Notes from Unit 1
Finite Difference Time Domain

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1 Introduction

Finite Difference Time Domain (FDTD) method is a numerical analysis technique used to approximate solutions of differential equations.

Pros	Cons
Simple and intuitive;	High processing time for 3D problems;
Works on Field variables;	Complex geometry demands a lot of work.
Do not invert matrices;	
Double precision;	
Directly models nonlinear phenomenons and several mediums.	

1.1 Identification of Electromagnetic Problems

Continuous problems are caregorized based on:

- the solution region;
- the equation that describes the problem;
- border conditions associated to the problem.

Electromagnetic problems can usually be classified based on the equations that describe them. These can be integral or/and differential equations. Most of them can be defined by the following equation:

$$L\Phi = g \tag{1}$$

where L is a differential, integral, or integral-diferential operator, g is the source (usually known) and Φ is the function to be solved.

$$a\frac{\partial^2\Phi}{\partial x^2} + b\frac{\partial^2\Phi}{\partial x \partial y} + c\frac{\partial^2\Phi}{\partial y^2} + d\frac{\partial\Phi}{\partial x} + e\frac{\partial\Phi}{\partial y} + f\Phi = g \tag{2}$$

$$L = a\frac{\partial^2}{\partial x^2} + b\frac{\partial^2}{\partial x \partial y} + c\frac{\partial^2}{\partial y^2} + d\frac{\partial}{\partial x} + e\frac{\partial}{\partial y} + f \tag{3}$$

where the coeficients a,b,c are functions of x and y, and and might be dependent on Φ, in case the Partial Diferential Equation (PDE) is nonlinear. Additionally, the PDE is homogeneous if $g(x, y) = 0$ and non homogeneous if $g(x, y) \neq 0$.

1.2 Second Order Differential Equations

Derived from the equation $ax + bxy + cy^2 + dx + ey + f = 0$, the second order PDEs are classified by:

condition	case
$b - 4ac < 0$	eliptic
$b - 4ac > 0$	hyperbolic
$b - 4ac = 0$	parabolic

Table 1: Classification of Electromagnetic Problems

Case	Equation	Variables	Associated to
Eliptic	$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} = g(x, y)$	$a = c = 1$ and $b = 0$	Stationary phenomenons
Hyperbolic	$\frac{\partial^2\Phi}{\partial x^2} = \frac{1}{u^2}\frac{\partial^2\Phi}{\partial t^2}$	$a = u^2$, $b = 0$ and $c = -1$	Propagation problems
Parabolic	$\frac{\partial^2\Phi}{\partial x^2} = k\frac{\partial\Phi}{\partial t}$	$a = 1$ and $b = c = 0$	Slow change phenomenons

The solution Φ in a bounded region R is required to satisfy given conditions S, these are:

Table 2: Boundary Conditions

Condition	Equation
Dirichlet Condition	$\Phi(\boldsymbol{r}) = 0$
Neumann Condition	$\frac{\partial\Phi(\boldsymbol{r})}{\partial n} = 0$
Neumann Condition	$\frac{\partial\Phi(\boldsymbol{r})}{\partial n} + \Phi(\boldsymbol{r}) + h(\boldsymbol{r}) = 0$