1 Analyze Local Differential Privacy

1.1 Fairness

In this section, I compare the probability that participants communicate with their intended recipients.

1.1.1 Maximum Difference

Before adding noise, the probability that a participant P_i communicates with their desired recipient is $\frac{1}{x}$, where x is the number of participants who share the same target as P_i.

After adding noise, the probability becomes $\frac{e^\epsilon}{n-1+e^\epsilon} \times \frac{1}{x}$, where x remains the same, n is the total number of participants in the network, and ϵ is the security parameter.

1.1.1.1 How Epsilon effect

Case 1: 10 participants in the protocol, with 5 of them wanting to communicate with the same person.
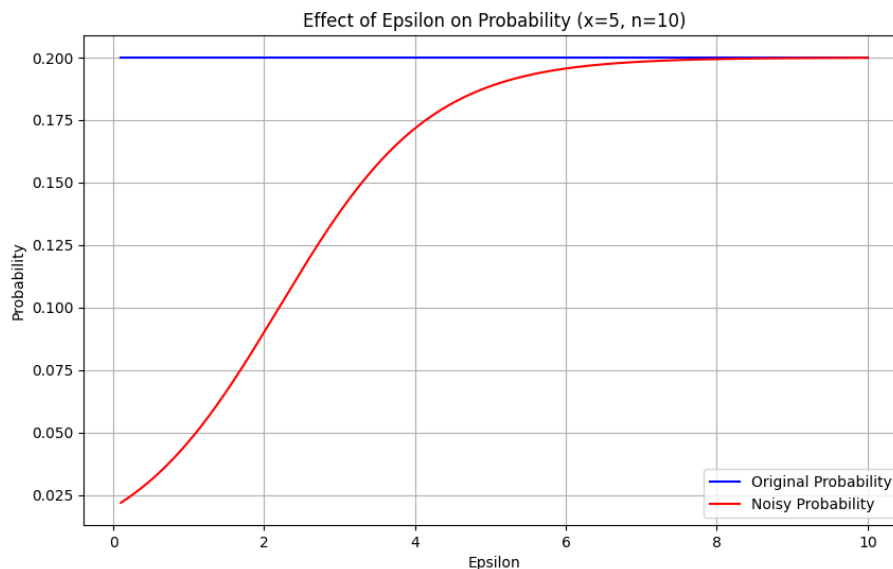
Effect of Epsilon on Probability (x=5, n=10)

Case 2: 100 participants in the protocol, with 5 of them wanting to communicate with the same person.

Effect of Epsilon on Probability (x=5, n=100)
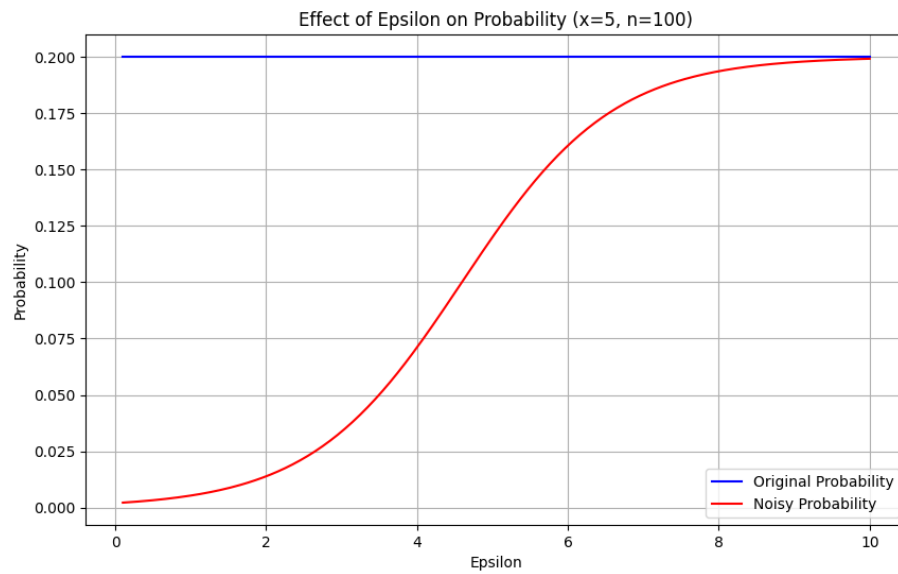
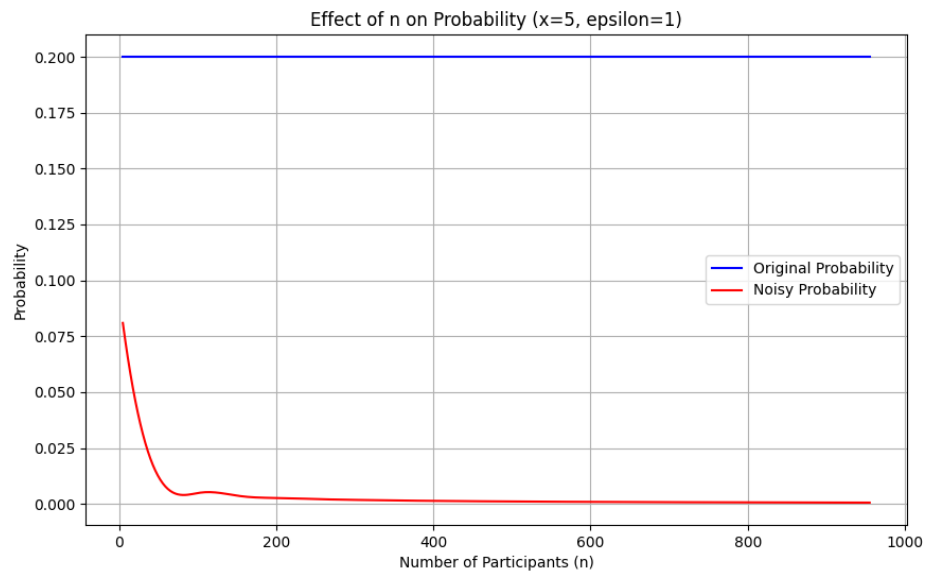**Case 3: 100 participants in the protocol, with 50 of them wanting to communicate with the same person.**

Effect of Epsilon on Probability (x=50, n=100)
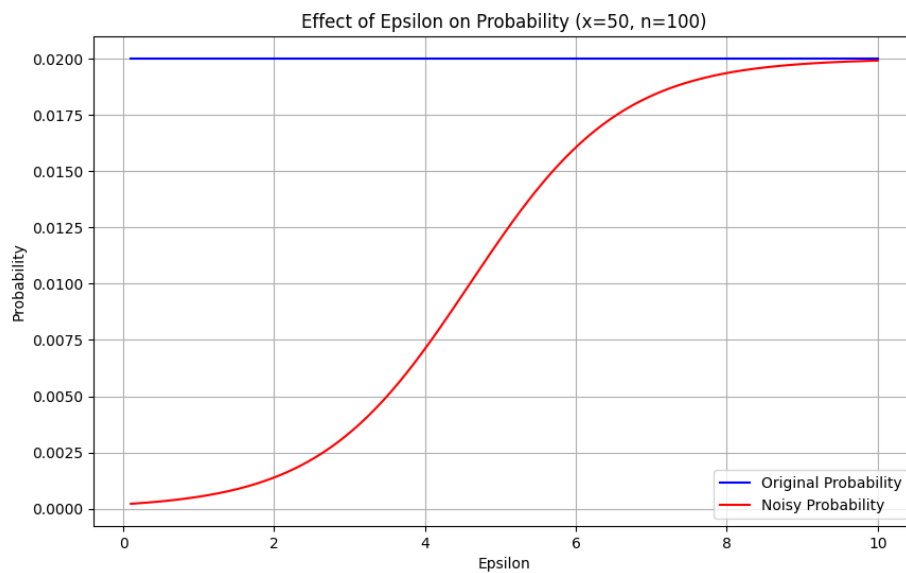
1.1.1.2 How n effect

Case 1: 5 of participants wants to communicate with the same person, and $\epsilon = 1$.

Effect of n on Probability (x=5, epsilon=1)

Case 2: 5 of participants wants to communicate with the same person, and $\epsilon = 5$.

Effect of n on Probability (x=5, epsilon=5)
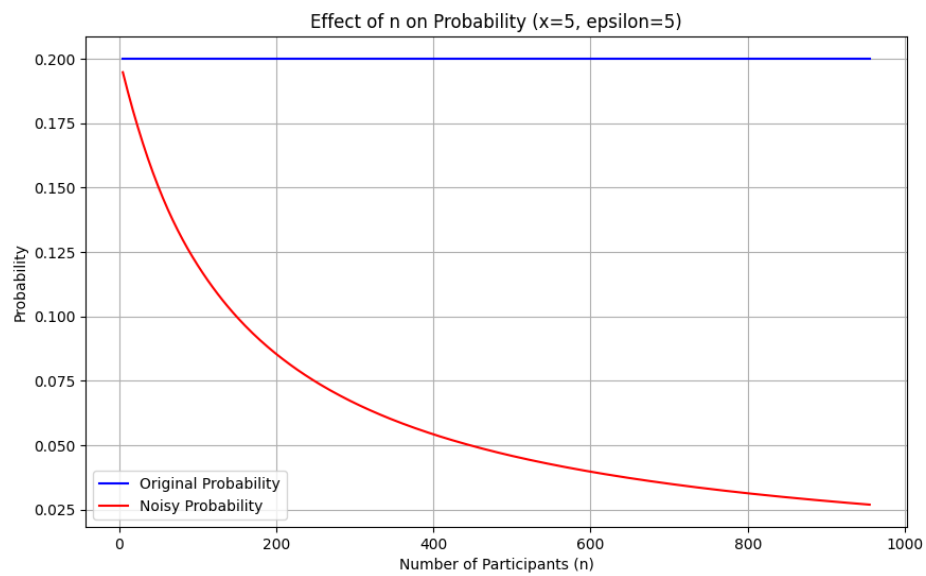
Case 3: 50 of participants wants to communicate with the same person, and $\epsilon = 5$.

Effect of n on Probability (x=50, epsilon=5)
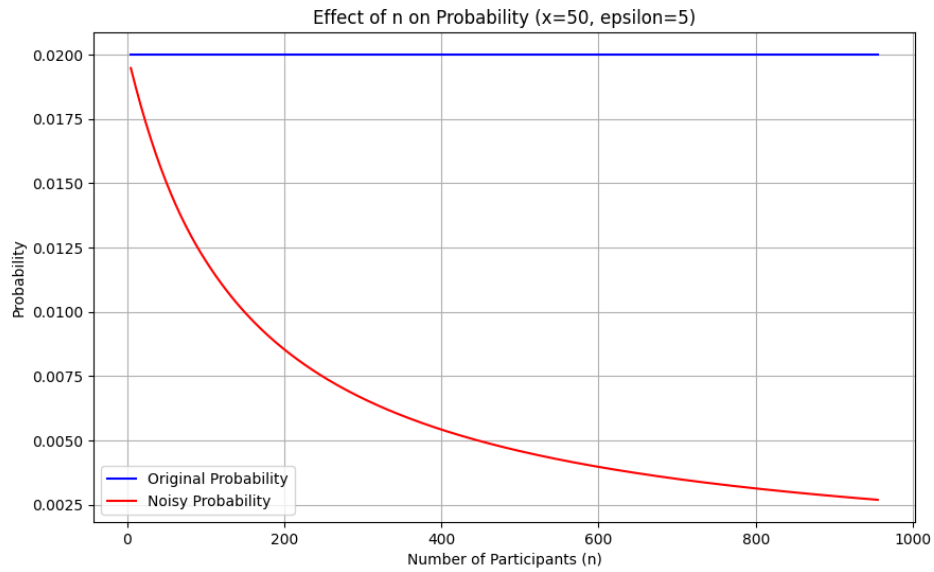
1.1.1.3 How x effect

Case 1: 100 participants in the protocol and $\epsilon = 1$

Effect of x on Probability (n=100, epsilon=1)

Case 2: 100 participants in the protocol and $\epsilon = 5$
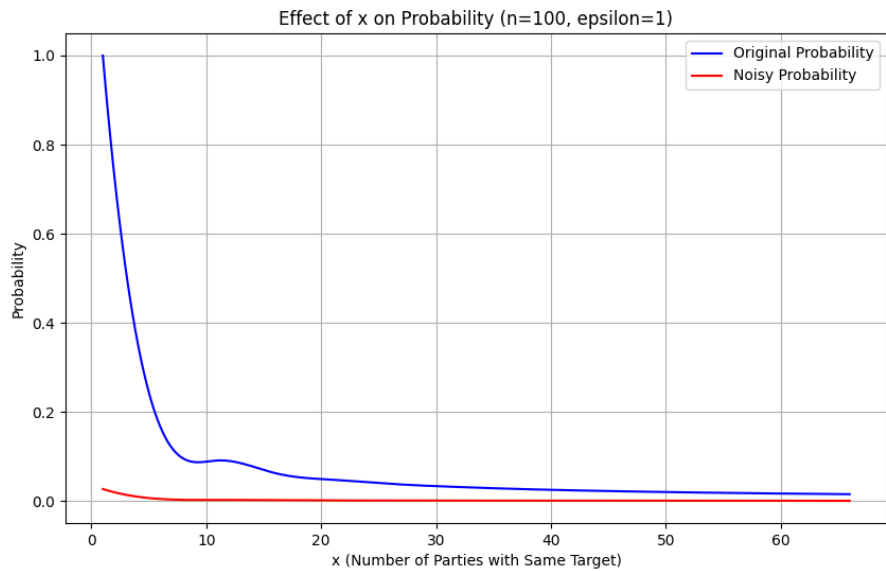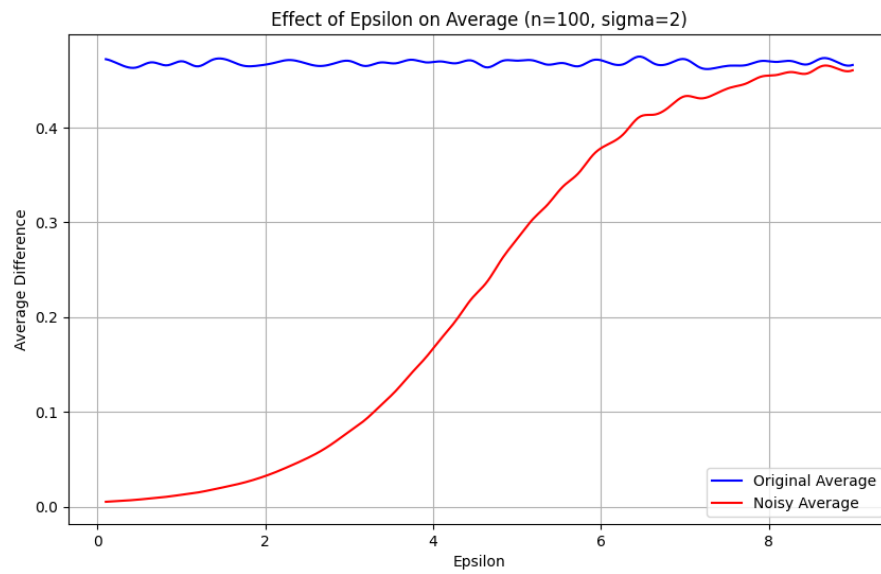
Effect of x on Probability (n=100, epsilon=5)
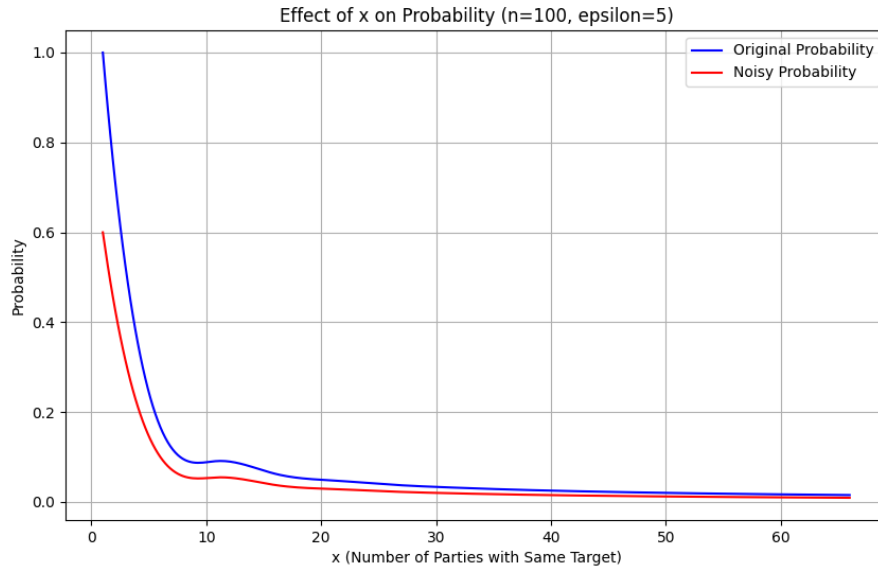
1.1.2 Average Difference
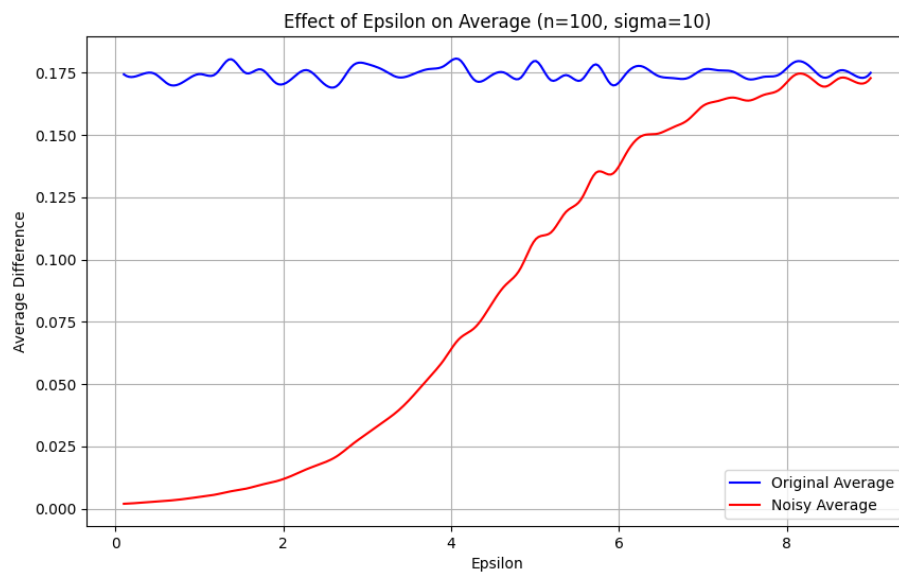
The average probability is influenced by several factors. It depends on the number of participants n in the protocol and the security parameter ϵ. Additionally, it is affected by how we generate the input, specifically in terms of the distribution of participants wanting to communicate with the same invitee. We use a Gaussian distribution to model the protocol's communication pattern (i.e., how many people want to communicate with P_1, how many want to communicate with P_2, and so on). The mean of the Gaussian distribution is set to 1, as each participant can only have one invitee. The parameter σ controls the variance in the distribution, which affects how concentrated the communication is. A higher σ means that more participants are likely to want to communicate with a smaller, more concentrated set of invitees.

1.1.2.1 How ϵ effect
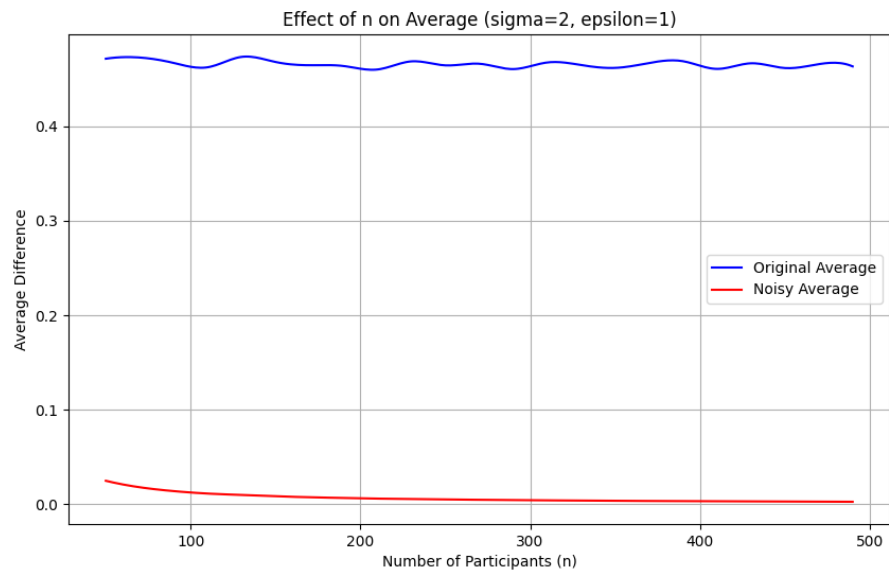
Case 3: n=100, σ=2

Effect of Epsilon on Average (n=100, sigma=2)

Case 2: n=100, σ = 10

Effect of Epsilon on Average (n=100, sigma=10)

1.1.2.2 How n effect

Case 1: $\sigma = 2, \epsilon = 1$

Effect of n on Average (sigma=2, epsilon=1)

Case 2: $\sigma = 2, \epsilon = 8$

Effect of n on Average (sigma=2, epsilon=8)

Case 3: $\sigma = 8, \epsilon = 8$

Effect of n on Average (sigma=8, epsilon=8)

1.1.2.3 How σ effect

Case 1: $n = 100, \epsilon = 1$

Effect of Sigma on Average (n=100, epsilon=1)
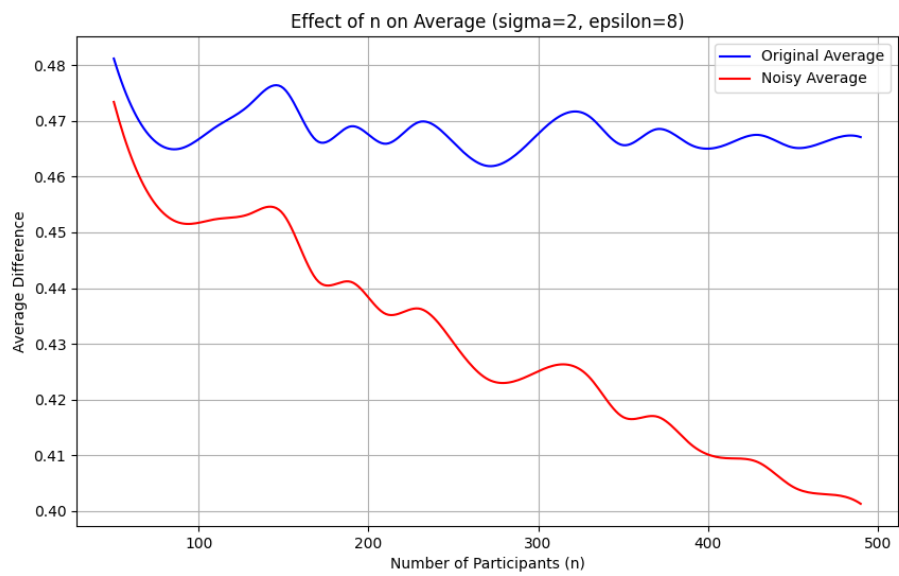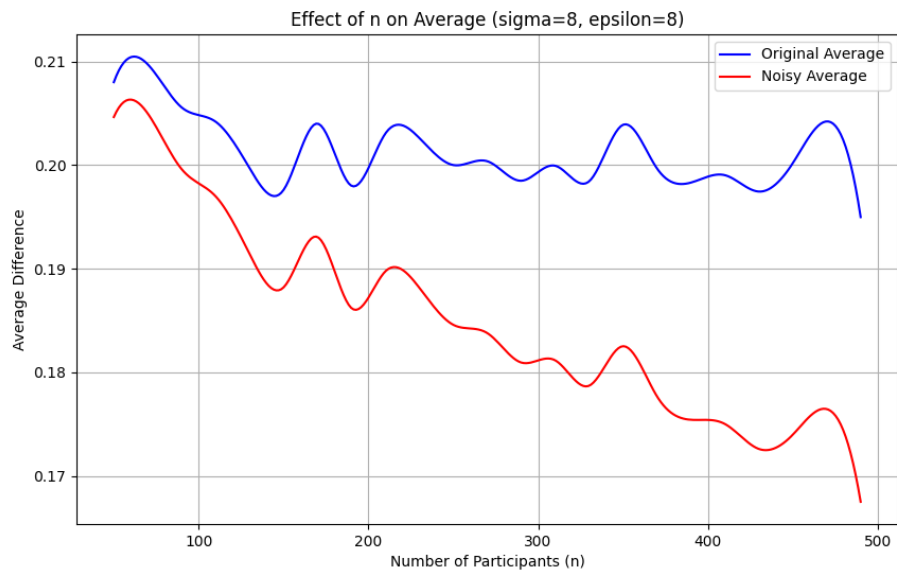
Case 2: $n = 100, \epsilon = 8$
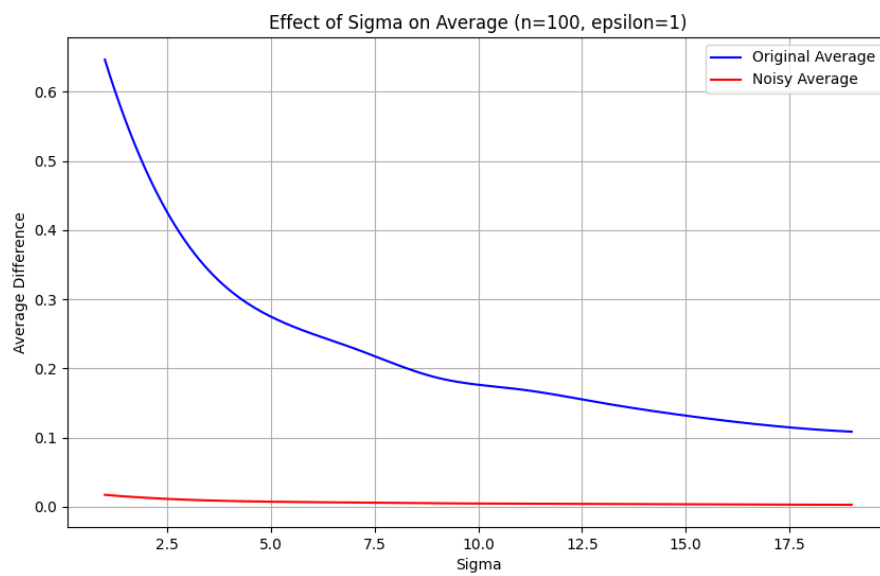
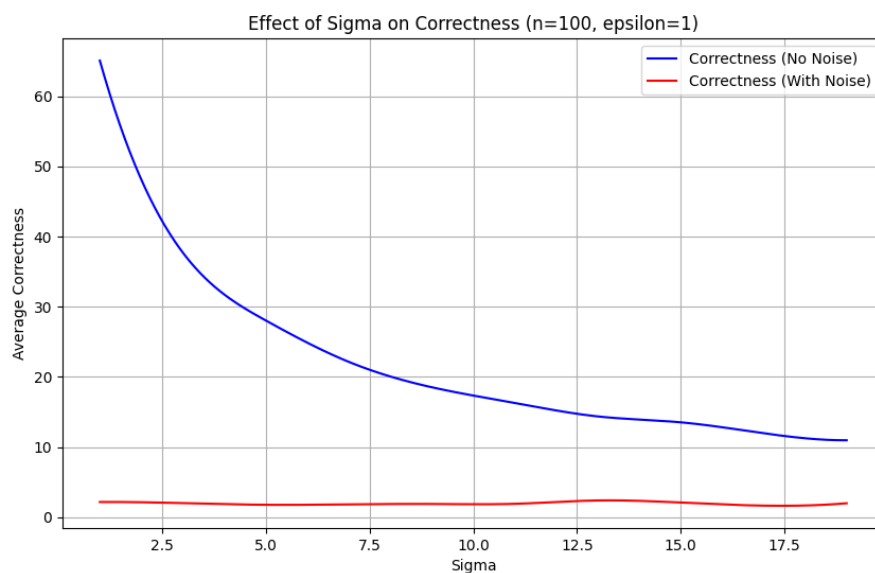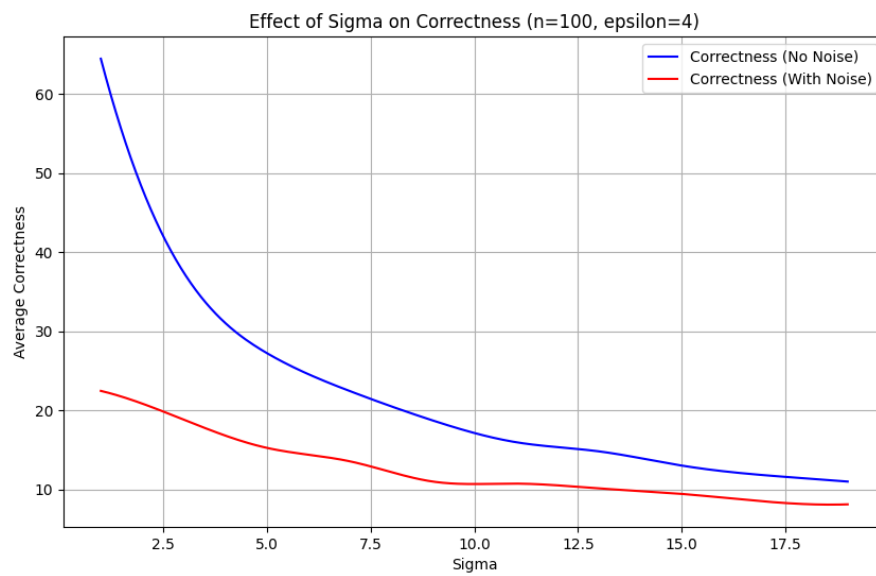Effect of Sigma on Average (n=100, epsilon=8)
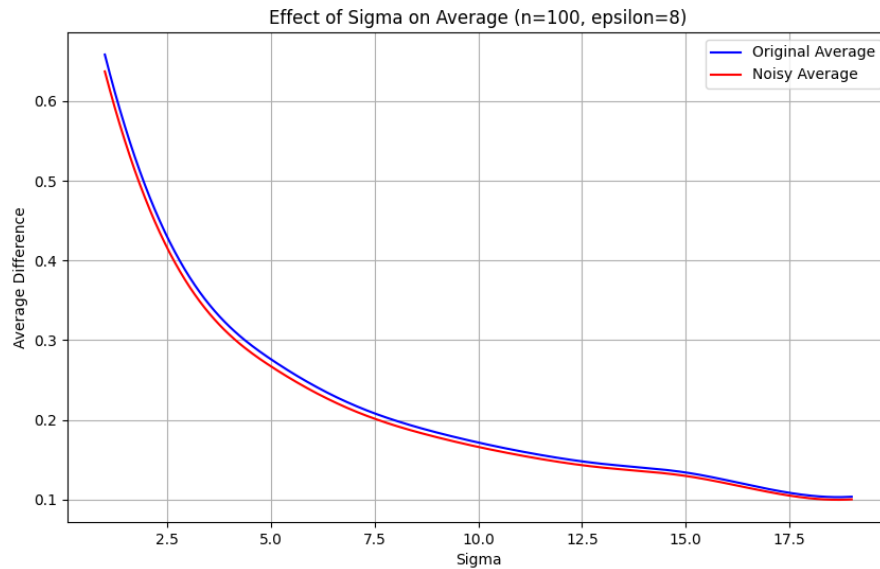
1.2 Correctness

Similar to the average difference, correctness is influenced by three factors: the number of participants n, the security parameter ϵ, and how we generate the input. We use a Gaussian distribution to model the communication pattern (i.e., how many people want to communicate with each invitee). The mean is set to 1, as each participant has only one invitee, and σ controls the variance. A higher σ means more participants tend to communicate with a smaller, concentrated group of invitees.

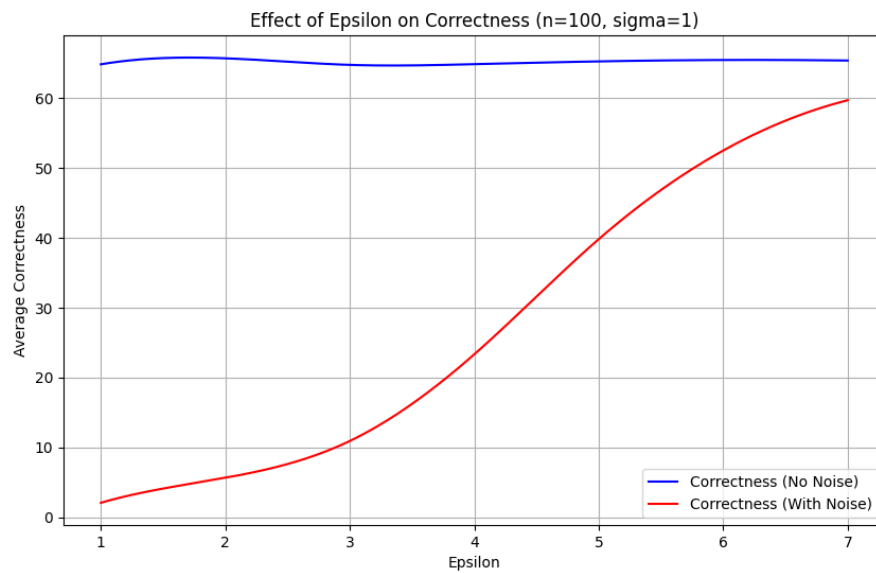## 1.2.1 How σ effect

Case 1: $n = 100, \epsilon = 1$

Effect of Sigma on Correctness (n=100, epsilon=1)

Case 1: $n = 100, \epsilon = 4$

Effect of Sigma on Correctness (n=100, epsilon=4)

1.2.2 How ϵ effect

Case 1: $n = 100, \sigma = 1$

Effect of Epsilon on Correctness (n=100, sigma=1)

Case 2: $n = 100, \sigma = 8$

Effect of Epsilon on Correctness (n=100, sigma=8)

1.2.3 How n effect

Case 1: $\sigma = 2, \epsilon = 1$

Effect of n on Correctness (sigma=2, epsilon=1)

Case 2: $\sigma = 2, \epsilon = 8$

Effect of n on Correctness (sigma=2, epsilon=8)

Case 3: $\sigma = 8, \epsilon = 6$

Effect of n on Correctness (sigma=8, epsilon=6)