interval length is equal to channel order in all simulations. Regarding power distribution, we simulate a severe near-far scenario where each interferer has a power level 20 dB above the desired user, that is, near-far ratio (NFR) is equal to 20 dB.

In Fig. 2 we assess the desired user BER performance of the analyzed receivers versus E_b/N_0 . The system has K=12 users. In this case, the multipath gains are randomly drawn from a zero-mean complex Gaussian random variable and

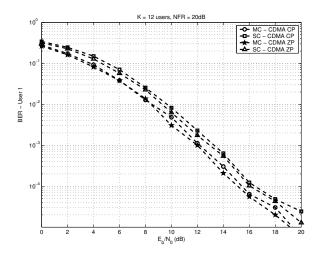


Fig. 2. BER versus E_b/N_0 (dB)

kept fixed throughout the experiment. These gains are normalized in order to have $\|\boldsymbol{h}\|^2 = 1$. The results are an average of 1000 experiments.

Fig. 3 compares the analytical performance expressions given in section 4 with simulation results for the MC-CDMA ZP receiver. The simulated system has K = 8 users and the desired user has a power level corresponding to $E_b/N_0 =$ 15 dB. The SINR is plotted as a function of the number of observation samples (N) used for estimating the detector. We also plot the SINR value obtained supposing perfect knowledge of the correlation matrix R, denoted Perfect Estimation. Here, the multipath gains are also randomly drawn from a zero-mean complex Gaussian random variable but are normalized such that $E[\|\boldsymbol{h}\|^2] = 1$. The results are an average of 1000 experiments. As in [7], it is noticed here that as $N \to \infty$, the semi-analytical result tends to the perfect estimation SINR level. The same behavior was observed for the other systems, but these figures are omitted due to lack of space.

As the simulation results showed a good agreement with the analytical ones, in Fig. 4, the output SINR obtained from (17) is plotted versus E_b/N_0 for all systems. The results in this figure are obtained under the same channel condition described in connection with Fig. 2 Three cases were considered: perfect correlation matrix estimation and estimation using 1000 and 5000 samples. We can note that as E_b/N_0 increases, so does the degradation in SINR performance due to finite-data-sample parameter estimation.

Fig. 5 depicts the average BER performance versus an estimated channel order. Here, the channel estimation procedure uses the given length G of the guard interval as the unknown channel order, that is, G = L' - 1. The channel length is fixed with coefficients h=[0.5957 + j0.0101; -0.3273 - j0.3472; 0.2910-j0.0533; 0.1285-j0.5599] T . The results are an average of 1000 experiments. The receiver,

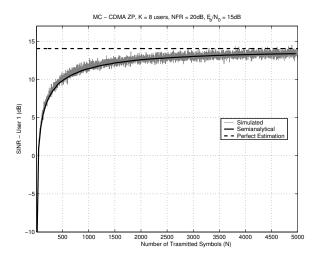


Fig. 3. SINR versus number of transmitted symbols

for multicarrier transmission, was not as sensitive to overestimation as their single carrier counterparts.

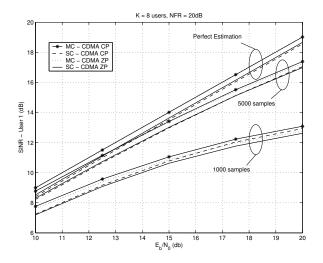


Fig. 4. Output SINR versus E_b/N_0 (dB)

6. CONCLUSIONS

In this paper we have compared single and multicarrier block transmission CDMA-based multiple access systems. The comparison was carried out in several performance measures with a minimum variance receiver. The effect of finite-data-samples estimation was also considered. Under the test conditions, it is show that in terms of BER and SINR, MC-CDMA-ZP performs slightly better than the other systems, MC-CDMA-CP, SC-CDMA-CP, and SC-CDMA-ZP. Also, we concluded that the CMV receiver for multicarrier transmission was less sensitive to channel order overestimation than their single carrier counterparts.

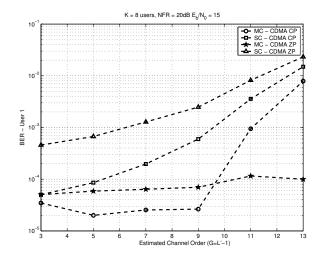


Fig. 5. BER versus Estimated Channel Order

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