Calculation of Band Diagrams Through the Finite-Difference Time-Domain Method

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This paper presents the bandgap diagram computation of photonic crystals using FDTD. The analysis of 2D and 3D non disperssive structures was carried out and compared to the well known PWE solutions.

***Index Terms*—Photonic crystals, bandgap, FDTD, PWEM.**

I. Introduction

A photonic crystal is a periodic lattice of dielectric materials with cell dimensionscorresponding to the wavelength of visible light. The periodicity and symmetry of thepatterned material manifests itself as a periodic change of its dielectric constant. Forvisible light the required dimensions are around 500 nm, about three orders ofmagnitude higher than the atomic spacing of an ordinary crystal. Thus, the behavior oflight in a photonic crystal can be well described by the Maxwell equations. Based onsolid-state physics analogy, similarity can be deduced between the behavior ofelectrons in ordinary crystal lattices and the propagation of electromagnetic fields in photonic crystals. In both cases, as a result of Bragg reflections there are certainfrequencies that cannot propagate in the lattice and gaps will appear in the frequencyspectrum. The interesting optical properties of photonic crystals are consequences ofthe existing photonic bands (Joannopouloset al., 1995; Sakoda, 2001; Poole and Owens,2003).The finite difference time domain (FDTD) method (Taflove, 1995; Sullivan, 2000;Ward and Pendry, 1998) is widely used to determine the photonic bands of thesestructures. Passing a light pulse of Gaussian distribution through the photonic crystaland analyzing the transmitted wave can explore the photonic bands. Since thedielectric constant in real optical materials is a function of frequency, this dispersionshould be considered in the process of determining the accurate band structure. In contrast to PWE method, the FDTD provides the possibility of the refractive index vriation during the computation process, which allows to take into account losses and nonlinearity when computing the band structure.

II. Problem Definition

The band diagram of a photonic crystal can be defined as a Maxwell eigenproblem. Employing the dirac notation to provide an independent representation for the fields and inner products, the source-free Maxwell's equations for a linear dielectric $\epsilon = \epsilon(\vec{\boldsymbol{r}})$ can be written in terms of only the magnetic field $|H\rangle$

III. The Finite-Difference Time-Domain Approach

$$\vec{\boldsymbol{E}}(\vec{\boldsymbol{r}}) = \vec{\boldsymbol{A}}(\vec{\boldsymbol{r}})e^{j\vec{\boldsymbol{\beta}}\cdot\vec{\boldsymbol{r}}} \tag{1}$$

$$\vec{\boldsymbol{A}}(\vec{\boldsymbol{r}} + \vec{\boldsymbol{t}}_{pqr}) = \vec{\boldsymbol{A}}(\vec{\boldsymbol{r}}) \tag{2}$$

$$\epsilon(\vec{\boldsymbol{r}} + \vec{\boldsymbol{t}}_{pqr}) = \epsilon(\vec{\boldsymbol{r}}) \tag{3}$$

$$\vec{\boldsymbol{t}}_{pqr} = p\vec{\boldsymbol{t}}_1 + q\vec{\boldsymbol{t}}_2 + r\vec{\boldsymbol{t}}_3 \tag{4}$$

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A. Bloch Periodic Boundary Conditions

$$\vec{\boldsymbol{E}}(x \pm \Lambda_x) = \vec{\boldsymbol{E}}(x)e^{j\pm\beta_x\Lambda_x} \tag{5}$$
$$\vec{\boldsymbol{E}}(y \pm \Lambda_y) = \vec{\boldsymbol{E}}(y)e^{j\pm\beta_y\Lambda_y} \tag{6}$$
$$\vec{\boldsymbol{E}}(z \pm \Lambda_z) = \vec{\boldsymbol{E}}(z)e^{j\pm\beta_z\Lambda_z} \tag{7}$$
$$\vec{\boldsymbol{H}}(x \pm \Lambda_x) = \vec{\boldsymbol{H}}(x)e^{j\pm\beta_x\Lambda_x} \tag{8}$$
$$\vec{\boldsymbol{H}}(y \pm \Lambda_y) = \vec{\boldsymbol{H}}(y)e^{j\pm\beta_y\Lambda_y} \tag{9}$$
$$\vec{\boldsymbol{H}}(z \pm \Lambda_z) = \vec{\boldsymbol{H}}(z)e^{j\pm\beta_z\Lambda_z} \tag{10}$$

B. Initial Conditions

To compute the photonic bandgap diagram for a given PhC, the time-dependent response of its structure by a source that excites all modes should be found at any point of the computational domain. Therefore, several distinct poralized impressed current sources with wide spectrum gaussian waveforms should be randomly placed throughout the computational domain to ensure that all modes of interest are excited. Moreover, as there is no absorption due to perioic boundary conditions, radiation will indefinetely exist.

C. Structure Response Analysis

The spectral analysis of the time dependent response can be carried out by the Fourier transform. Although the accuracy of the method achieves its maximum with infinite computtion time, given finite resources, the computation time should still be considerably large. The eigen-states of the structure are found searching for local maximas at the response spectrum. A more detailed analysis should be made to assure that no

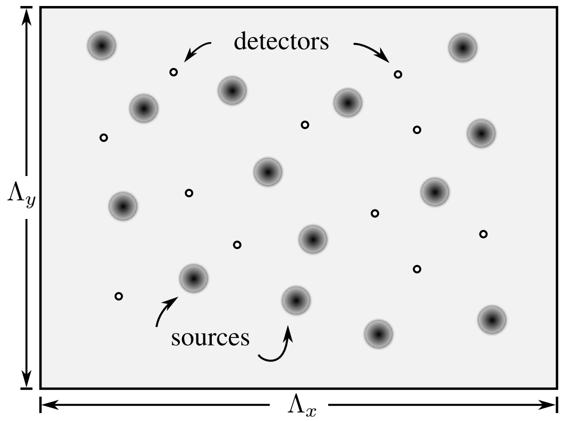

Fig. 1. Random distribution of sources and detectors.

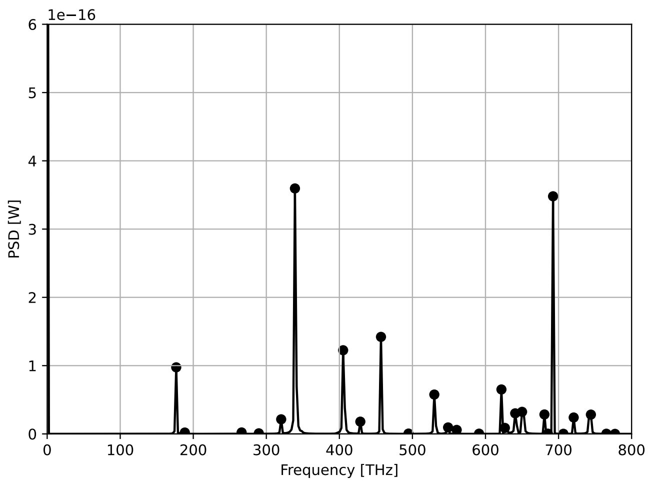

Fig. 2. Power spectral density and eigen-states peaks.

spurious solutions are considered, which usually appear as inessential peaks the the spectrum.

FFT the record arrays to calculate the power spectral density recorded at each record point.

$$PSD_p(\omega) = \left|\mathcal{F}\left\{\vec{\boldsymbol{E}}(t)|^p\right\}\right|^2 \tag{11}$$

The power spectral densities from all detector points are added

$$PSD(\omega) = \sum_p PSD_p(\omega) \tag{12}$$

frequencies corresponding to Bloch modes are identified as sharp peaks in the overall PSD.

TODO: graph of peaks

IV. Results

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V. Conclusion

The conclusion goes here.

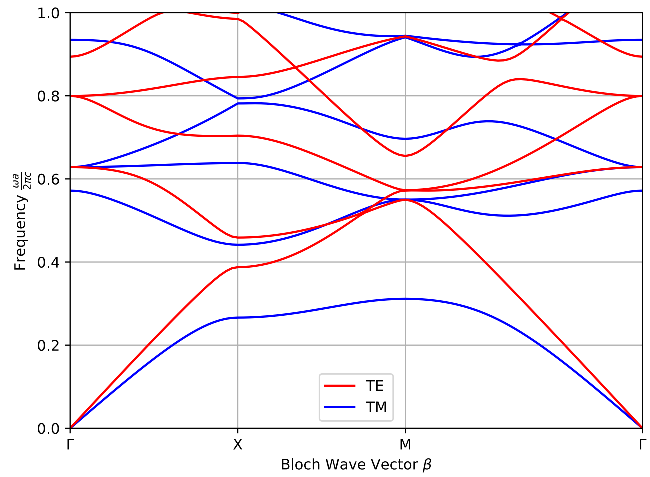

Fig. 3. PWE 2D

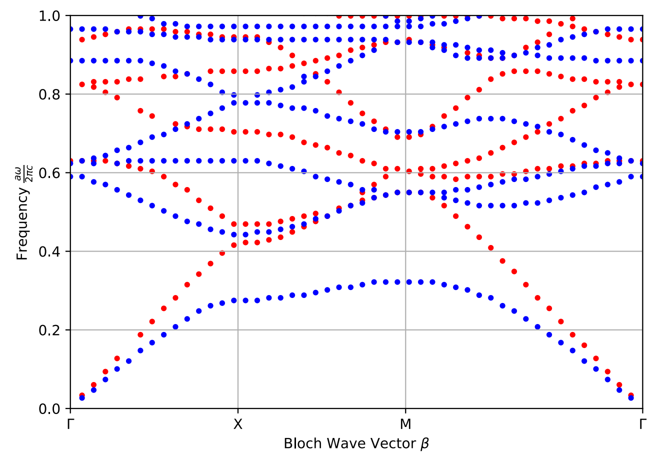

Fig. 4. FDTD 2D

Appendix A
Proof of the First Zonklar Equation

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Appendix B

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Acknowledgment

The authors would like to thank...

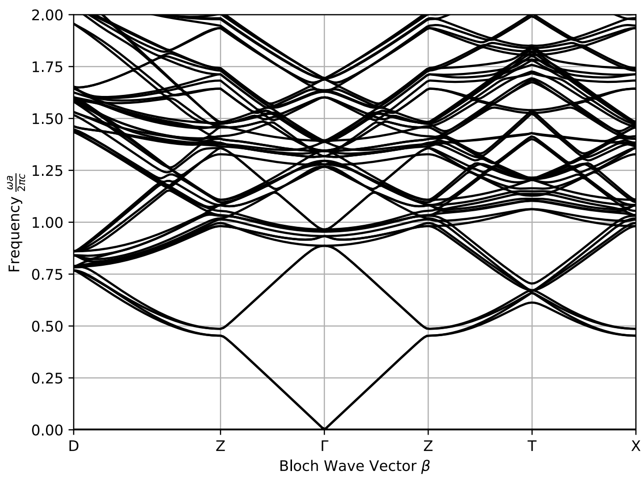

Fig. 5. PWE 3D

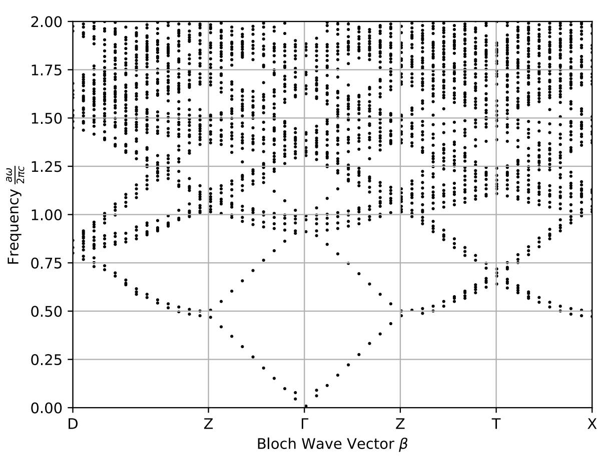

Fig. 6. FDTD 3D