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Incorporating Learnable Membrane Time Constant to Enhance Learning of Spiking Neural Networks

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Abstract

Spiking Neural Networks (SNNs) have attracted enormous research interest due to temporal information processing capability, low power consumption, and high biological plausibility. However, the formulation of efficient and high-performance learning algorithms for SNNs is still challenging. Most existing learning methods learn the synaptic-related parameters only, and require manual tuning of the membrane-related parameters that determine the dynamics of single spiking neuron. These parameters are typically chosen to be the same for all neurons, which limits the diversity of neurons and thus the expressiveness of the resulting SNNs. In this paper, we take inspiration from the observation that membrane-related parameters are different across brain regions, and propose a training algorithm that is capable of learning not only the synaptic weights but also the membrane time constants of SNNs. We show that incorporating learnable membrane time constants can make the network less sensitive to initial values and can speed up learning. In addition, we reevaluate the pooling methods in SNNs and find that max-pooling is able to increase the fitting capability of SNNs in temporal tasks, as well as reduce the computation cost. We evaluate the proposed method for image classification tasks on both traditional static MNIST, Fashion-MNIST, CIFAR-10 datasets, and neuromorphic N-MNIST, CIFAR10-DVS, DVS128 Gesture datasets. The experiment results show that the proposed method outperforms the state-of-the-art accuracy on nearly all datasets, using fewer time-steps.

1. Introduction

Spiking Neural Networks (SNNs) are viewed as the third generation of neural network models, which is closer to biological neurons in the brain [38]. Together with neuronal and synaptic states, the importance of spike timing is also considered in SNNs. Due to their distinctive properties, such as temporal information processing capability, low power consumption [50], and high biological plausibility [16], SNNs increasingly arouse researchers' great interest in recent years. Nevertheless, it remains challenging to formulate efficient and high-performance learning algorithms for SNNs.

Generally, the learning algorithms for SNNs can be divided into unsupervised learning, supervised learning, reward-based learning, and Artificial Neural Network (ANN) to SNN conversion methodologies. Either way, we find that most existing learning methods only consider to learn the synaptic-related parameters like synaptic weights and treat the membrane-related parameters as hyperparameters. These membrane-related parameters like membrane time constants, which determine the dynamics of single spiking neuron, are typically chosen to be the same for all neurons. Note, however, there exist different membrane time constants for spiking neurons across brain regions [39, 9], which are proved to be essential for representation of working memory and formulation of learning [20, 54]. Thus simply ignoring different time constants in SNNs will limit the heterogeneity of neurons and thus the expressiveness of the resulting SNNs.

In this paper, we propose a training algorithm that is capable of learning not only the synaptic weights but also membrane time constants of SNNs. As illustrated in Fig. 1, we find that adjustments of the synaptic weight and the membrane time constants have different effects on neuronal dynamics. We show that incorporating learnable membrane time constants is able to enhance the learning of SNNs.

The main contributions of this paper can be summarized as follows:

1) We propose the backpropagation-based learning algorithm using spiking neurons with learnable membrane
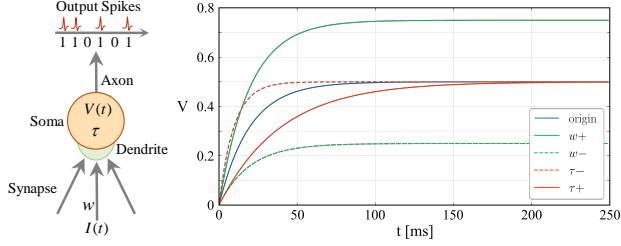
(a) Spiking neuron (b) The membrane potential of a LIF neuron

Figure 1. (a) A Leaky Integrat-and-Fire (LIF) neuron with membrane potential V, membrane time constant τ, input $I(t)$ and synaptic weight w. (b) The membrane potential V of the LIF neuron when constant input is received. Increasing or decreasing τ will stretch the $v = f(t)$ curve in the t direction while increasing or decreasing w will stretch the $v = f(t)$ curve in the V direction.

parameters, referred to as Parametric Leaky Integrate-and-Fire (PLIF) spiking neurons, which better represent the heterogeneity of neurons and thereby enhancing the expressiveness of the SNNs. We show that the SNNs consist of PLIF neurons are more robust to initial values and can learn faster than SNNs consist of neurons with a fixed time constant.

2) We reevaluate the pooling methods in SNNs and discredit the previous conclusion that the max-pooling results in significant information loss. We show that the max-pooling can increase the fitting capability of SNNs in temporal tasks and reduce the computation cost by introducing the winner-take-all mechanism in the spatial domain and time-variant topology in the temporal domain.

3) We evaluate our methods on both traditional static MNIST [31], Fashion-MNIST [60], CIFAR-10 [30] datasets widely used in ANNs as benchmarks, and neuromorphic N-MNIST [44], CIFAR10-DVS [35], DVS128 Gesture [1] datasets that are applied to verify the network's temporal information processing capability. The proposed method exceeds state-of-the-art accuracy on nearly all tested datasets, using fewer time-steps.

2. Related Works

Unsupervised learning of SNNs The unsupervised learning methods of SNNs are based on biological plausible local learning rules, like Hebbian learning [22] and Spike-Timing-Dependent Plasticity (STDP) [3]. Existing approaches exploited self-organization principle [57, 11, 28], and STDP-based expectation-maximization algorithm [43, 17]. However, these methods are only suitable for shallow SNNs, and the performance is far below state-of-the-art ANN results.

Reward-based learning of SNNs Reward-based learn-

ing of SNNs mimics the way the human brain learns by taking advantage of the reward or punishment signals induced by dopaminergic, serotonergic, cholinergic, or adrenergic neurons [13, 6, 41]. Despite the methods that arise in reinforcement learning, like policy gradient [53, 27], temporal-difference learning [47, 14] and Q-learning [6], some heuristic phenomenological models based on STDP [15, 63] were proposed recently.

ANN to SNN conversion ANN to SNN conversion (ANN2SNN) converts a trained non-spiking ANN to an SNN by using the firing rate of each spiking neuron to approximate the corresponding ReLU activation of an analog neuron [24, 7, 51]. It can get near loss-less inference results as an ANN [52], but there is a trade-off between accuracy and latency. To improve accuracy, longer inference latency is needed [19]. ANN2SNN is restricted to rate-coding, which lost the processing capability in temporal tasks. As far as we know, ANN2SNN only works for static datasets, not neuromorphic datasets.

Supervised learning of SNNs SpikeProp [5] was the first supervised learning method for SNNs based on backpropagation, which used a linear approximation to overcome the non-differentiable threshold-triggered firing mechanism of SNNs. Subsequent works included Tempotron[18], ReSuMe [46], and SPAN [40], but they could only be applied to single-layer SNNs. Recently, the surrogate gradient method was proposed and provided another solution to training multi-layer SNNs [34, 25, 65, 58, 55, 33, 26]. It utilized surrogate derivatives to define the derivative of the threshold-triggered firing mechanism. Thus the SNNs could be optimized with gradient descent algorithms as ANNs. Experiments have showed that SNNs optimized by the surrogate gradient methods show competitive performance with ANNs [42]. Compared to ANN2SNN, the surrogate gradient method has no restrictions on simulating time-steps because it is not based on rate-coding [59, 64].

Spiking neurons and layers of deep SNNs Spiking neuron and layer models play an essential role in SNNs. Wu et al. [59] proposed neuron normalization to balance each neuron's firing rate to avoid severe information loss. Cheng et al. [8] added the lateral interactions between neighboring neurons and get better accuracy and stronger noise-robustness. Zimmer et al. [66] firstly adopt the learnable time constants in LIF neurons for the speech recognition task. Bellec et al. [2] proposed the adaptive threshold spiking neuron to enhance computing and learning capabilities of SNNs, which was improved by [62] with learnable time constants. Rathi et al. [48] suggested to use a learnable membrane leak and firing threshold to finetune SNNs converted from ANNs. Despite this, no systematic research on the effects of learning membrane time constants to SNNs has been conducted so far, which is exactly the aim of this paper.

3. Method

In this section, we first briefly review the Leaky Integrate-and-Fire model in Sec. 3.1, and analyze the effect of synaptic weight and membrane time constant in Sec. 3.2. The Parametric Leaky Integrate-and-Fire model and the network structure of the SNNs are then introduced in Sec. 3.3 and Sec. 3.4. At last, we describe the spike max-pooling and the learning algorithm of SNNs in Sec. 3.5 and Sec. 3.6.

3.1. Leaky Integrate-and-Fire model

The basic computing unit of an SNN is the spiking neuron. Neuroscientists have built several spiking neuron models for describing the accurate relationships between input and output signals of the biological neuron. The Leaky Integrate-and-Fire (LIF) model [16] is one of the simplest spiking neuron models used in SNNs. The subthreshold dynamics of the LIF neuron is defined as:

$$\tau \frac{\mathrm{d}V(t)}{\mathrm{d}t} = -(V(t) - V_{reset}) + X(t), \qquad (1)$$

where $V(t)$ represents the membrane potential of the neuron at time t, $X(t)$ represents the input to neuron at time t, τ is the membrane time constant. When the membrane potential $V(t)$ exceeds a certain threshold V_{th} at time t^f, the neuron will elicit a spike and then the membrane potential $V(t)$ goes back to a reset value $V_{reset} < V_{th}$. The LIF neuron achieves a balance between computing cost and biological plausibility.

3.2. Function comparison of synaptic weight and membrane time constant

In most of the previous learning algorithms for SNNs, the membrane time constant τ is regarded as a hyperparameter and chosen to be the same for all neurons before learning. The learning of SNNs is only to optimize the synaptic weights. However, it cannot be ignored that the behavior of a spiking neuron for given inputs depends not only on the weights of connected synapses but also on the neuron's inherent dynamics controlled by the membrane time constant τ.

In order to compare the effects of synaptic weight and membrane time constant to the neuronal dynamics, we consider a simple case where the LIF neuron z_i receives weighted input $X(t) = wI(t)$ from a presynaptic neuron z_j (Fig. 1(a)). The reset potential V_{reset} is set to 0. When the input is constant, namely, $I(t) = I$, the membrane potential of the LIF neuron z_i changes over time is showed in Fig. 1(b) (blue curve), which is computed according to Eq. (1). Increasing or decreasing w, as showed by the $w+$ and $w-$ curves, will stretch the $v = f(t)$ curve in the V direction. On the contrary, increasing or decreasing τ will stretch the $v = f(t)$ curve in the t direction, and
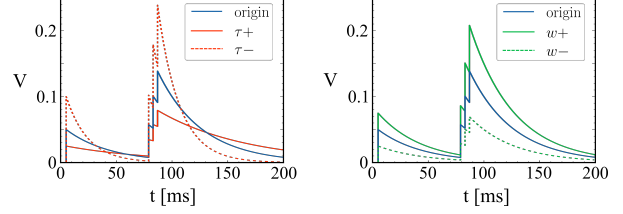

Figure 2. The membrane potential V of a LIF neuron when instant spikes at $t = 5, 80, 85, 90$ are received.

will not change the steady-state voltage of the neuron z_i as $V(+\infty) = wI(t)$. Fig. 2 illustrates the response of the neuron z_i to instant input spikes at time $t = \{5, 80, 85, 90\}$ms, namely, $X(t) = w(\delta(t-5)+\delta(t-80)+\delta(t-85)+\delta(t-90))$ [1]. The neuron's response to instant input spike at $t = 5$ indicates that a smaller τ (the $\tau-$ curve) leads to faster charge to the steady-state voltage and faster decay to the resting value, making the LIF neuron more sensitive to an instant spike. This sensitivity helps the neuron to capture instant variety in the input. In contrast, a smaller w (the $w-$ curve) leads to a slower charge to the steady-state voltage without affecting decaying speed. When there are three successive input spikes, the membrane potential of the neuron with a smaller τ (the $\tau-$ curve) will reach a higher value at a faster rate, which makes it easier to fire.

To some extent, the effect of decreasing τ is similar to that of increasing w. Nevertheless, adjusting both τ and w can bring some superior additional benefits. As mentioned above, changing both τ and w can stretch the $v = f(t)$ curve, namely the neuron's response to a given input, in both t direction and V direction, which endows the neuron better fitting ability.

3.3. Parametric Leaky Integrate-and-Fire model

We propose the Parametric Leaky Integrate-and-Fire (PLIF) spiking neuron model to learn both the synaptic weights and the membrane time constants of SNNs. The dynamics of the PLIF neuron can be described by Eq. (1), and is differentiated from the LIF neuron by that the membrane time constant τ is a learnable parameter.

The SNNs with PLIF neurons follow the three rules:

(1). The membrane time constant τ is optimized automatically during training, rather than being set as a hyperparameter manually before training.

(2). The membrane time constant τ is shared within the neurons in the same layer in SNNs, which is biologically plausible as the neighboring neurons have similar properties.

(3). The membrane time constant τ of neurons in different layers are distinct, making diverse phase-frequency responsiveness of neurons.

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[1] $\delta(t)$ represents Dirac delta function. If $x \neq 0$, then $\delta(t) = 0$. $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$.
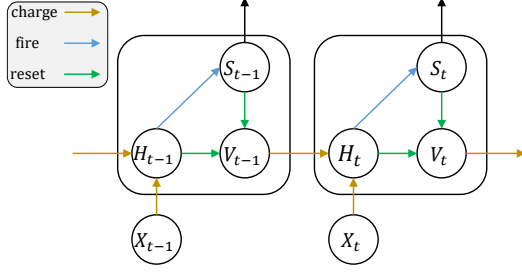
Figure 3. The general discrete spiking neuron model.

In fact, the proposed rules are able to increase the heterogeneity of neurons and the expressiveness of the resulting SNNs while effectively controlling computation costs.

For numerical simulations of PLIF neurons in SNNs, we need to consider a version of the parameters dynamics that is discrete in time. Specifically, by including the threshold-triggered firing mechanism and the reset of the membrane potential after firing, we can describe the dynamics of all kinds of spiking neurons with the following equations:

$$H_t = f(V_{t-1}, X_t), \tag{2}$$

$$S_t = \Theta(H_t - V_{th}), \tag{3}$$

$$V_t = H_t(1 - S_t) + V_{reset} S_t. \tag{4}$$

To avoid confusion, we use H_t and V_t to represent the membrane potential after neuronal dynamics and after the trigger of a spike at time-step t, respectively. X_t denotes the external input, and V_{th} denotes the firing threshold. S_t denotes the output spike at time t, which equals 1 if there is a spike and 0 otherwise. As showed in Fig. 3, Eqs. (2) - (4) build a general model to describe the discrete spiking neuron's action: charging, firing, and resetting. Specifically, Eq. (2) describes the neuronal dynamics, and different spiking neuron models have different functions $f(\cdot)$. For example, the function $f(\cdot)$ for the LIF neuron and PLIF neuron is

$$H_t = V_{t-1} + \frac{1}{\tau}(-(V_{t-1} - V_{reset}) + X_t). \tag{5}$$

Eq. (3) describes the spike generative process, where $\Theta(x)$ is the Heaviside step function and is defined by $\Theta(x) = 1$ for $x \geq 0$ and $\Theta(x) = 0$ for $x < 0$. Eq. (4) illustrates that the membrane potential returns to V_{reset} after eliciting a spike, which is called *hard reset* and widely used in deep SNNs [32].

For PLIF neurons, directly optimizing the membrane time constant τ in Eq. (5) may induce numerical instability as τ is in the denominator. Besides, Eq. (5), as the discrete version of Eq. (1), is true only when the time-step dt is smaller than τ, that is, $\tau > 1$. To avoid the above problems, we reformulate Eq. (5) to the following equation with a trainable parameter a:

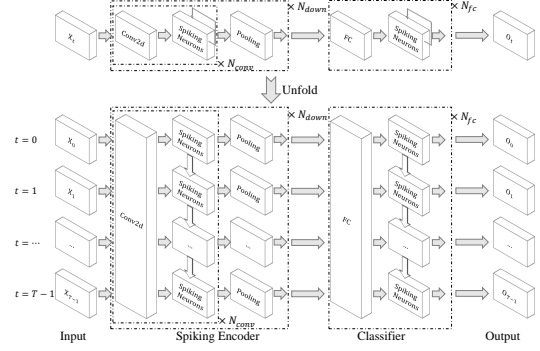$$H_t = V_{t-1} + k(a)(-(V_{t-1} - V_{reset}) + X_t). \tag{6}$$

Figure 4. The general formulation of our networks and its unfolded formulation. $\times N_{conv}$ indicates there are N_{conv} {*Conv2d-Spiking Neurons*} connected sequentially. $\times N_{down}$ and $\times N_{fc}$ have the same meaning. Note that the network's parameters are shared at all time-steps.

Here $k(a)$ denotes the clamp function and $k(a) \in (0, 1)$, which ensures that $\tau = \frac{1}{k(a)} \in (1, +\infty)$. In our experiments, $k(a)$ is the sigmoid activation function, that is, $k(a) = \frac{1}{1+\exp(-a)}$.

3.4. Network Formulation

Fig. 4 illustrates the general formulation of the SNNs used in this paper, which includes a spiking encoder network and a classifier network. The spiking encoder network consists of stacks of convolutional layers, spiking neuron layers, and pooling layers, which can extract features from inputs and convert them into the firing spikes at different time-steps. The classifier network consists of fully connected layers and spiking neuron layers. Similar to [59, 48], the input is directly fed to the network without being first converted to spikes, e.g., through a Poisson encoder. Note that synaptic connections, including convolutional layers and fully connected layers, are stateless, while the spiking neuron layers have self-connections in the temporal domain, as the unfolded network formulation showed in Fig. 4. All parameters are shared at all time-steps.

3.5. Spike Max-Pooling

The pooling layer is widely used to reduce the size of feature maps and to extract compact representation in convolutional ANNs, as well as SNNs. Most previous studies [52, 8, 49] preferred to use the average-pooling in SNNs as they found that max-pooling in SNNs leads to significant information loss. We argue that the max-pooling is consistent with the SNNs' temporal information processing ability and can increase SNNs' fitting capability in temporal tasks and reduce the computation cost for the next layer.

Specifically, the max-pooling layers are behind spiking neuron layers in our model (Fig. 4), and the max-pooling operation is carried on spikes. Different from all neurons transmit information to the next layer equally in the average-pooling window, only the neuron fires a spike in

4

(a) Spike max-pooling
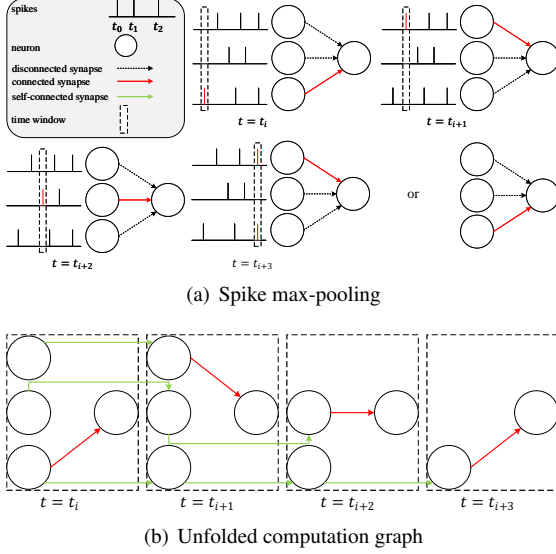

(b) Unfolded computation graph

Figure 5. Spike max-pooling regulates connections dynamically. (a) An example of three presynaptic neurons and one postsynaptic neuron with spike max-pooling. At every time-step, only the neuron that fires a spike can connect to the postsynaptic neuron. When more than one neuron fire at the same time-step, the neuron that can connect to the postsynaptic neuron is randomly selected. (b) The unfolded computation graph of (a).

the max-pooling window can transmit information to the next layer. Therefore, the max-pooling layer introduces the winner-take-all mechanism, allowing the most activated neuron to communicate with the next layer and ignoring other neurons in the pooling window. Another attractive property is that the max-pooling layer will regulate connections dynamically (Fig. 5). The spiking neuron's membrane potential V_t will return to V_{reset} after firing a spike. It is hard for a spiking neuron to fire again as recharging needs time. However, if the neurons in the max-pooling window fire asynchronously, they will be connected to the postsynaptic neuron in turn, which makes the postsynaptic neuron resembles to connect a continuously firing presynaptic neuron and easier to fire. The winner-take-all mechanism in the spatial domain and time-variant topology in the temporal domain achieved by max-pooling can increase SNNs' fitting capability in temporal tasks, such as classifying the CIFAR10-DVS dataset. It is worth noting that the outputs of the max-pooling layer are still binary, while the outputs of the average-pooling layer are float. The matrix multiplication and element-wise multiplication operation on spikes can be accelerated by replacing *multiplication* $*$ with *logical AND* $\&$, which is also the advantage of SNNs compared with ANNs.

3.6. Training Framework

Here we combine the neuron model (Fig. 3) and network formulation (Fig. 4) to drive the backpropagation training

algorithm for SNNs. Denote the simulating time-steps as T and classes number as C, the output $\boldsymbol{O} = [o_{t,i}]$ is a $C \times T$ tensor. For a given input with label l, we encourage the neuron that represents class l to have the highest excitatory level while other neurons should remain silent. So the target output is defined by $\boldsymbol{Y} = [y_{t,i}]$ with $y_{t,i} = 1$ for $i = l$ and $y_{t,i} = 0$ for $i \neq l$. The loss function is defined by the mean squared error (MSE) $L = MSE(\boldsymbol{O}, \boldsymbol{Y}) = \frac{1}{T}\sum_{t=0}^{T-1} L_t = \frac{1}{T}\sum_{t=0}^{T-1} \frac{1}{C}\sum_{i=0}^{C-1}(o_{t,i} - y_{t,i})^2$. And the predicted label l_p is regarded as the index of the neuron with the maximum firing rate $l_p = \arg\max_i \frac{1}{T}\sum_{t=0}^{T-1} o_{t,i}$.

Here we suppose that a^i represents the learnable parameter of the PLIF neurons in the i-th layer in the network. At time-step t, the vectors $\boldsymbol{H}_t^i$ and $\boldsymbol{V}_t^i$ represent the membrane potential after neuronal dynamics and after reset, the vector $\boldsymbol{V}_{th}^i$ and $\boldsymbol{V}_{reset}^i$ represents the threshold and reset potential, respectively. The weighted inputs from the previous layer are $\boldsymbol{X}_t^i = \boldsymbol{W}^{i-1}\boldsymbol{I}_t^i$. $\boldsymbol{S}_t^i = [s_{t,j}^i]$ denotes the output spike at time-step t, where $s_{t,j}^i = 1$ if the j-th neuron fires a spike, else $s_{t,j}^i = 0$. The gradients backward from the next layer are $\frac{\partial L_t}{\partial \boldsymbol{S}_t^i}$. We can calculate the gradients recursively:

$$\frac{\partial L}{\partial \boldsymbol{H}_t^i} = \frac{\partial L}{\partial \boldsymbol{H}_{t+1}^i}\frac{\partial \boldsymbol{H}_{t+1}^i}{\partial \boldsymbol{H}_t^i} + \frac{\partial L_t}{\partial \boldsymbol{H}_t^i} \tag{7}$$

$$\frac{\partial \boldsymbol{H}_{t+1}^i}{\partial \boldsymbol{H}_t^i} = \frac{\partial \boldsymbol{H}_{t+1}^i}{\partial \boldsymbol{V}_t^i}\frac{\partial \boldsymbol{V}_t^i}{\partial \boldsymbol{H}_t^i} \tag{8}$$

$$\frac{\partial L_t}{\partial \boldsymbol{H}_t^i} = \frac{\partial L_t}{\partial \boldsymbol{S}_t^i}\frac{\partial \boldsymbol{S}_t^i}{\partial \boldsymbol{H}_t^i} \tag{9}$$

According to Eq. (6), Eq. (3), and Eq. (4) we can get

$$\frac{\partial \boldsymbol{S}_t^i}{\partial \boldsymbol{H}_t^i} = \Theta'(\boldsymbol{H}_t^i - \boldsymbol{V}_{th}^i) \tag{10}$$

$$\frac{\partial \boldsymbol{H}_{t+1}^i}{\partial \boldsymbol{V}_t^i} = 1 - k(a^i) \tag{11}$$

$$\frac{\partial \boldsymbol{V}_t^i}{\partial \boldsymbol{H}_t^i} = 1 - \boldsymbol{S}_t + (\boldsymbol{V}_{reset}^i - \boldsymbol{H}_t^i)\frac{\partial \boldsymbol{S}_t^i}{\partial \boldsymbol{H}_t^i} \tag{12}$$

$$\frac{\partial \boldsymbol{H}_t^i}{\partial \boldsymbol{X}_t^i} = k(a^i) \tag{13}$$

$$\frac{\partial \boldsymbol{H}_t^i}{\partial a^i} = (-(\boldsymbol{V}_{t-1}^i - \boldsymbol{V}_{reset}^i) + \boldsymbol{X}_t^i)k'(a^i)$$
$$+ \frac{\partial \boldsymbol{H}_t^i}{\partial \boldsymbol{V}_{t-1}^i}\frac{\partial \boldsymbol{V}_{t-1}^i}{\partial \boldsymbol{H}_{t-1}^i}\frac{\partial \boldsymbol{H}_{t-1}^i}{\partial a^i} \tag{14}$$

Finally, we can get the gradients of the learnable param-

eters:

$$\frac{\partial L}{\partial a^i} = \sum_{t=0}^{T-1} \frac{\partial L}{\partial \boldsymbol{H}_t^i} \frac{\partial \boldsymbol{H}_t^i}{\partial a^i} \qquad (15)$$

$$\frac{\partial L}{\partial \boldsymbol{W}^{i-1}} = \sum_{t=0}^{T-1} \frac{\partial L}{\partial \boldsymbol{H}_t^i} \frac{\partial \boldsymbol{H}_t^i}{\partial \boldsymbol{X}_t^i} \boldsymbol{I}_t^i \qquad (16)$$

Note that $\frac{\partial *}{\partial \boldsymbol{S}_t^i} = 0$ when $t \geq T$, $\boldsymbol{V}_{-1}^i = \boldsymbol{V}_{reset}^i$, $\Theta'(x)$ is defined by the surrogate function $\Theta'(x) = \sigma'(x)$ and $k(x)$ is the clamp function.

4. Experiments

We evaluate the performance of SNNs with PLIF neurons and spike max-pooling for classification tasks on both traditional static MNIST, Fashion-MNIST, CIFAR-10 datasets, and neuromorphic N-MNIST, CIFAR10-DVS, and DVS128 Gesture datasets. More details of the training can be found in the supplementary.

4.1. Preprocessing

Static Datasets. We apply data normalization on all static datasets to ensure that input images have zero mean and unit variance. Besides, random horizontal flipping and cropping on MNIST and CIFAR-10 are conducted to avoid the problem of over-fitting. We do not use these augmentations on Fashion-MNIST because images in this dataset are tidy.

Neuromorphic Datasets. The data in neuromorphic datasets usually take the form of address event representation (AER) $E(x_i, y_i, t_i, p_i)$ $(i = 0, 1, ..., N-1)$ to represent the event location in the asynchronous stream, the timestamp, and the polarity. As the number of events is large, e.g. more than one million in CIFAR10-DVS, we split the events into T slices with nearly the same number of events in each slice and integrate events to frames. The new representation $F(j, p, x, y)$ ($0 \leq j \leq T-1$) is the summation of event data in the j-th slice:

$$F(j, p, x, y) = \sum_{i=j_l}^{j_r-1} \mathcal{I}_{p,x,y}(p_i, x_i, y_i), \qquad (17)$$

where $\mathcal{I}_{p,x,y}(p_i, x_i, y_i)$ is an indicator function and it equals 1 only when $(p, x, y) = (p_i, x_i, y_i)$. j_l and j_r are the minimal and the maximal timestamp indexes in the j-th slice. $j_l = \lfloor \frac{N}{T} \rfloor \cdot j$, $j_r = \lfloor \frac{N}{T} \rfloor \cdot (j+1)$ if $j < T-1$ and N if $j = T-1$. Here $\lfloor \cdot \rfloor$ is the floor operation. Note that T is also the number of time-step in our experiments.

4.2. Network Structure

The network structures of SNNs for different datasets are showed in Tab. 1. We set *kernel size* = 3, *stride* = 1 and *padding* = 1 for all convolutional layers. The *out channels*

Dataset	N_{conv}	N_{down}	N_{fc}
*MNIST	1	2	2
CIFAR-10	3	2	2
CIFAR10-DVS	1	4	2
DVS128 Gesture	1	5	2

Table 1. Network structures for different datasets. N_{conv}, N_{down} and N_{fc} are defined in Fig. 4. *MNIST* represents MNIST, Fashion-MNIST and N-MNIST datasets.

of convolutional layers is 256 for CIFAR-10 dataset and 128 for all other datasets. A batch normalization (BN) layer is added after each convolutional layer as [32] has showed that BN can make SNNs converge faster. All pooling layers are max-pooling with *kernel size* = 2 and *stride* = 2. A time-invariant dropout layer [33] is placed before each fully connected layer. The mask of time-invariant dropout is initiated at the first time-step and remaining constant during the whole duration, which can guarantee that some neurons are permanently silenced at every time-step. It is consistent with our intention to use the dropout. A voting layer after the output spiking neurons layer is used to boost classifying robustness. The voting layer is implemented by average-pooling with *kernel size* = M and *stride* = M, which means each class is represented by the output spikes of a population of M neurons. We set $M = 10$ for all datasets.

4.3. Performance

Quantitative evaluation. Tab. 2 shows the accuracies of the proposed and other comparing methods on both traditional static MNIST, Fashion-MNIST, CIFAR-10 datasets, and neuromorphic N-MNIST, CIFAR10-DVS, DVS128 Gesture datasets. We set the same training hyperparameters for all datasets (see supplementary). For a fair comparison, we report two types of accuracy. The accuracy-A is obtained by training on the training set, testing on the test set alternately, and recording the maximum test accuracy. The accuracy-B is obtained by splitting the origin training set into a new training set and validation set, training on the new training set, testing on the validation set alternately, and recording the test accuracy on the test set only once with the model that achieved the maximum validation accuracy. We are aware that method B is by far the best practice since method A overestimates generalization accuracy. Unfortunately, most of the previous works used method A (see Tab. 2). Thus we decided to include it to allow fair comparisons. We utilize 85% samples of each class in the origin training set as the new training set and set the rest 15% as the validation set. As showed in Tab. 2, whether with or without validation set, our methods achieve the highest accuracies on MNIST, Fashion-MNIST, N-MNIST, CIFAR10-DVS, and DVS128 Gesture. The accuracy on CIFAR-10 is slightly lower than [19], which is based on ANN2SNN conversion. However, they only ap-
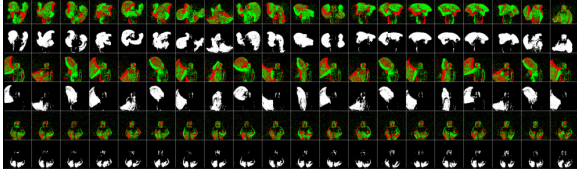
Figure 6. Visualization of the learnable spiking encoder. Three samples from the DVS128 Gesture dataset labeled *random other gestures*, *right hand clockwise*, *drums* are showed in row 1, 3, 5. The corresponding output spikes from channel 59 of the first PLIF neurons layer are showed in row 2, 4, 6.

plied to static images as ANNs are ill-suited to neuromorphic datasets. Different from them, our method is also applicable to neuromorphic datasets and outperforms the SOTA accuracy.

Tab. 3 compares the number of time-steps of our method and the previous works that achieve the best performance on each dataset. It can be found that the proposed method takes fewer time-steps than all the other methods. For example, our method uses up to $256\times$ fewer inference time-steps compared to ANN2SNN conversion [19]. Thus our method can not only decrease the memory consumption and the training time but also increase inference speed greatly.

Visualization of the spiking encoder. As illustrated in Sec. 3.4, the conventional layers with spiking neurons can be seen as a learnable spiking encoder. Here we present the visualization of the spiking encoder (Fig. 6). Three samples labeled *random other gestures*, *right hand clockwise*, *drums* at $t = 0, 1, ..., 19$ from the DVS128 Gesture dataset are showed in row 1, 3, 5 of Fig. 6. For comparison, the corresponding output spikes from channel 59 of the PLIF neurons in the first conventional layer are showed in row 2, 4, 6. One crucial difference is that the output almost only includes the gesture's response spikes, indicating that the spiking neurons implement efficient and accurate filtering on both spatial-variant and temporal-variant input data, reserving the gesture but discarding the player (see supplementary for more results).

Comparison of PLIF and LIF. We compare the test accuracy-A of PLIF neurons and LIF neurons on four challenging datasets to illustrate the benefits of PLIF neurons (Fig. 7). As can be seen, if the initial membrane time constant τ_0 of PLIF neurons is set equal to the membrane time constant τ of LIF neurons, the test accuracy of the SNNs with PLIF neurons is always higher than that with LIF neurons. This is due to the membrane time constants of PLIF neurons in different layers can be different after learning, which better represents the heterogeneity of neurons. Furthermore, the accuracy and convergence speed of the SNNs with LIF neurons decrease seriously if the initial value of the membrane time constant is not reasonable (red curve). In contrast, the PLIF neurons can learn the ap-

(a) Fashion-MNIST   (b) CIFAR-10

PLIF, $\tau_0 = 2$ PLIF, $\tau_0 = 16$ LIF, $\tau = 2$ LIF, $\tau = 16$
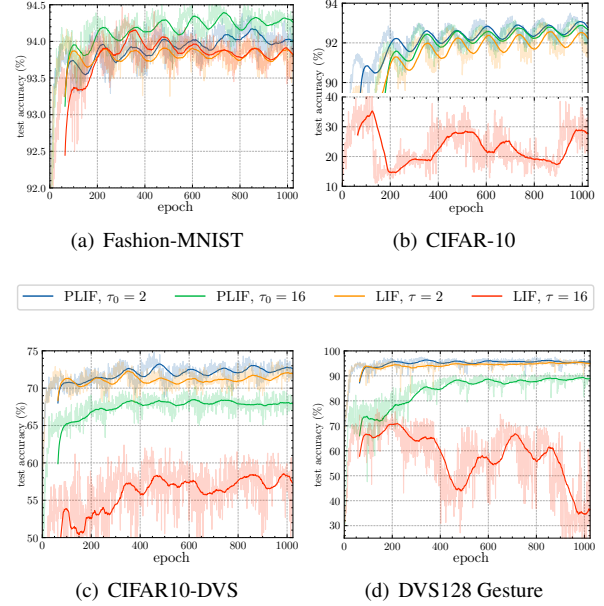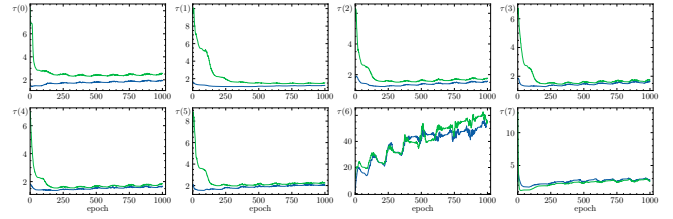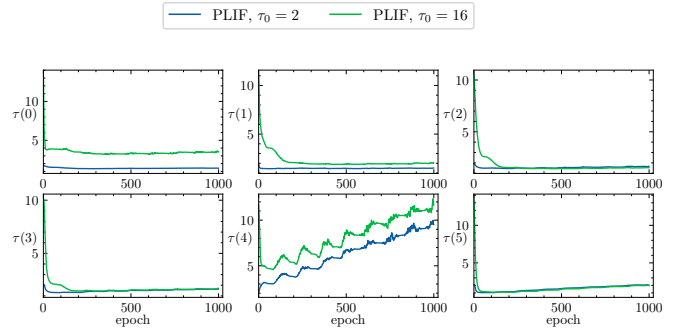
(c) CIFAR10-DVS   (d) DVS128 Gesture

Figure 7. The test accuracy-A of PLIF v.s. LIF neurons on different datasets during training. The shaded curves indicate the origin data. The solid curves are 64-epoch moving averages.

(a) The change of $\tau(i)$ during training on CIFAR-10.

PLIF, $\tau_0 = 2$ PLIF, $\tau_0 = 16$

(b) The change of $\tau(i)$ during training on CIFAR10-DVS.

Figure 8. The change of membrane time constants in different layers during training with different initial values. $\tau(i)$ represents the membrane time constant τ of the i-th PLIF neurons layer.

propriate membrane time constants and achieve better performance (green curve). To analyze the influence of initial values in PLIF neurons, we show how the membrane time constants of the neurons in each layer change during learning with respect to different initial values. As showed in

Model	Method	Accuracy MNIST	Accuracy Fashion-MNIST	Accuracy CIFAR-10	Accuracy N-MNIST	Accuracy CIFAR10-DVS	Accuracy DVS128 Gesture	Accuracy Type
[24]	ANN2SNN	98.37%	-	82.95%	-	-	-	Unknow
[51]	ANN2SNN	99.44%	-	88.82%	-	-	-	Unknow
[52]	ANN2SNN	-	-	91.55%	-	-	-	A
[19]	ANN2SNN	-	-	**93.63%**	-	-	-	Unknow
[34]	Spike-based BP	99.31%	-	-	98.74%	-	-	A
[58]	Spike-based BP	99.42%	-	-	98.78%	50.7%	-	Unknow
[55]	Spike-based BP	99.36%	-	-	99.2%	-	93.64%	Unknow
[26]	Spike-based BP	-	-	-	96%	-	**95.54%**	Sub-Dataset
[25]	Spike-based BP	99.49%	-	-	98.84%	-	-	Unknow
[65]	Spike-based BP	**99.62%**	90.13%	-	-	-	-	Unknow
[59]	Spike-based BP	-	-	90.53%	**99.53%**	**60.5%**	-	A
[33]	Spike-based BP	99.59%	-	90.95%	99.09%	-	-	A
[8]	Spike-based BP	99.5%	**92.07%**	-	99.45%	-	-	Unknow
[36]	Spike-based BP	-	-	-	96.3%	32.2%	-	Unknow
[61]	Spike-based BP	-	-	-	-	-	92.01%	A
[12]	Spike-based BP	99.46%	-	-	99.39%	-	96.09% (10 classes)	A
[21]	Spike-based BP	-	-	-	98.28%	-	93.40%	Unknow
[48]	ANN2SNN and Spike-based BP	-	-	92.64%	-	-	-	Unknow
[56]	HATS	-	-	-	99.1%	52.4%	-	Unknow
[4]	GCN	-	-	-	99.0%	54.0%	-	Unknow
Ours **Without Validation**	Spike-based BP	99.72%	94.38%	93.50%	99.61%	74.80%	97.57%	A
15% Validation	Spike-based BP	99.63%	93.85%	92.58%	99.57%	69.00%	96.53%	B

Table 2. Performance comparison between the proposed method and the state-of-the-art methods on different datasets.

Dataset	SOTA	SOTA's T	ours T
MNIST	[65]	400	8
Fashion-MNIST	[8]	20	8
CIFAR-10	[19]	2048	8
N-MNIST	[59]	59-64	10
CIFAR10-DVS	[59]	230-292	20
DVS128 Gesture	[26]	500(training) 1800(testing)	20

Table 3. The time-steps of previous SOTA works and ours on each dataset. The SOTA's T for N-MNIST and CIFAR10-DVS are calculated manually according to [59] that the time resolution is reduced by accumulating the spike train within every 5 ms.

Fig. 8, the membrane time constants with different initial values in each layer tend to gather during training, which indicates that the PLIF neurons are robust to initial values.

Tab. 4 summarizes the accuracy-A of SNNs with different neuron models and pooling layers on all six datasets. The performance of different methods on simple datasets, such as MNIST and N-MNIST, is very close and not notable. When the dataset is complicated, the accuracies of SNNs with PLIF neurons are superior to that with LIF neurons in general. The last row in Tab. 4 shows that the performance of max-pooling is similar to that of average-pooling, which proves that the previous suspect about using max-pooling in SNNs is not reasonable. It is remarkable that the max-pooling gets slightly higher accuracies on CIFAR-10, CIFAR10-DVS, and DVS128 Gesture datasets, showing its better fitting capability in complex tasks.

5. Conclusion

In this work, we proposed the Parametric Leaky Integrate-and-Fire (PLIF) neuron to incorporate the learnable membrane time parameter into SNNs. We show that the SNNs with the PLIF neurons outperform state-of-the-art comparing methods on both static and neuromorphic datasets. Besides, we prove that the SNNs consist of PLIF neurons are more robust to initial values and can learn faster than SNNs consist of LIF neurons. We also reevaluate the performance of max-pooling and average-pooling in SNNs and find the previous works underestimate the performance of max-pooling. We recommend using max-pooling in SNNs for its lower computation cost, higher temporal fitting capability, and the characteristic to receive spikes and output spikes rather than floating values as average-pooling.

A. Supplementary Materials

A.1. Reproducibility

All experiments are implemented by SpikingJelly[2], which is an open-source deep learning framework based on PyTorch [45] for SNNs. All of the source codes, training logs are available at https://www.researchgate.net/publication/346410859_Source_CodesIncorporating_Learnable_Membrane_Time_Constant_to_Enhance_Learning_of_Spiking_Neural_Networks. To

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[2]https://github.com/fangwei123456/spikingjelly

Neuron	Pooling layer	Accuracy MNIST	Accuracy Fashion-MNIST	Accuracy CIFAR-10	Accuracy N-MNIST	Accuracy CIFAR10-DVS	Accuracy DVS128 Gesture
PLIF($\tau_0 = 2$)	Max	99.72%	94.38%	**93.50%**	99.61%	**74.80%**	**97.57%**
PLIF($\tau_0 = 16$)	Max	**99.73%**	94.65%	93.23%	99.53%	70.50%	92.01%
LIF($\tau = 2$)	Max	99.69%	94.17%	93.03%	99.64%	73.60%	96.88%
LIF($\tau = 16$)	Max	99.49%	94.47%	47.50%	99.15%	62.40%	76.74%
PLIF($\tau_0 = 2$)	Average	99.71%	**94.74%**	93.30%	**99.66%**	72.70%	97.22%

Table 4. Accuracy-A of SNNs with different spiking neurons and pooling layers. The highest accuracy on each dataset is in bold.

maximize reproducibility, we use identical seeds in all codes.

A.2. RNN-like Expression of LIF and PLIF

The LIF and PLIF neurons have a similar function as recurrent neural networks. Specifically, when $V_{reset} = 0$, the neuronal dynamics of the LIF neuron and PLIF neuron (Eq. (5) in the main text) can be written as:

$$H_t = \left(1 - \frac{1}{\tau}\right) V_{t-1} + \frac{1}{\tau} X_t, \qquad (18)$$

where the integration progress $\frac{1}{\tau} X_t$ makes the LIF and PLIF neurons able to remember current input information, while the leakage progress $(1 - \frac{1}{\tau})V_{t-1}$ can be seen as forgetting some information from the past. Eq. (18) shows that the balance between remembrance and forgetting is controlled by the membrane time constant τ, which plays an analogous role as the gates in Long Short-Term Memory (LSTM) networks [23].

A.3. Introduction of the Datasets

MNIST The MNIST dataset of handwritten digits comprises 28×28 gray-scale images and labeled from 0 to 9. The MNIST dataset has 60,000 training images and 10,000 test images.

Fashion-MNIST Similar to the MNIST dataset, the Fashion-MNIST dataset consists of a training set of 60,000 examples and a test set of 10,000 examples. Each example in the Fashion-MNIST dataset is also a 28×28 gray-scale image with a label from 10 classes.

CIFAR-10 The CIFAR-10 dataset consists of 60,000 natural images in 10 classes, with 6,000 images per class. The number of the training images is 50,000, and that of the test images is 10,000.

N-MNIST The Neuromorphic-MNIST (N-MNIST) dataset is a spiking version of the MNIST dataset recorded by the neuromorphic sensor. It was converted from MNIST by mounting the ATIS sensor on a motorized pan-tilt unit and moving the sensor while recording MNIST examples on an LCD monitor. It consists of 60,000 training examples and 10,000 test examples.

Dataset	Network Structure
*MNIST	{c128k3s1-BN-PLIF-MPk2s2}*2-DP-FC2048-PLIF-DP-FC100-PLIF-APk10s10
CIFAR-10	{{c256k3s1-BN-PLIF}*3-MPk2s2}*2-DP-FC2048-PLIF-DP-FC100-PLIF-APk10s10
CIFAR10-DVS	{c128k3s1-BN-PLIF-MPk2s2}*4-DP-FC512-PLIF-DP-FC100-PLIF-APk10s10
DVS128 Gesture	{c128k3s1-BN-PLIF-MPk2s2}*5-DP-FC512-PLIF-DP-FC110-PLIF-APk10s10

Table 5. Detailed network structures for different datasets. *MNIST* represents MNIST, Fashion-MNIST, and N-MNIST datasets.

CIFAR10-DVS The CIFAR10-DVS dataset is the neuromorphic version of the CIFAR-10 dataset. It is composed of 10,000 examples in 10 classes, with 1000 examples in each class. As the CIFAR10-DVS dataset does not provide division for training and testing.

DVS128 Gesture The DVS128 Gesture dataset contains 11 kinds of hand gestures from 29 subjects under 3 illumination conditions recorded by a DVS128 camera.

A.4. Network Structure Details

Tab. 5 illustrates network structures details for different datasets. *c128k3s1* represents the convolutional layer with *output channels* = 128, *kernel size* = 3 and *stride* = 1. *BN* is the batch normalization. *MPk2s2* is the max-pooling layer with *kernel size* = 2 and *stride* = 2. *PLIF* is the PLIF spiking neurons layer. *DP* represents the time-invariant dropout proposed in [33]. *FC2048* represents the fully connected layer with *output features* = 2048. The symbol {}* indicates the repeated structure. For example, {*c128k3s1-BN-PLIF-MPk2s2*}*2 means that there are two {*c128k3s1-BN-PLIF-MPk2s2*} modules connected sequentially. The last layer *APk10s10* is the voting layer, which is implemented by an average-pooling layer with *kernel size* = 10 and *stride* = 10.

9

A.5. Training Algorithm to Fit Target Output

After defining the derivative of the spike generative process, the parameters of SNNs can be trained by gradient descent algorithms as that in ANNs. Classification, which is the task in this paper, as well as other tasks for both ANNs and SNNs, can be seen as optimizing parameters of the network to fit a target output when given a specific input. The direct gradient descent algorithm for SNNs to fit a target output is derived in the main text (Eq. 15 and Eq. 16), which can be described as follows:

Algorithm 1 Gradient Descent Algorithm for SNNs to Fit Target Output

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Require: learning rate ϵ, network's parameter $\boldsymbol{\theta}$, total simulating time-steps T, input $\boldsymbol{X} = \{\boldsymbol{X}_0, \boldsymbol{X}_1, ..., \boldsymbol{X}_{T-1}\}$, target output $\boldsymbol{Y} = \{\boldsymbol{Y}_0, \boldsymbol{Y}_1, ..., \boldsymbol{Y}_{T-1}\}$, loss function $L = \mathcal{L}(\boldsymbol{O}, \boldsymbol{Y})$
initialize $\boldsymbol{\theta}$
create an empty list $\boldsymbol{S} = \{\}$
for $t \leftarrow 0, 1, ...T - 1$
  input $\boldsymbol{X}_t$ to network, get output spikes $\boldsymbol{S}_t$
  append $\boldsymbol{S}_t$ to $\boldsymbol{S} = \{\boldsymbol{S}_0, \boldsymbol{S}_1, ..., \boldsymbol{S}_{t-1}\}$
calculate loss $L = \mathcal{L}(\boldsymbol{Y}, \boldsymbol{O})$
update parameter $\boldsymbol{\theta} = \boldsymbol{\theta} - \epsilon \cdot \nabla_{\boldsymbol{\theta}} L$

Here the loss function $L = \mathcal{L}(\boldsymbol{O}, \boldsymbol{Y})$ is a distance measurement between $\boldsymbol{Y}$ and $\boldsymbol{S}$, e.g., the mean squared error (MSE) in the main text.

A.6. Hyper-Parameters

We use the Adam [29] optimizer with the learning rate 0.001 and the cosine annealing learning rate schedule [37] with $T_{schedule} = 64$. The *batch size* is set to 16 to reduce memory consumption. The drop probability p for dropout layers is 0.5. The clamp function for PLIF neurons is $k(a) = \frac{1}{1+e^{-a}}$ and the surrogate gradient function is $\sigma(x) = \frac{1}{\pi} \arctan(\pi x) + \frac{1}{2}$, thus $\sigma'(x) = \frac{1}{1+(\pi x)^2}$. We set $V_{reset} = 0$ and $V_{th} = 1$ for all neurons. We notice that some previous works, e.g., [58], [59], fine-tuned V_{th} for different tasks, which is unnecessary. To be specific, as $\Theta(V - V_{th}) = \Theta(V_{th}(\frac{V}{V_{th}} - 1)) = \Theta(\frac{V}{V_{th}} - 1)$ and V is directed influenced by trainable weights, setting $V_{th} = 1$ implements an implicit normalization for weights, which can mitigate the exploding and vanishing gradient problem. As discovered by Zenke and Vogels [64], ignoring the neuronal reset when computing gradients by detaching them from the computational graph can improve performance, we also detach S_t in the neuronal reset.

A.7. Visualization of Spiking Encoder

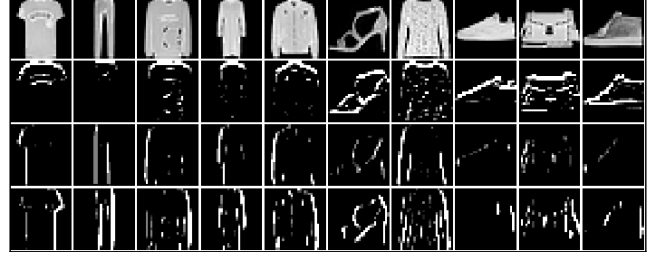As mentioned in the main text, the convolutional layers with spiking neurons can be seen as a spiking encoder that extracts features from the analog input and converts them to spikes. Different from the Poisson encoder that is widely used in SNNs [11, 33, 55], the spiking encoder is a learnable encoder. To evaluate the encoder, we give inputs $\boldsymbol{x}_t$ to the trained network and show the output spikes $\boldsymbol{S}_t^n(c)$ and the firing rates $\boldsymbol{F}_{T_s}^n(c) = \frac{1}{T_s} \sum_{t=0}^{T_s-1} \boldsymbol{S}_t^n(c)$ from channel c in the n-th layer, which is similar to [10]. Although the output spikes from deeper spiking neurons layers contain more semantic features, they are harder to read and understand. Thus we only show the spikes from the first spiking neurons layer, that is, $n = 2$.

Fig. 9 illustrates 10 input images from static Fashion-MNIST dataset (row 1) and the firing rates $\boldsymbol{F}_{T_s=8}^2$ of three typical channel (45, 75 and 76) of the first PLIF neurons layer (row 2, 3 and 4). One can find that the firing rates from channel 45, 75 and 76 detect upper, left, right edges of the input images. Fig. 10(a) shows a 2-D grid flatten across channels from the 3-D tensor $\boldsymbol{S}_{t=0}^2(c = 0, 1, ..., 127)$ when given an input sample labeled *horse*, which illustrates the features extracted by the spiking encoder at $t = 0$. As the CIFAR10-DVS dataset is converted from the static CIFAR-10 dataset, the firing rates accumulated from spikes can reconstruct the images filtered by the convolutional layer. Fig. 10(b) illustrates the firing rates $\boldsymbol{F}_{T_s=19}^2$ of all 128 channels ($c = 0, 1, ..., 127$), which have clearer texture than binary output spikes in Fig. 10(a). Fig. 11(a) shows the input $\boldsymbol{x}_t$ (row 1) and the corresponding output spikes $\boldsymbol{S}_t^2$ of channel 40 and 103 (row 2 and 3) at $t = 0, 1, ..., 19$, and Fig. 11(b) shows the mean input $\boldsymbol{x}(T_s) = \frac{1}{T_s} \sum_{t=0}^{T_s-1} \boldsymbol{x}_t$ (row 1) and the corresponding firing rates $\boldsymbol{F}_{T_s}^2$ of channel 40 and 103 (row 2 and 3) at $T_s = 0, 1, ..., 19$. One can find that as T_s increases, the texture constructed by firing rates $\boldsymbol{F}_{T_s}^2$ becomes more distinct, which which is similar to the use of the Poisson encoder.

Figure 9. Ten samples from the Fashion-MNIST dataset and the corresponding firing rates $\boldsymbol{F}_{T_s=8}^2$ from channel 45,75 and 76 ($c = 45, 75, 76$) of the first PLIF neurons layer are shown in row 1-4. Each column represents a sample and corresponding firing rates.

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(a) $\boldsymbol{S}^2_{t=0}(c = 0, 1, ..., 127)$
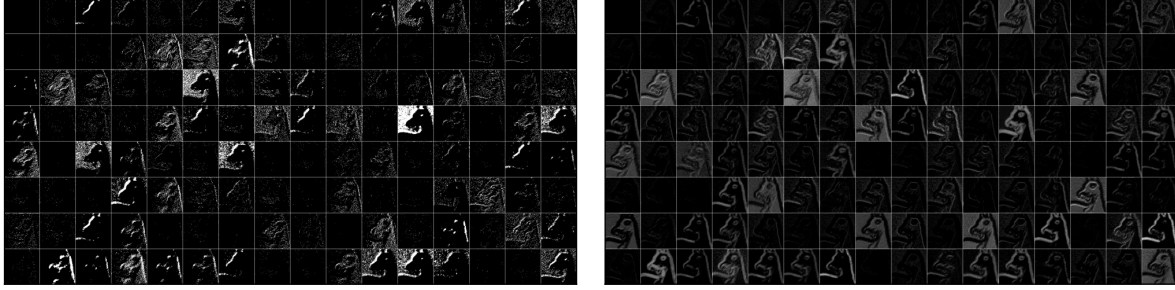
(b) $\boldsymbol{F}^2_{T_s=19}(c = 0, 1, ..., 127)$

Figure 10. Given a sample labeled *horse* from CIFAR10-DVS, (a) shows spikes from all 128 channels of the first spiking neurons layer at $t = 0$, and (b) shows firing rates of these neurons at $T_s = 19$.
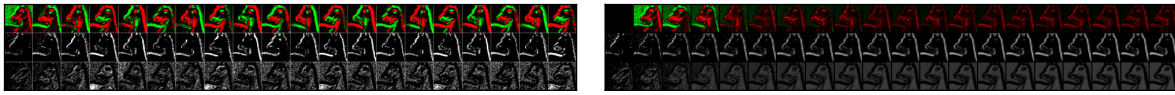
(a) $\boldsymbol{x}_t$ and $\boldsymbol{S}^2_t(c = 40, 103)$ at $t = 0, 1, ..., 19$

(b) $\boldsymbol{x}(T_s)$ and $\boldsymbol{F}^2_{T_s}(c = 40, 103)$ at $T_s = 0, 1, ..., 19$

Figure 11. Given the sample sample as Fig. 10, the input data and output spikes of channel 40 and 103 at each time-step are showed in (a) at row 1, 2, 3, respectively. The mean input data and firing rates of channel 40 and 103 at each time-step are showed in (b).

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